



Dedicated to the 85th anniversary of Professor P.P. Firstov

Evaluation of the Quality of Periodic Variation Extraction in Model Time Series Under Various Levels of Flicker Noise $1/f^\alpha$

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Abstract. A comparison of seven methods for extracting the seasonal component was performed on model data simulating tiltmeter observations in Kamchatka. The seasonal signal was a smoothed meander (period 365 days, duty cycle 0.6), and the noise was flicker noise with exponent $\alpha = 2.9$. The signal-to-noise ratio (SNR) was varied from -25 to $+20$ dB, with 30 realizations generated for each of the 12 SNR levels. Quality was assessed using RMSE, correlation, standard deviation ratio, and a developed residual periodicity metric. The mean seasonal function (MSF) method consistently outperformed all other methods across all metrics over the entire SNR range. GPR and FFT showed acceptable results but with limitations (GPR unstable under strong noise; FFT requires precise knowledge of the period). STL, SARIMA, SSA, and the Chebyshev filter are not recommended for standalone seasonal extraction due to systematic distortions or increasing error. For processing Kamchatka data, MSF is optimal. The conclusions are specific to the smoothed meander model, known period, absence of trend, and $\alpha = 2.9$.

Key words: seasonal component extraction; flicker noise; smoothed meander; tiltmeter observations; Kamchatka; comparison of decomposition methods; mean seasonal function; Chebyshev filter; residual periodicity metric.

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Оценка качества выделения периодической составляющей в модельных временных рядах при различных уровнях фликкер-шума $1/f^\alpha$

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Аннотация. Проведено сравнение семи методов выделения сезонной компоненты на модельных данных, имитирующих наклонные наблюдения на Камчатке. Сезонный сигнал представлял собой сглаженный меандр (период 365 суток, коэффициент заполнения 0.6), а шум — фликкер-шум с показателем $\alpha = 2.9$. Отношение сигнал/шум (SNR) варьировалось от -25 до $+20$ дБ, для каждого из 12 уровней SNR генерировалось по 30 реализаций. Качество оценивалось по RMSE, корреляции, отношению стандартных отклонений и разработанной метрике остаточной периодичности. Метод среднего сезонного хода (MSF) стабильно превосходил все остальные методы по всем метрикам во всём диапазоне SNR. GPR и FFT показали приемлемые результаты, но с ограничениями (GPR неустойчив при сильном шуме; FFT требует точного знания периода). STL, SARIMA, SSA и фильтр Чебышева не рекомендуются для самостоятельного выделения сезонности из-за систематических искажений или роста ошибки. Для обработки камчатских данных оптимален MSF. Выводы ограничены моделью сглаженного меандра, известным периодом, отсутствием тренда и $\alpha = 2.9$.

Ключевые слова: выделение сезонной компоненты; фликкер-шум; сглаженный меандр; наклонные наблюдения; Камчатка; сравнение методов декомпозиции; средний сезонный ход; фильтр Чебышева; метрика остаточной периодичности.

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1 Introduction

Extraction of the seasonal component is a mandatory step in processing geophysical time series prior to the analysis of anomalous deviations. Real data are generally non-stationary due to climatic, hydrogeological, and anthropogenic factors. Classical parametric decomposition methods perform poorly under these conditions, so adaptive nonparametric approaches are more commonly used. However, their choice remains subjective: it is based on theoretical recommendations and the researcher's experience rather than on quantitative assessment. Furthermore, a geophysical time series represents a single realization of a stochastic process (ergodicity is absent), which makes direct ensemble averaging impossible. All of this creates obstacles for objective comparison of seasonal extraction methods and for estimating their errors.

An additional complication arises from the fact that noise in geophysical data often obeys a power law ($1/f^\alpha$) (flicker noise). This is a universal phenomenon observed in physical, biological, and technical systems [1]. In geophysics, flicker noise has been identified, for example, in electrometric variations at the Garm test site [2] and in hydrogeochemical data from Kamchatka [3]. The parameters of such noise (power-law exponent $\alpha = 2.9$) in the present work were determined from real tiltmeter observations from the Kamchatka network [4,5]. Under these conditions, standard seasonal extraction methods that assume stationarity or weak noise dependence may produce systematic errors. A systematic comparison of approaches to seasonal extraction against ($1/f$) noise remains insufficiently studied.

In real data, the true seasonal component is unknown, so direct comparison of methods on field records is impossible. The only way to quantitatively assess extraction quality is through numerical experiments on model data with fully known signal and noise parameters.

The aim of this work is to compare, using model data with flicker noise, seven approaches based on different methods (MSF, FFT, SSA, STL, GPR, SARIMA, Chebyshev filter) for extracting the seasonal component at different SNR levels. No linear trend is used in the additive model — this simplification allows assessment of seasonal signal extraction quality in its pure form, without errors introduced by trend separation.

2 Theoretical Justification

Tiltmeter measurements in Kamchatka are conducted at a network of sites that differ in geological, climatic, and engineering conditions. As shown in [7,8], this leads to a diversity of seasonal manifestations: oscillation amplitude decreases with sensor installation depth; noise levels at some stations increase in winter due to soil heave; diurnal variation amplitude increases in summer due to thawing of upper horizons; the annual cycle shape deviates from sinusoidal; and a long-term trend is present.

For quantitative comparison of seasonal extraction approaches, a model experiment was chosen because the true component is unknown in real data. An additive model

was used [9]:

$$\text{Model}(t) = S(t) + kN(t), \quad (1)$$

where $S(t)$ is the seasonal signal, $N(t)$ is flicker noise, and k is a coefficient defining the noise level.

Extracting the trend would require introducing additional assumptions, which would complicate interpretation of the results and limit their generality. Therefore, trend is not considered in this study.

Noise parameters were taken from real observations. The signal shape is a smoothed meander, reflecting the asymmetry of the annual cycle (different durations of warm and cold seasons).

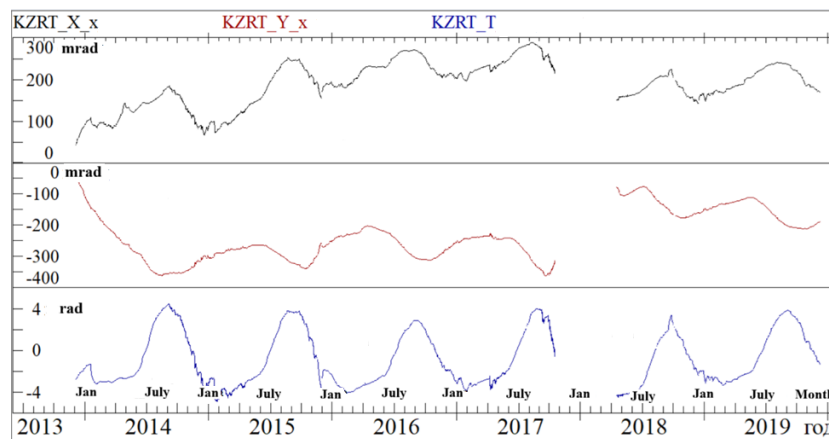


Fig. 1. Plots of tilt variations and temperature at station KZRT (“Kozyrevsk”)

Figure 1 shows plots of ground surface tilts at station KZRT [10] and observation chamber temperature, illustrating the actual seasonal variation.

