



Dedicated to the 85th anniversary of Professor P.P. Firstov

Modeling of Particle Electrification in a Fractal Cloud Environment Taking into Account Charge Leakage Due to Conductivity

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Abstract. This study develops a mathematical model of cloud particle electrification that integrates fractal cloud geometry and charge leakage through atmospheric conductivity. Using fractional calculus with the Gerasimov–Caputo operator of variable order α , related to the fractal dimension by $\alpha = 6 - 2D_f$, the model captures multiscale memory effects missing in classical approaches. Analytical solutions for hailstones and shed droplets are obtained via Mittag-Leffler functions. Numerical simulations over a 50-second interval for $\alpha = 0.1, 0.3, 0.5, 0.7, 0.9$ reveal strong sensitivity of charge dynamics to the fractality parameter: for $\alpha < 0.3$ charging is smooth and monotonic, whereas for $\alpha > 0.7$ it rapidly approaches saturation; the most probable scenario with $\alpha \approx 0.5$ yields accumulation times of 20–40 s, consistent with field observations. By determining fractal dimension parameters and leakage coefficients, the model predicts thunderstorm probability and intensity, improves lightning forecasting, supports critical infrastructure protection, and facilitates the inclusion of atmospheric electricity in climate models under increasing extreme weather conditions.

Key words: mathematical model, fractional equation, Gerasimov–Caputo derivative, thunderstorm cloud, fractal medium, charge generation.

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Моделирование электризации частиц в фрактальном облаке с учётом утечки заряда за счет проводимости

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Аннотация. В работе предложена математическая модель электризации облачных частиц, учитывающая фрактальную геометрию грозового облака и утечку заряда за счёт атмосферной проводимости. На основе дробного исчисления с оператором Герасимова — Капуто переменного порядка α , связанного с фрактальной размерностью среды соотношением $\alpha = 6 - 2D_f$, модель описывает многомасштабные эффекты памяти, отсутствующие в классических подходах. Для градин и отделяющихся капель получены аналитические решения через функции Миттаг — Леффлера; численное моделирование на интервале 50 с при значениях $\alpha = 0.1, 0.3, 0.5, 0.7, 0.9$ выявило высокую чувствительность динамики заряда к параметру фрактальности. Показано, что при $\alpha < 0.3$ зарядка носит плавный монотонный характер, а при $\alpha > 0.7$ быстро выходит на насыщение; наиболее вероятному сценарию соответствует $\alpha \approx 0.5$, при котором времена накопления заряда согласуются с натурными данными (20–40 с). Определение фрактальной размерности и коэффициентов утечки позволяет прогнозировать вероятность и интенсивность гроз, улучшать предупреждение молниевой активности, защиту критически важной инфраструктуры и учёт атмосферного электричества в климатических моделях в условиях роста экстремальных погодных явлений.

Ключевые слова: математическая модель, дробное уравнение, производная Герасимова — Капуто, грозовое облако, фрактальная среда, генерация заряда.

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Introduction

Cloud electrification is a complex physical process underlying the formation of atmospheric electricity, including lightning – one of the most hazardous natural phenomena. Understanding the mechanisms of charge separation and accumulation in clouds is of key importance for weather forecasting, ensuring aviation and infrastructure safety, and analyzing climate change [1–6]. However, accurate modeling of these processes remains challenging due to the complex cloud structure and multiple factors that influence charge dynamics and are overlooked in classical models of atmospheric processes. Therefore, the relevance of this work stems from the need to overcome the limitations of classical models, which fail to account for the fractal nature of the cloud environment. Contemporary challenges, such as the rising frequency of extreme weather events and the advancement of high-tech industries, necessitate the development of more precise mathematical models that incorporate the multiscale fractal nature of clouds and memory-dependent processes.

A crucial aspect frequently overlooked in classical models is the cloud environment's fractal nature. Clouds exhibit a fractal dimension of $D_f = 2.3\text{--}2.7$, associated with turbulence and vapor condensation [7]. Clouds, specifically cumulonimbus formations, manifest a self-similar structural topology across a wide range of spatial scales, consistent with fractal properties. Such topology influences the dynamics and interaction of intra-cloud particles, thereby altering collision probabilities, charge exchange, and electric field distribution. Accounting for fractal properties enables a more precise description for environmental chaotic behavior, particle concentration inhomogeneity, and their trajectories, which is critically important for understanding charge generation mechanisms.