In this work, the seasonal period is taken as fixed and equal to 365 samples (days), which corresponds to real tiltmeter observations in Kamchatka. Possible minor phase variations that may exist in real signals were not modeled, since they do not change the overall pattern of seasonal variation. This assumption is based on the fact that the propagation velocity of a thermal wave in soil, even under sharply contrasting conditions, does not exceed the first few tens of centimeters per day, and tiltmeter network sensors are typically installed at depths of 5 m or more. Therefore, phase changes in temperature variations, unless extreme, cannot significantly affect the recorded seasonal tilt variation. This statement holds not only for tiltmeter measurements but also for a number of other long-term geophysical measurements, including borehole observations.

In the experiment, the coefficient k is varied, corresponding to changes in SNR. Seven approaches to seasonal component extraction are compared: mean seasonal function (MSF), fast Fourier transform (FFT), singular spectrum analysis (SSA), LOESS decomposition (STL), Gaussian processes (GPR), seasonal ARIMA (SARIMA), and the Chebyshev bandpass filter (baseline). Each approach is tuned to the problem conditions: absence of trend, known period (365 samples), and a signal shape in the form of a smoothed meander. For FFT, only odd harmonics (1, 3, 5, ...) are extracted,

since the shape of the real seasonal signal is close to an asymmetric meander, and even harmonics are negligibly small. SSA was applied with a window length that is a multiple of the period, followed by grouping of eigenvectors corresponding to the seasonal component. STL was used in a mode extracting only the seasonal component without estimating the trend. GPR and SARIMA were also configured without a trend component. The Chebyshev filter was used as a baseline approach with a passband covering the main harmonics of the seasonal signal.

Resource-intensive methods (neural networks, iterative algorithms) are excluded from the comparison, since one of the objectives was to extract the seasonal component with acceptable quality in minimal time.

The numerical experiment was conducted for a smoothed meander, flicker noise with $\alpha = 2.9$, absence of trend, and known period. For other signal types, noise, or additive model parameters (presence of trend, unknown period), different results are highly likely. Therefore, the conclusions of the analysis are limited to the experimental conditions and cannot claim universality.

3 Experimental Methodology

3.1 Generation of the Seasonal Signal

The model signal is a smoothed meander with period $T = 365$ samples and zero mean. For all $t = 0, \dots, L - 1$ ($L = 3650 - 10$ periods (years)), a rectangular meander is generated:

$$M(t) = \begin{cases} +1, & 0 \leq \{t/T\} < d, \\ -1, & d \leq \{t/T\} < 1, \end{cases}$$

where $\{\cdot\}$ is the fractional part, and $d = 0.6$ is the duty cycle (the winter season is longer). For edge smoothing, a 2nd-order Butterworth filter with cutoff frequency $f_{\text{cut}} = 4/T$ is applied. Its frequency response is:

$$H(f) = \frac{1}{\sqrt{1 + (f/f_{\text{cut}})^4}}.$$

The value $4f_0$ ($f_0 = 1/T$) lies between the third and fifth harmonics, providing slight attenuation of the third harmonic and noticeable suppression of the fifth and higher harmonics. Filtering is bidirectional (zero phase lag) [11]. After filtering, positive peaks fall below +1, and negative peaks fall below -1 in absolute value. To restore symmetric amplitude, separate normalization is performed:

$$S(t) = \begin{cases} \frac{S_{\text{filter}}(t)}{\max(S_{\text{filter}}(t))}, & S_{\text{filter}}(t) > 0, \\ \frac{S_{\text{filter}}(t)}{\min(S_{\text{filter}}(t))}, & S_{\text{filter}}(t) < 0. \end{cases}$$

As a result, the positive and negative half-waves are scaled to unit amplitude, and the mean value of the signal remains close to zero. To avoid edge effects, the original meander is generated with a buffer of 365 samples on each side; after filtering and normalization, the buffer is discarded. The final series length is 3650 samples.

3.2 Generation of the Noise Component

Flicker noise with power-law exponent $\alpha = 2.9$ (Fig. 3b) was generated using the shaping filter method in the frequency domain. The amplitude spectrum $A(f)$ was formed proportional to $f^{-\alpha/2}$, and the harmonic phases were uniformly distributed in the interval $[0, 2\pi)$. Applying the inverse Fourier transform to the resulting spectrum yielded a time realization $N_{\text{raw}}(t)$. Since the power spectral density is related to the amplitude spectrum by $S(f) = |A(f)|^2$, the resulting spectrum of the realization corresponds to the power law $S(f) \propto f^{-\alpha}$ in the frequency range from $f_{\min} = 1/T$ to $f_{\max} = 0.1$ cycles/day. The upper frequency limit was chosen based on the stability of the α exponent estimate using Welch's method [12]: at higher frequencies, the estimate is systematically underestimated.

It should be particularly noted that the non-stationarity of this process for $\alpha \geq 1$ is not due to the formal infinity of the theoretical variance, but to the divergence of the power spectral density integral in the low-frequency region ($f \rightarrow 0$). This leads to the variance of the process growing without bound with increasing observation time, which violates one of the key requirements of stationarity — the invariance of variance over time. At the same time, for a finite realization of fixed length ($N = 3650$ samples), the lower boundary frequency is nonzero ($f_{\min} = 1/N$), and therefore the sample variance remains finite and well-defined.

For comparability of realizations, the noise is normalized to unit standard deviation:

$$N(t) = \frac{N_{\text{raw}}(t)}{\sigma_{\text{raw}}}, \quad \sigma_{\text{raw}} = \sqrt{\text{Var}(N_{\text{raw}}(t))}.$$

Normalization does not introduce systematic error when comparing methods. The spectral properties ($\alpha = 2.9$) are preserved. Control of α was performed using Welch's method. For 30 realizations, the mean $\alpha = 2.89 \pm 0.07$, which corresponds to the target value (Fig. 2).