Furthermore, a prevailing deficiency in many current models is the omission of charge leakage induced by ambient conductivity. The fractal topology amplifies localized fields, consistent with empirical observations of cumulonimbus electrification [8,9]. Conversely, elevated conductivity (e.g., due to the presence of ions) counteracts this effect, inhibiting electrical breakdown [10]. Atmospheric conductivity induced by ions and free electrons, as well as the surface conductivity of hydrometeors, facilitates charge dissipation over time. This process can substantially influence the kinetics of electrification, decelerating charge accumulation or inhibiting the attainment of thresholds necessary for discharge initiation. The incorporation of conductivity effects into the model brings theoretical predictions closer to actual atmospheric conditions, wherein charge leakage plays a vital role.

The objective of this study is the development of a cloud particle electrification model incorporating both environmental fractality and conductivity-driven charge leakage.

The remainder of the paper is organized as follows. Section 2 formulates the mathematical model of charge dynamics for hailstones and shed droplets in a fractal cloud medium, introduces the Gerasimov–Caputo fractional derivative with variable order, and derives analytical solutions in terms of the Mittag-Leffler function. Section 3 describes the numerical implementation, presents simulation results for different values

of the fractality parameter α , and discusses their physical implications in comparison with observational data. Finally, Section 4 summarises the main findings and outlines directions for future research, including a more detailed treatment of charge sign separation and quantitative validation against field measurements.

Problem Formulation and Solution Method

A sphere of radius R is considered, assuming that it is filled with hailstones of uniform radius r_i at a constant concentration n . Based on [11], the charge of the sphere over a given time interval, incorporating charge leakage due to conductivity $\frac{dQ_{i\lambda}}{dt}$, is determined as follows:

$$\frac{dQ_i}{dt} = \frac{4}{3}\pi R^3 n A r_i^2 v_i E - \omega \frac{r_d}{r_i} Q_i - \frac{dQ_{i\lambda}}{dt}, \quad (1)$$

where r_i is the hailstone radius; r_d is the droplet radius; v_i, v_d is the terminal fall velocity of the hailstone and droplet; A is a constant [12]; R is the cloud sphere radius; $t \in [0, T]$ is the shedding frequency of droplets from the hailstone; $t \in [0, T]$ is time; $T > 0$ is the simulation time; and n is the hailstone concentration (*const*).

Considering that the leakage current from hailstones is defined as follows:

$$\frac{dQ_{i\lambda}}{dt} = \pi \lambda_f Q_i,$$

where λ_f denotes the effective conductivity, we rewrite Equation (1) in the following form:

$$\frac{dQ_i}{dt} = K_{i1} - K_{i2} Q_i, \quad (2)$$

where $K_{i1} = \frac{4}{3}\pi R^3 n A r_i^2 v_i E$; $K_{i2} = \omega \frac{r_d}{r_i} + \pi \lambda_f$.

Since a thundercloud represents a self-affine fractal, its characteristics undergo temporal variations [13]. Therefore, the rate of change of the sphere's charge over time dt , incorporating the fractal properties of the cloud medium and charge leakage due to conductivity, is assumed to obey the following law:

$$\partial_{0t}^\alpha Q_i(t) + g_i Q_i(t) = f_i, \quad 0 < \alpha < 1, \quad (3)$$

where $g_i = K_{i2}\theta^{\alpha-1}$, $f_i = K_{i1}\theta^{\alpha-1}$. The fractional derivative operator is defined in the Gerasimov-Caputo sense [14, 15]:

$$\partial_{0t}^\alpha Q_i(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\dot{Q}_i(\tau) d\tau}{(t-\tau)^\alpha}, \quad 0 < \alpha < 1, \quad (4)$$

where $\Gamma(\cdot)$ is Euler's gamma function.