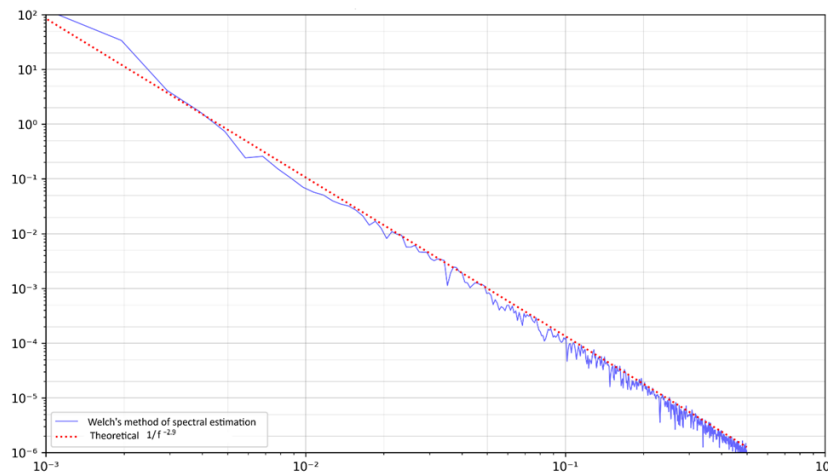


Fig. 2. Power spectral density in log-log scale. The estimated scaling exponent $\alpha = 2.89$ is within 0.34% of the target value $\alpha = 2.9$

3.3 Noise Levels Used

The signal power is $\overline{S^2(t)} = 1$. The noise is centered, so $\overline{N^2(t)} = \sigma_N^2 = 1$. In model (1), the noise power is k^2 . The signal-to-noise power ratio is:

$$\text{SNR} = \frac{\overline{S^2(t)}}{(kN(t))^2} = \frac{1}{k^2}.$$

In logarithmic scale:

$$\text{SNR}_{\text{dB}} = -20 \log_{10} k.$$

The noise scaling coefficient is:

$$k = 10^{-\text{SNR}_{\text{dB}}/20}.$$

Thus, the proposed model (Fig. 3) reproduces three key aspects of real tiltmeter observations in Kamchatka: the annual cycle shape, approximated by a smoothed meander with a period of 365 days and a duty cycle of 0.6; a wide range of signal-to-noise ratios covering the range from -25 to $+20$ dB; and flicker noise with exponent $\alpha = 2.9$, corresponding to the spectral characteristics of real recordings from the Kamchatka network. For each of the twelve SNR levels (Table 1), 30 independent realizations of the additive model (1) were generated, totaling 360 model series. Each realization was processed by all considered seasonal component extraction methods. For each method and each realization, quality metrics were computed. Then, for each method and each SNR level, the metrics were averaged over the 30 realizations. This data volume ensures statistical significance when comparing the robustness and efficiency of the approaches.

3.4 Methods Used

For objective assessment of the effectiveness of seasonal component extraction under different noise levels and specified noise characteristics [13], seven widely used time series decomposition methods were selected. Each requires preliminary tuning to the parameters of the model data. In this case, adaptation consists of bringing all methods to uniform conditions: absence of trend in the data and known period of the seasonal signal (365 samples). For the FFT-based approach, the signal shape property close to a meander was exploited: the spectrum contains only odd harmonics, which improved noise robustness.

At $\alpha = 2.9$, the power spectral density of flicker noise decays with frequency as $f^{-2.9}$. At the seasonal signal frequency (approximately 0.0027 cycles/day), the noise level is several orders of magnitude lower than in the high-frequency region. The signal is concentrated in a narrow band near this frequency and its odd harmonics. From this spectral separation, it follows that even a simple bandpass filter can potentially extract the seasonal component with acceptable accuracy. Therefore, the Chebyshev filter was chosen as the baseline method: it contains no data-adaptive mechanisms and does not rely on statistical noise properties; its result is unambiguously determined by the cutoff frequency and filter order.

Below is a brief description of the algorithmic basis of each approach and implementation details.

Table 1

Parameters of model series realizations with different noise levels (SNR)

SNR, dB	SNR, linear	k	Explanation
-25	0.00316	17.78	Hypothetical lower limit
-20	0.01	10.0	Extremely unfavorable conditions
-15	0.0316	5.62	Testing applicability limits
-10	0.1	3.16	Strong noise. Robustness assessment
-6	0.25	2.00	Noise four times more powerful than signal
-3	0.5	1.41	Noise twice as powerful as signal (lower bound of real data)
0	1.0	1.00	Equal powers. Typical conditions
3	2.0	0.71	Signal twice as powerful as noise (upper bound of real data)
6	4.0	0.50	Signal four times more powerful than noise
10	10	0.32	Moderate noise
15	31.6	0.18	Weak noise
20	100	0.10	Ideal conditions

3.4.1 Chebyshev Bandpass Filter (baseline method)

Seasonal component extraction is performed by low-pass filtering using a Type I Chebyshev filter [14]. The spectrum of the smoothed meander contains the first harmonic at frequency $1/365 \approx 0.00274$ cycles/sample and odd higher harmonics ($3/365 \approx 0.00822$, $5/365 \approx 0.01370$, ...). To preserve signal shape, the filter must pass at least three harmonics. The cutoff frequency was set to 0.015 cycles/sample — above the third harmonic (0.00822) but below the fifth (0.01370), ensuring passage of the first and third harmonics and suppression of higher ones. The filter order was taken as 4 (a compromise between cutoff steepness and stability). Passband ripple — 1 dB. Filtering is performed with zero phase shift (forward and reverse pass). The transient response of a 4th-order filter at cutoff frequency 0.015 lasts about 50–100 samples; therefore, the first and last 100 days are excluded when calculating quality metrics. Reconstructed series are preserved in full (3650 samples). Method limitation: the cutoff frequency is fixed and does not adapt to noise level; under strong noise ($k \geq 1.4$), outliers at the edges may occur.

3.4.2 GPR (Gaussian Process Regression)

Gaussian regression is a probabilistic data analysis method [14]. It is based on the assumption that any finite samples follow a multivariate normal distribution. The key role is played by the covariance function (kernel), which determines the degree of similarity between observations. The method allows automatic model complexity tuning and error estimation (RMSE, MAE, R^2 , etc.).

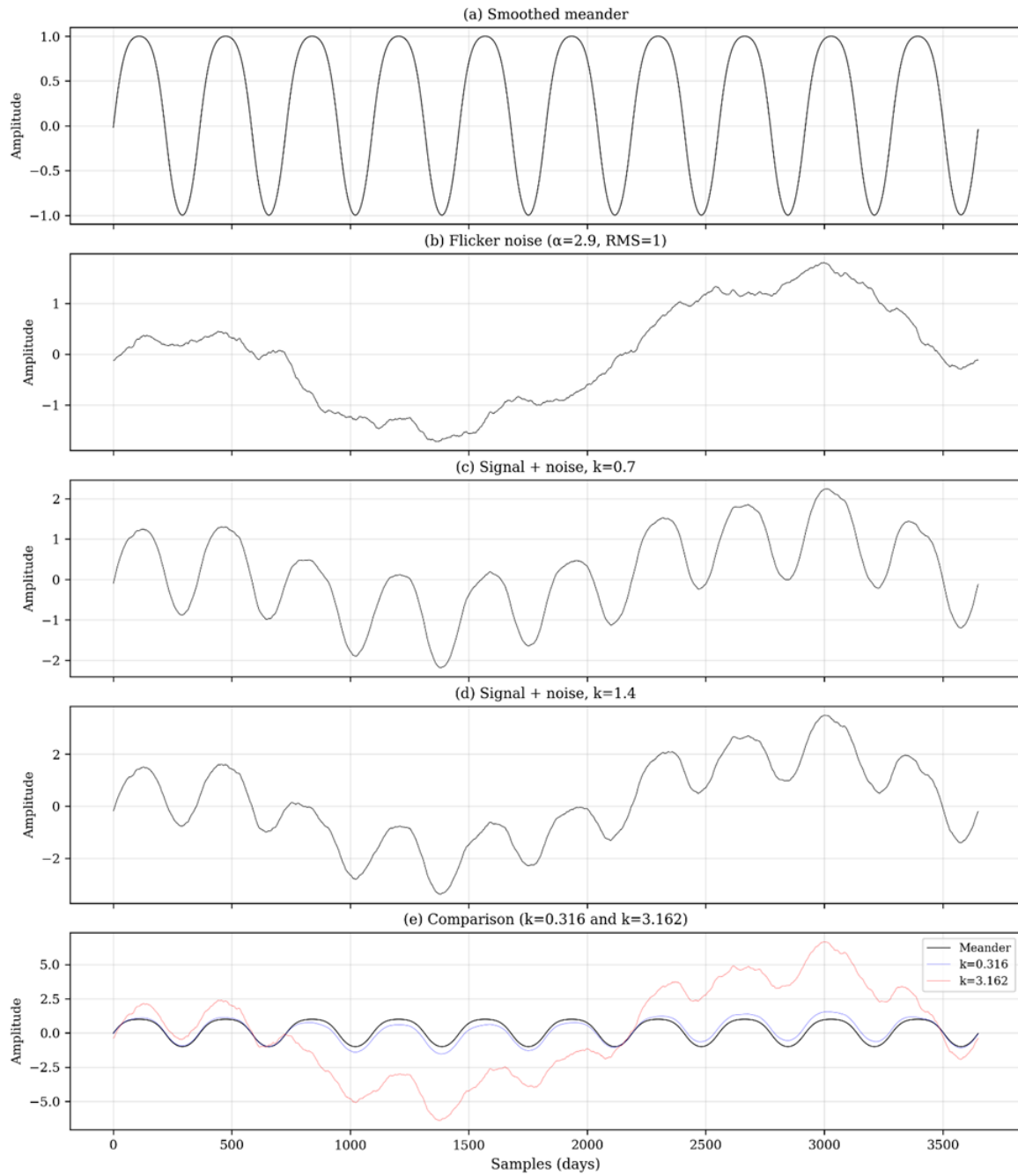


Fig. 3. Examples of the initial data: (a) smoothed meander with a period of 365 days and a duty cycle of 0.6, used as the seasonal signal; (b) realization of correlated noise with $\alpha = 2.9$, normalized to σ ; (c, d) realizations of the additive model with $k = 0.7$ (3 dB) and 1.4 (-3 dB); (e) overlay of the series shows that at $k = 3.162$ (-10 dB) the signal is heavily masked by noise, while at $k = 0.316$ (10 dB) the seasonal structure remains identifiable

When dealing with seasonal fluctuations, GPR correctly analyzes variations from period to period and effectively handles missing data.

In the model experiment, a Gaussian process with a periodic kernel (StdPeriodic) was applied. The kernel was specified with parameters: period 365.25 days, variance

0.5 (50% seasonal component contribution), length scale 50 days (shape smoothness). The period was additionally optimized within 335–398 days with a prior lognormal distribution. Initial kernel parameter values were fixed, then refined using the BFGS method for 5000 iterations. Sparse approximation was not used.

3.4.3 FFT (Fast Fourier Transform)

The fast Fourier transform implements the discrete Fourier transform for seasonal component extraction [15]. With a known target period (365 days), the algorithm searches for the fundamental frequency in the range of 365 ± 15 days by maximum spectral power, which improves robustness at low SNR. Since the model signal is a smoothed meander, its spectrum contains predominantly odd harmonics; therefore, reconstruction is performed using harmonics 1 and 3. The DC component is preserved. A key limitation is edge distortions: with a period determination error of ± 1 day over 10 years, a phase shift of up to 10 days accumulates. Therefore, the first and last 100 days are excluded when assessing quality.

3.4.4 SARIMA (Seasonal Autoregressive Integrated Moving Average)

The SARIMA method accounts for short-term and long-term dependencies [16]. In the absence of a trend, the differencing parameters (d and D) are set to zero. The seasonal period s is set to 36, corresponding to 360 days after preliminary aggregation of the original data at a 10-day interval. Parameter tuning is adapted to the noise level k (related to SNR by $\text{SNR} = 1/k^2$). Under strong noise ($k > 5.0$), the seasonal autoregressive and moving components are disabled ($P = 0, Q = 0$), with non-seasonal order = (1,0,1), seasonal_order = (0,0,0,36), and maxiter = 50. Under moderate noise ($0.5 \leq k \leq 5.0$), order = (1,0,1), seasonal_order = (1,0,1,36), and maxiter = 75. Under weak noise ($k < 0.5$), non-seasonal autoregression is increased to (2,0,2) while maintaining seasonal_order = (1,0,1,36) and maxiter = 100. Stationarity and invertibility checks are disabled; linear trend is not considered. The obtained seasonal component has a resolution of 10 days; to return to the daily grid, interpolation is performed, followed by smoothing with a Savitzky–Golay filter [17] (window 61 days, polynomial degree 2). Edge distortions are excluded (first and last 100 days).

3.4.5 SSA (Singular Spectrum Analysis)

The SSA method [18] belongs to nonparametric approaches. The algorithm includes construction of the trajectory matrix, singular value decomposition, grouping of singular triples, and diagonal averaging. The window length L was chosen as 1095 days (slightly less than one-third of the series length $N = 3650$), providing sufficient resolution for extracting seasonal oscillations with a period of 365 days. Seasonal component reconstruction is performed using the first four reconstructed components (RCs), since the smoothed meander concentrates most of the variance in the first few singular triples. A specific limitation of SSA is edge distortions (about 100 points on each side); therefore, the first and last 100 days are excluded when calculating metrics.