To establish a direct physical link between the fractional order α and the cloud's fractal geometry, we use the empirically derived relation

$$\alpha = 6 - 2 D_f,$$

where D_f is the time-dependent fractal dimension of the cloud medium. For typical cumulonimbus clouds, D_f ranges from 2.3 to 2.7 [7, 13], which yields α between 0.6 and 1.4. After normalisation to the interval (0,1) for the fractional derivative, this relation ensures that the memory effects correctly reflect the evolving self-similarity of the turbulent cloud structure. The specific form $\alpha = 6 - 2D_f$ is motivated by our analysis of experimental data and is consistent with known fractal models of atmospheric turbulence.

The fractional derivative of variable order (4) in model equation (3) emerges formally by analogy with [16], wherein the integer derivative is replaced by the following relationship:

$$\frac{d}{dt} \rightarrow \frac{1}{\theta^{1-\alpha}} \partial_{0t}^{\alpha},$$

where θ – is a parameter possessing the dimension of time and characterizing a specific intrinsic process time within the fractal structure of the thundercloud. We can also say that θ characterises the typical lifetime of large-scale eddies in the cloud; it can be associated with the correlation time of the turbulent velocity field, which for convective clouds lies in the range of 10–100 s. In our numerical simulations we set $\theta = 1$ s for simplicity, but the model allows for a direct estimate from in-situ measurements of turbulence spectra.

It should be noted that at $\alpha = 1$, model equation (3) reduces to equation (1) of the classical hailstone charge dynamics model, which accounts for charge dissipation via conductivity, and the following initial condition holds for it:

$$Q_i(0) = Q_{i0}. \quad (5)$$

This equation is analyzed in [17], wherein a general solution for problem (3)–(5) under the conditions $g_i = -g_i = \text{const}$ and $\alpha(t) = \text{const}$ is derived in the following form:

$$Q_i(t) = \sum_{k=1}^n Q_k |t - a|^{n-k} E_{1/\alpha(t)} \left(-g_i(t) |t - a|^{\alpha(t)}; n - k + 1 \right) + \\ + \text{sign}(a - t) \int_a^t f_i(\xi) |t - a|^{\alpha(t)-1} E_{1/\alpha(t)} \left(-g_i(t) |t - \xi|^{\alpha(t)}; \alpha(t) \right) d\xi.$$

In our case, using a formula of the form:

$$\partial_{0t}^{\alpha} Q_i(t) = D_{0t}^{\alpha} Q_i(t) - \sum_{k=1}^n \frac{t^{k-\alpha-1}}{\Gamma(k-\alpha)} Q_i^{(k-1)}(0). \quad (6)$$

The general form of Equation (3) is rewritten as follows:

$$D_{0t}^{\alpha} Q_i(t) - \sum_{k=1}^n \frac{t^{k-\alpha-1}}{\Gamma(1-\alpha)} Q_i^{(k-1)}(0) + g_i Q_i(t) = f_i. \quad (7)$$

From (7), we find:

$$Q_i(t) = F * t^{\alpha-1} E_{\alpha, \alpha}(-g_i t^{\alpha}) + \sum_{k=1}^n a_k t^{\alpha-k} E_{\alpha, \alpha-k+1}(-g_i t^{\alpha}) =$$

$$= f_i * t^{\alpha-1} E_{\alpha, \alpha}(-g_i t^\alpha) + \sum_{k=1}^n Q^{(k-1)}(0) \frac{t^{k-\alpha-1}}{\Gamma(k-\alpha)} * t^{\alpha-1} E_{\alpha, \alpha}(-g_i t^\alpha), \quad (8)$$

where $E_{\alpha, \alpha}(-g_i t^\alpha) = \sum_{k=1}^n \frac{(-g_i t^\alpha)^k}{\Gamma(\alpha k + \alpha)}$ represents the Mittag-Leffler function.

When $Q_i(t)$ pertains to the absolutely continuous class, the condition $\lim_{t \rightarrow 0} D_{0t}^{\alpha-1} Q_i(t) = a_1 = 0$ must be added, whereby (8) assumes the following form:

$$Q_i(t) = f_i t^\alpha E_{\alpha, \alpha+1}(-g_i(t) t^\alpha) + Q_{i0} E_{\alpha, 1}(-g_i t^\alpha). \quad (9)$$

Expression (9) describes the hailstone charging dynamics in cumulonimbus clouds, incorporating charge dissipation driven by conductivity.