3.4.6 STL (Seasonal and Trend decomposition using LOESS)

The STL method [19] is a nonparametric robust algorithm for decomposition into trend, seasonality, and residual. The seasonal period is explicitly set to 365 days, and the seasonal parameter is also set to 365. Since the model data contain no trend, the trend parameter is set to zero. Robust mode is enabled (`robust = True`), providing resistance to outliers. Other parameters are left at default values. Limitations: edge effects associated with moving windows — exclusion of the first and last 100 days.

3.4.7 MSF (Mean Seasonal Function)

The mean seasonal function method is based on averaging the series values within a single period [9, 20]. The seasonal period is set to 365 days. For each day of the year (0...364), the arithmetic mean of all observations falling on that day across the entire series length is calculated. If for a given day the proportion of valid values (non-missing) falls below 50%, the corresponding MSF value is considered undefined. The resulting mean seasonal function (length 365 samples) is repeated over the entire 10-year interval. In the present implementation, there are no stages of preliminary smoothing, detrending, or additional filtering — this is dictated by the model experiment conditions (no trend, signal shape does not require smoothing). Edge effects are absent because the series length is a multiple of the period.

3.5 *Quality Metrics for Reconstruction*

For quantitative assessment of seasonal component extraction quality, three standard metrics were used: root mean square error (RMSE), Pearson correlation coefficient (Corr), and the ratio of standard deviations of the reconstructed signal to the true one (`std_ratio`). RMSE characterizes overall reconstruction accuracy. Corr evaluates waveform similarity, remaining invariant to amplitude and constant offset. `std_ratio` shows how accurately the method preserves the amplitude of the original meander (ideal — 1.0). For each metric, the mean, standard deviation, and 95% confidence intervals based on the t-distribution were computed over 30 realizations.

Additionally, a metric Q was introduced to assess the degree of suppression of the periodic component in the residuals. It is defined as the ratio of the power spectral density at the seasonal frequency $f_{\text{seas}} = 1/365$ cycles/day:

$$Q = \frac{\text{PSD}_{\text{res}}(f_{\text{seas}})}{\text{PSD}_n(f_{\text{seas}})},$$

where PSD_{res} is the power spectral density of the residual (the difference between the original noisy series and the reconstructed signal), and PSD_n is the power spectral density of the original noisy series. Normalization by PSD_n ensures correct comparison at different noise levels. The ideal value $Q = 0$ corresponds to complete absence of periodicity in the residual. Values $Q \leq 0.05$ can be considered excellent, $0.05 < Q \leq 0.20$ — satisfactory, and $Q > 0.5$ — unsatisfactory.

Power spectral density estimation was performed using the Lomb–Scargle periodogram [21,22], which is robust to window function artifacts. The frequency grid was chosen in the range from 0.001 to 0.1 cycles/day with a resolution no worse than 5×10^{-4} cycles/day. For each realization, the periodogram is computed, then Q values are averaged over 30 realizations with 95% confidence intervals.

4 Results

Comparison was performed for seven methods of periodic component extraction: Chebyshev filter (used as the baseline method), FFT, GPR, SARIMA, SSA, STL, and MSF. Examples of the reconstructed signal are shown in Figs. 4 and 5 (the original meander is shown in gray) for two noise levels ($k = 0.708$ and $k = 1.413$). Reconstruction quality was assessed by comparison with the true meander. The comparison was conducted on synthetic series with sequentially increasing noise intensity.

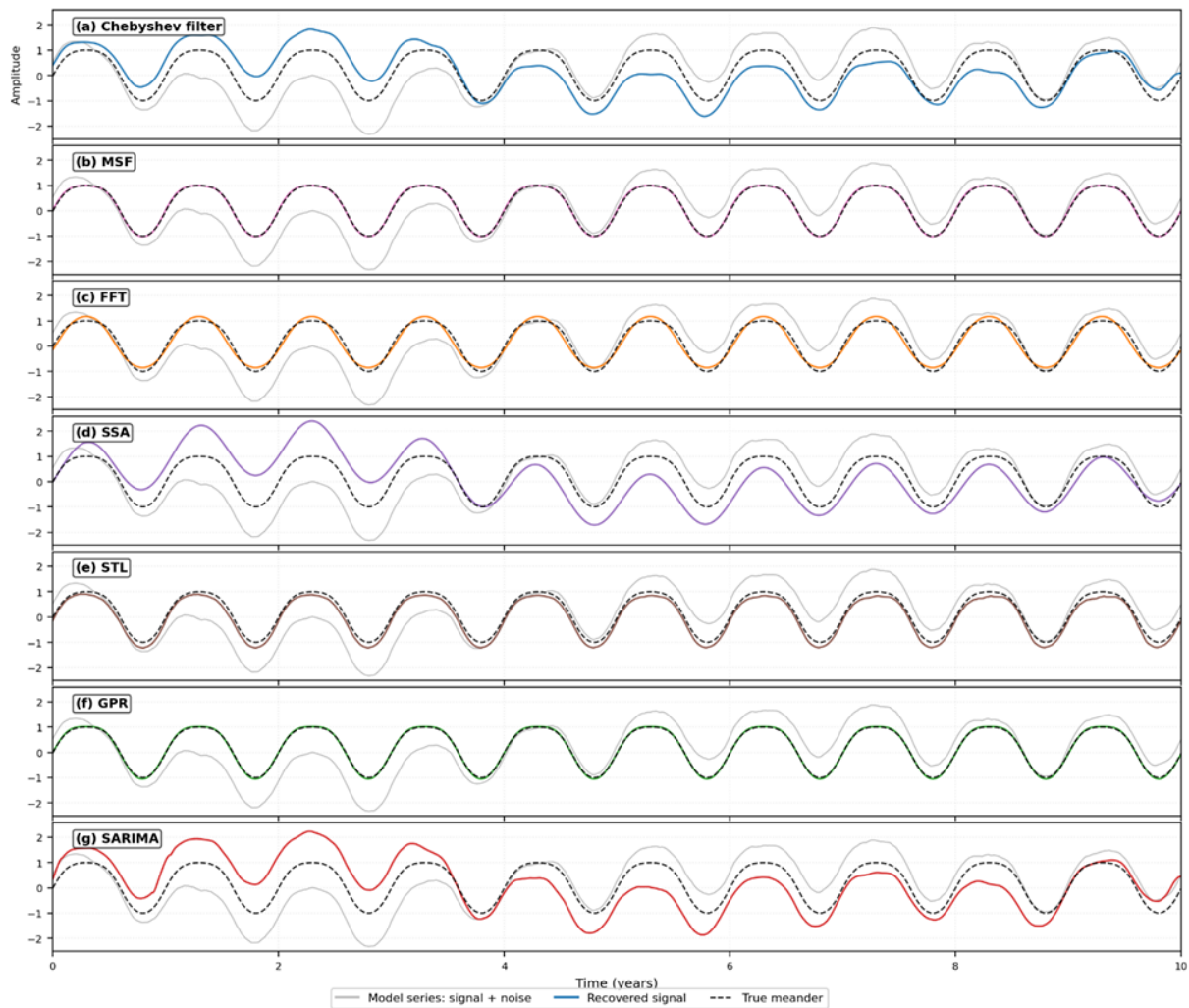


Fig. 4. Examples of the original signal and the seasonality recovered by various methods at a signal-to-noise ratio of 3 dB ($k = 0.708$)

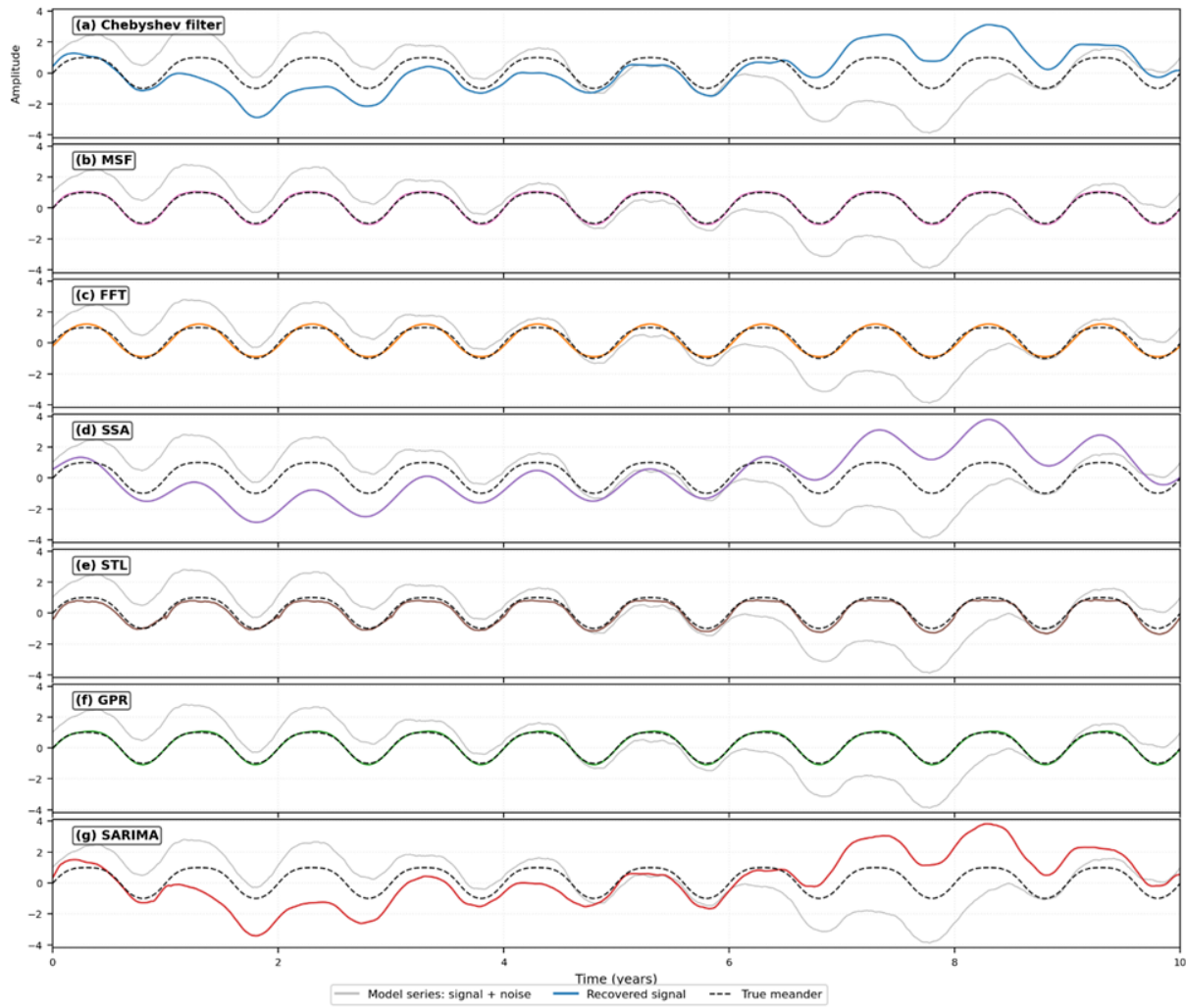


Fig. 5. Examples of the original signal and the seasonality recovered by various methods at a signal-to-noise ratio of -3 dB ($k = 1.413$)

For each of the 12 noise levels (k), 30 realizations of the model series were generated. Quality metrics were computed for each realization, then averaged over all realizations. As a result, for each method, averaged metric values were obtained as a function of noise level.

The Chebyshev bandpass filter is used as the baseline. Its operation is transparent: seasonal component extraction reduces to frequency filtering with a fixed passband. The method does not adapt to the noise level and does not rely on statistical data properties. This allows it to be considered as the simplest baseline approach, relative to which the effectiveness of the other methods is assessed. The main focus is on the noise level range $k = 0.3$ – 1.413 (signal-to-noise ratio from 10 to -3 dB), which corresponds to the conditions of real geophysical observations that served as the starting point for addressing the problem.

A quantitative comparison of the reconstructed signals with the original meander in terms of correlation coefficient is given in Table 2.