Following the scheme described above, the charge of droplets shedding from hailstones Q_d within the studied fractal region R is given by the following expression [11]:

$$\partial_{0t}^\alpha Q_d(t) + g_d Q_d(t) = f_d, \quad 0 < \alpha < 1, \quad (10)$$

with the initial condition:

$$Q_d(0) = Q_{d0}, \quad (11)$$

where $g_d = K_{d2} \theta^{\alpha-1}$, $f_d = K_{d1} \theta^{\alpha-1}$, $K_{d1} = \omega \frac{r_d}{r_i} \frac{v_i}{v_d} R^2 E$; $K_{d2} = \omega \frac{r_d}{r_i} \frac{v_i}{v_d} + \pi \lambda_f$, равен under the condition that the leakage currents from droplets and hailstones are proportional to their aggregate charges, while the total discharge current through the sphere equals:

$$\partial_{0t}^\alpha Q_{i\lambda}(t) + \partial_{0t}^\alpha Q_{d\lambda}(t) = \pi \lambda_f R^2 E \theta^{\alpha-1}.$$

Equation (10) and initial condition (11) constitute a Cauchy problem for the fractional equation governing the charging dynamics of droplets in the considered model.

The solution to the thundercloud fractal model (10)–(11) is given by:

$$Q_d(t) = f_d t^\alpha E_{\alpha, \alpha+1}(-g_d t^\alpha) + Q_{d0} E_{\alpha, 1}(-g_d t^\alpha). \quad (12)$$

When $\alpha(t)$ represents a piecewise constant function:

$$\alpha(t) = \begin{cases} \alpha_1, t \in [0, t_1], \\ \alpha_2, t \in (t_1, t_2], \\ \dots \\ \alpha_k, t \in (t_{k-1}, t_k], \end{cases} \quad (13)$$

where k is the number of subintervals of the time interval $t \in [0, T]$; in the simplest case, relationship (13) determines the self-affinity property of the fractal thundercloud, and the fractional derivative order can be linked to the medium's fractal dimension [18]. Based on the foregoing, the fractional derivative order can be a function of time, for instance, according to (13).

Subsequently, the values of the fractional derivative order are assumed to be increasing: $\alpha_1 < \alpha_2 < \dots < \alpha_k$. Obtaining an analytical solution for the thundercloud fractal models (3), (5) and (10), (11) incorporating the Caputo derivative with an $\alpha(t)$ -dependence represents an exceptionally challenging problem owing to the

non-linear parameter behavior. For this purpose, the corresponding Cauchy problem must be solved with varying parameter values α_k , g_{i_k} , f_{i_k} at each time subinterval k . Consequently, this yields a complex analytical expression formulated in terms of the special Mittag-Leffler function [17] as follows [12]:

$$Q_k(t) = Q_{k-1}(t_{k-1}) E_{\alpha_k}(-\lambda(t - t_{k-1})^{\alpha_k}) + \int_{t_{k-1}}^t \left[f - \sum_{i=1}^{k-1} \int_{t_{i-1}}^{t_i} \frac{Q_{i-1}(s)(\vartheta - s)^{-\alpha_i}}{\Gamma(1-\alpha_i)} ds \right] (t - \vartheta)^{\alpha_k-1} E_{\alpha_k, \alpha_k}(-\lambda(t - \vartheta)^{\alpha_k}) d\vartheta, \quad (14)$$

which remains highly challenging to implement computationally for visualization purposes. Visualization of the derived solutions for problems (3), (5) and (10), (11) is carried out using the Python programming language.

Results and Discussion

The proposed integrated visualization model enables monitoring the charging dynamics of cloud droplets and hailstones, incorporating charge dissipation due to conductivity as a function of temporal parameters and medium properties. Herein, the calculations incorporate the following values from the table [11, 19].

Table. Model Parameters.