Table 2

Correlation coefficient							
k	FFT	GPR	SARIMA	SSA	STL	MSF	Cheb.
0.1	$0.985 \pm 1.6 \times 10^{-5}$	$1.000 \pm 4.0 \times 10^{-6}$	$0.978 \pm 1.1 \times 10^{-3}$	$0.990 \pm 2.0 \times 10^{-3}$	$1.000 \pm 2.8 \times 10^{-5}$	$1.000 \pm 3.0 \times 10^{-6}$	$0.992 \pm 3.9 \times 10^{-4}$
0.178	$0.985 \pm 2.9 \times 10^{-5}$	$1.000 \pm 1.1 \times 10^{-5}$	$0.960 \pm 2.0 \times 10^{-3}$	$0.972 \pm 8.5 \times 10^{-3}$	$1.000 \pm 8.9 \times 10^{-5}$	$1.000 \pm 1.0 \times 10^{-5}$	$0.977 \pm 1.2 \times 10^{-3}$
0.316	$0.985 \pm 6.0 \times 10^{-5}$	$1.000 \pm 3.9 \times 10^{-5}$	$0.910 \pm 4.0 \times 10^{-3}$	$0.909 \pm 4.3 \times 10^{-3}$	$1.000 \pm 2.8 \times 10^{-4}$	$1.000 \pm 3.1 \times 10^{-5}$	$0.933 \pm 3.1 \times 10^{-3}$
0.501	$0.985 \pm 1.2 \times 10^{-4}$	$1.000 \pm 9.4 \times 10^{-5}$	$0.815 \pm 6.1 \times 10^{-3}$	$0.822 \pm 8.0 \times 10^{-3}$	$0.999 \pm 6.9 \times 10^{-4}$	$1.000 \pm 7.9 \times 10^{-5}$	$0.853 \pm 5.8 \times 10^{-3}$
0.708	$0.985 \pm 2.0 \times 10^{-4}$	$0.999 \pm 1.9 \times 10^{-4}$	$0.711 \pm 8.4 \times 10^{-3}$	$0.721 \pm 1.0 \times 10^{-2}$	$0.998 \pm 1.4 \times 10^{-3}$	$1.000 \pm 1.6 \times 10^{-4}$	$0.757 \pm 8.4 \times 10^{-3}$
1	$0.984 \pm 3.7 \times 10^{-4}$	$0.999 \pm 3.7 \times 10^{-4}$	$0.586 \pm 1.0 \times 10^{-2}$	$0.598 \pm 1.0 \times 10^{-2}$	$0.996 \pm 2.7 \times 10^{-3}$	$0.999 \pm 3.2 \times 10^{-4}$	$0.636 \pm 1.0 \times 10^{-2}$
1.413	$0.984 \pm 7.0 \times 10^{-4}$	$0.998 \pm 7.3 \times 10^{-4}$	$0.459 \pm 1.0 \times 10^{-2}$	$0.470 \pm 2.0 \times 10^{-2}$	$0.991 \pm 5.1 \times 10^{-3}$	$0.998 \pm 6.4 \times 10^{-4}$	$0.505 \pm 1.0 \times 10^{-2}$
1.995	$0.983 \pm 1.4 \times 10^{-3}$	$0.997 \pm 1.5 \times 10^{-3}$	$0.347 \pm 1.0 \times 10^{-2}$	$0.351 \pm 2.0 \times 10^{-2}$	$0.983 \pm 9.8 \times 10^{-3}$	$0.997 \pm 1.3 \times 10^{-3}$	$0.385 \pm 2.0 \times 10^{-2}$
3.162	$0.979 \pm 3.4 \times 10^{-3}$	$0.992 \pm 3.7 \times 10^{-3}$	$0.231 \pm 2.0 \times 10^{-2}$	$0.191 \pm 3.0 \times 10^{-2}$	$0.959 \pm 2.0 \times 10^{-2}$	$0.992 \pm 3.3 \times 10^{-3}$	$0.257 \pm 2.0 \times 10^{-2}$
5.623	$0.967 \pm 1.0 \times 10^{-2}$	$0.976 \pm 1.0 \times 10^{-2}$	$0.138 \pm 2.0 \times 10^{-2}$	$0.065 \pm 3.0 \times 10^{-2}$	$0.886 \pm 6.0 \times 10^{-2}$	$0.977 \pm 1.0 \times 10^{-2}$	$0.151 \pm 2.0 \times 10^{-2}$
10	$0.930 \pm 4.0 \times 10^{-2}$	$0.929 \pm 4.0 \times 10^{-2}$	$0.081 \pm 2.0 \times 10^{-2}$	$0.018 \pm 2.0 \times 10^{-2}$	$0.726 \pm 1.6 \times 10^{-1}$	$0.928 \pm 4.0 \times 10^{-2}$	$0.089 \pm 2.0 \times 10^{-2}$
17.783	$0.812 \pm 1.3 \times 10^{-1}$	$0.542 \pm 4.3 \times 10^{-1}$	$0.049 \pm 2.0 \times 10^{-2}$	$0.009 \pm 2.0 \times 10^{-2}$	$0.515 \pm 2.8 \times 10^{-1}$	$0.791 \pm 1.5 \times 10^{-1}$	$0.054 \pm 2.0 \times 10^{-2}$

Correlation coefficient (Table 2, Fig. 6) characterizes the degree of similarity between the shape of the reconstructed signal and the original meander. This metric is insensitive to constant offset and signal scaling.

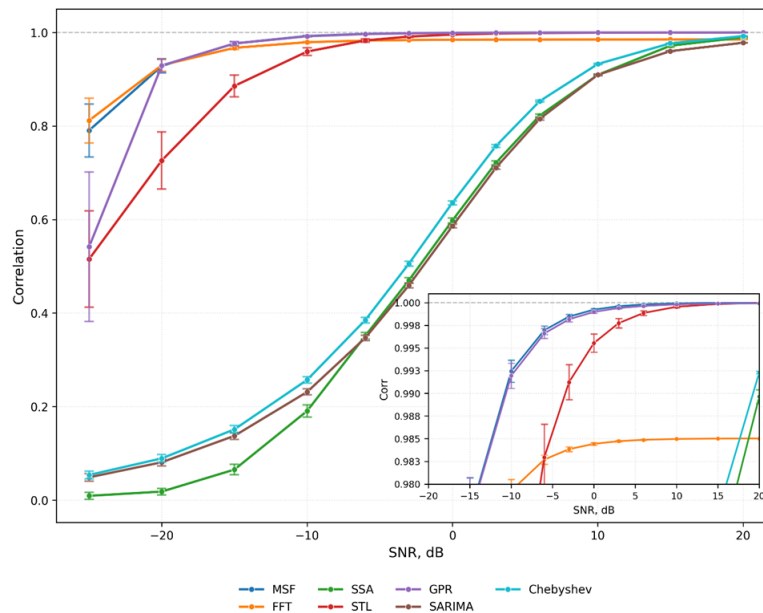


Fig. 6. Correlation between the reconstructed and true signals as a function of the noise level

At $k = 1.413$ (lower bound of the real range for Kamchatka data), MSF, GPR, and STL maintain correlation at the level of 0.998–0.991, while FFT remains stable (0.984), and SARIMA, SSA, and the Chebyshev filter decrease to 0.46–0.51. At $k = 3.2$ (strong noise region), MSF and GPR still yield Corr around 0.99, STL — 0.96, FFT — 0.98, while SARIMA and SSA drop to 0.23 and 0.19, respectively.