Parameters	Magnitudes
ω, s^{-1}	2
r_d, m	$2 \cdot 10^{-5}$
r_i, m	$2.5 \cdot 10^{-3}$
v_i, ms^{-1}	15
v_d, ms^{-1}	7
Q_{i0}, Kl	$1.2 \cdot 10^{-10}$
Q_{d0}, Kl	$1.2 \cdot 10^{-11}$
E, Vm^{-1}	$1.5 \cdot 10^4$
n_i, m^{-3}	5
n_d, m^{-3}	$5 \cdot 10^6$
$\lambda'_f, S/m$	10^{-13}

The numerical simulation results are depicted in Fig. 1 at different values of α , which are defined throughout the entire time interval $t \in [0, 50]$.

The main graphical plot of Fig. 1 displays the temporal evolution of the hailstone charge. The plot displays five trajectories that correspond to various values of the fractality parameter α (0.1, 0.3, 0.5, 0.7, 0.9). A distinct relationship emerges: for $\alpha < 0.3$, the process is characterized by a smooth monotonic charge increment, while for $\alpha > 0.7$, an exponential growth with swift approach to saturation occurs. The prevailing trajectory ($\alpha = 0.5$) represents the most probable scenario under ambient conditions. The black curve 6 (the principal trajectory) displays the actual process dynamics, wherein α varies smoothly with time in accordance with scheme (14),

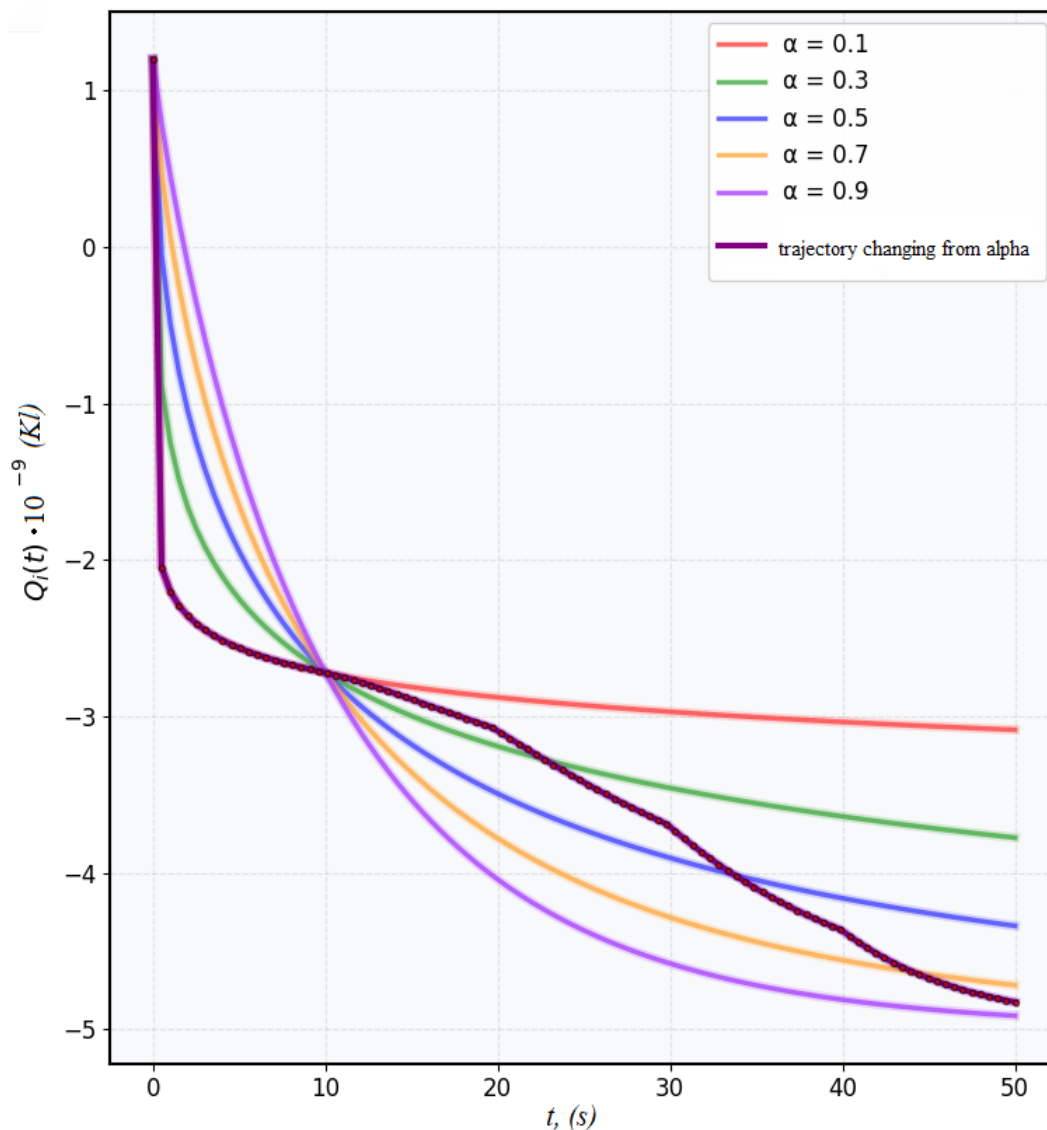


Fig. 1. Calculated curves of the parameter effect on the hailstone charge within a thundercloud

transitioning between consecutive values. This more realistic model is assumed to reflect real atmospheric processes, wherein conditions do not remain static.

The curves in Fig. 1 align with the findings of, wherein the collision of ice particles, specifically between hailstones (spherical ice particles) and fine supercooled droplets or ice crystals, represents the dominant charge separation mechanism.

Within specific temperature ranges ($-10\text{ }^{\circ}\text{C} \div -20\text{ }^{\circ}\text{C}$), steady charge separation occurs, wherein the hailstone acquires a negative charge, as in the present study, while the bouncing particles (droplets) acquire a positive charge. This process, recognized as riming electrification, results in spatial charge separation and possesses the potential to trigger lightning.

A wider discrepancy prolongs the charging process. Consequently, meteorologists are able not only to forecast the thunderstorm itself but also to assess its nature, varying from swift and severe to gradual and extended.

Fig. 2 illustrates the cloud droplet charging dynamics versus time for various values of the medium fractality parameter α . The plots exhibit a family of curves demonstrating the non-linear behavior of the charge over time. The curves are characterized by an abrupt exponential growth during the initial stage, subsequently reaching a saturation plateau.

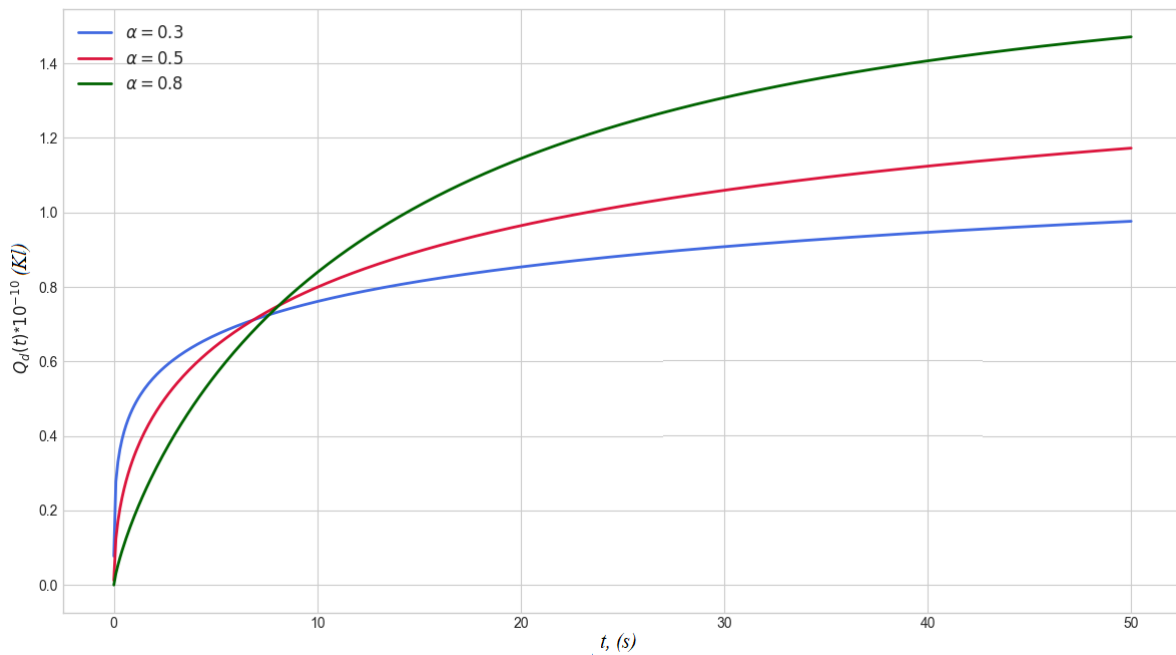


Fig. 2. Simulated cloud droplet charge curves for different α values

Consequently, curve analysis demonstrates the substantial impact of the parameter α on the process kinetics: smaller α values result in a more gradual charge increment with a prolonged timescale for achieving saturation, while increasing α accelerates the charging process and shortens the relaxation time of the system.

The numerical modeling results clearly illustrate that the charge dynamics of both hailstones and cloud droplets exhibit high sensitivity to variations in the non-linearity parameter α , which must be taken into account when constructing predictive models of thundercloud electrodynamics.

To assess the practical relevance of our model, we compare the predicted charging times with typical electric field evolution in convective clouds. According to in-situ measurements [1, 6], the charge separation in a growing cumulonimbus reaches a significant fraction of its maximum value within 20–40 s. Our calculations for $\alpha \approx 0.5$ (which corresponds to $D_f \approx 2.75$) show a similar time scale (see Fig. 1, curve 5). This qualitative agreement supports the validity of our approach, although a quantitative validation with specific field campaigns is planned for future studies.

The numerical modeling results clearly illustrate that the charge dynamics of both hailstones and cloud droplets exhibit high sensitivity to variations in the non-linearity parameter α , which must be taken into account when constructing predictive models of thundercloud electrodynamics.

It is important to emphasise that all simulations presented here describe the absolute value (magnitude) of the charge on the particles. The sign of the charge (positive or negative) is not considered in this study because our main goal is to analyse how the fractal parameter α affects the charging kinetics. In real thunderclouds, the riming process typically charges hailstones negatively and droplets positively [20, 21]. The inclusion of sign-dependent terms will be addressed in future work, as it is essential for predicting lightning polarity.

The characteristic charge accumulation times observed in real thunderstorms — typically on the order of tens of seconds — agree well with our predictions for $\alpha \approx 0.5$. This qualitative correspondence supports the validity of the model. However, a rigorous quantitative validation against direct field data (for instance, from electric field soundings) will be addressed in a subsequent study.

Conclusion

In conclusion, this work focuses on developing a mathematical model for the fractal charge dynamics of cloud particles, incorporating conductivity-driven charge dissipation. To attain this goal, the fractal medium is characterized via self-similar structure generation algorithms that replicate cloud particle distribution, while the charging dynamics during particle collisions are simulated by accounting for stochastic processes and non-uniformities induced by the fractal topology of the cloud environment.

The scientific novelty of this study resides in the integration of two pivotal factors, namely the fractal medium topology and charge leakage dynamics, which are previously analyzed separately. This enables obtaining new data on how the intricate cloud topology and environmental conductive properties affect charge separation efficiency and lightning initiation probability. The practical relevance resides in the enhancement of lightning activity predictions, which is critical for aviation safety, the energy sector, and climate studies.

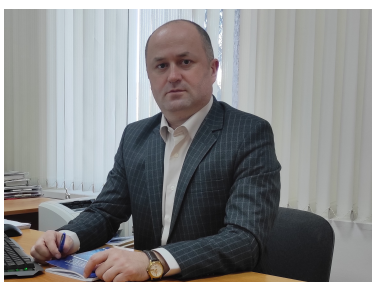
Consequently, the proposed model extends conventional approaches by providing a more realistic description of cloud electrification, thereby opening up avenues for future investigations into atmospheric electricity and the non-linear dynamics of atmospheric processes. электризации. Subsequent studies aim to include conductivity mechanisms governing the charge leakage rate and to analyze their impact on stationary and transient electrification modes.

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