At extremely high noise levels ($k = 17.8$), no method maintains high correlation. However, MSF and FFT remain at the level of 0.81–0.79, whereas GPR drops to 0.54 with a very large standard deviation ($\sigma = 0.43$), STL — to 0.52, and SARIMA, SSA, and the Chebyshev filter — to 0.05–0.09.

In terms of noise robustness in the range significant for the original data ($k = 0.3$ – 3.2), MSF and GPR show the best performance (standard deviation 10^{-4} – 10^{-3}). FFT and the Chebyshev filter have a scatter on the order of 10^{-3} – 10^{-2} , which remains

acceptable for practical use. STL demonstrates high correlation, but at $k \geq 5.6$ the scatter increases sharply (σ reaches 0.16), indicating instability of the method under strong noise. SARIMA and SSA are unstable already at $k \geq 1.413$.

Root mean square error (RMSE) allows quantitative assessment of the overall accuracy of seasonal component reconstruction, since it accounts for both amplitude deviations and signal shape mismatch. The smaller the RMSE value, the closer the reconstructed series is to the true meander. The results of this metric calculation for all considered methods and noise levels are given in Table 3.

Table 3

Comparison of methods by root mean square error (RMSE)

k	FFT	GPR	SARIMA	SSA	STL	MSF	Cheb.
0.1	$0.127 \pm 7.7 \times 10^{-5}$	$0.009 \pm 1.1 \times 10^{-4}$	$0.156 \pm 3.9 \times 10^{-3}$	$0.107 \pm 9.8 \times 10^{-3}$	$0.165 \pm 1.7 \times 10^{-4}$	$0.004 \pm 4.9 \times 10^{-5}$	$0.126 \pm 1.9 \times 10^{-3}$
0.178	$0.127 \pm 1.4 \times 10^{-4}$	$0.011 \pm 2.0 \times 10^{-4}$	$0.213 \pm 5.5 \times 10^{-3}$	$0.177 \pm 3.0 \times 10^{-2}$	$0.166 \pm 4.1 \times 10^{-4}$	$0.007 \pm 7.6 \times 10^{-5}$	$0.172 \pm 3.4 \times 10^{-3}$
0.316	$0.128 \pm 2.4 \times 10^{-4}$	$0.016 \pm 5.7 \times 10^{-4}$	$0.335 \pm 8.6 \times 10^{-3}$	$0.330 \pm 8.9 \times 10^{-3}$	$0.167 \pm 1.2 \times 10^{-3}$	$0.012 \pm 1.2 \times 10^{-4}$	$0.269 \pm 6.3 \times 10^{-3}$
0.501	$0.128 \pm 3.8 \times 10^{-4}$	$0.023 \pm 8.7 \times 10^{-4}$	$0.517 \pm 1.0 \times 10^{-2}$	$0.500 \pm 2.0 \times 10^{-2}$	$0.170 \pm 2.8 \times 10^{-3}$	$0.018 \pm 1.8 \times 10^{-4}$	$0.410 \pm 1.0 \times 10^{-2}$
0.708	$0.129 \pm 5.3 \times 10^{-4}$	$0.029 \pm 1.4 \times 10^{-3}$	$0.721 \pm 2.0 \times 10^{-2}$	$0.696 \pm 2.0 \times 10^{-2}$	$0.174 \pm 5.3 \times 10^{-3}$	$0.025 \pm 2.4 \times 10^{-4}$	$0.571 \pm 1.0 \times 10^{-2}$
1	$0.132 \pm 7.4 \times 10^{-4}$	$0.039 \pm 2.1 \times 10^{-3}$	$1.011 \pm 2.0 \times 10^{-2}$	$0.975 \pm 3.0 \times 10^{-2}$	$0.182 \pm 9.9 \times 10^{-3}$	$0.036 \pm 3.3 \times 10^{-4}$	$0.801 \pm 2.0 \times 10^{-2}$
1.413	$0.136 \pm 1.0 \times 10^{-3}$	$0.053 \pm 3.3 \times 10^{-3}$	$1.423 \pm 4.0 \times 10^{-2}$	$1.372 \pm 4.0 \times 10^{-2}$	$0.197 \pm 2.0 \times 10^{-2}$	$0.050 \pm 4.6 \times 10^{-4}$	$1.128 \pm 3.0 \times 10^{-2}$
1.995	$0.143 \pm 1.4 \times 10^{-3}$	$0.073 \pm 5.5 \times 10^{-3}$	$2.004 \pm 5.0 \times 10^{-2}$	$1.936 \pm 6.0 \times 10^{-2}$	$0.224 \pm 3.0 \times 10^{-2}$	$0.071 \pm 6.4 \times 10^{-4}$	$1.590 \pm 4.0 \times 10^{-2}$
3.162	$0.164 \pm 1.9 \times 10^{-3}$	$0.114 \pm 1.0 \times 10^{-2}$	$3.168 \pm 8.0 \times 10^{-2}$	$3.103 \pm 1.0 \times 10^{-1}$	$0.289 \pm 6.0 \times 10^{-2}$	$0.112 \pm 1.0 \times 10^{-3}$	$2.518 \pm 7.0 \times 10^{-2}$
5.623	$0.224 \pm 2.5 \times 10^{-3}$	$0.196 \pm 3.0 \times 10^{-2}$	$5.610 \pm 1.5 \times 10^{-1}$	$5.570 \pm 1.6 \times 10^{-1}$	$0.451 \pm 1.2 \times 10^{-1}$	$0.200 \pm 1.8 \times 10^{-3}$	$4.476 \pm 1.2 \times 10^{-1}$
10	$0.352 \pm 2.8 \times 10^{-3}$	$0.347 \pm 1.1 \times 10^{-1}$	$9.978 \pm 2.7 \times 10^{-1}$	$9.922 \pm 2.8 \times 10^{-1}$	$0.761 \pm 2.2 \times 10^{-1}$	$0.355 \pm 3.2 \times 10^{-3}$	$7.959 \pm 2.1 \times 10^{-1}$
17.783	$0.597 \pm 3.1 \times 10^{-3}$	$0.541 \pm 1.8 \times 10^{-1}$	$17.744 \pm 4.8 \times 10^{-1}$	$17.628 \pm 5.0 \times 10^{-1}$	$1.330 \pm 3.9 \times 10^{-1}$	$0.632 \pm 5.6 \times 10^{-3}$	$14.153 \pm 3.8 \times 10^{-1}$

A graphical representation of the RMSE dependence on signal-to-noise ratio is shown in Fig. 7.

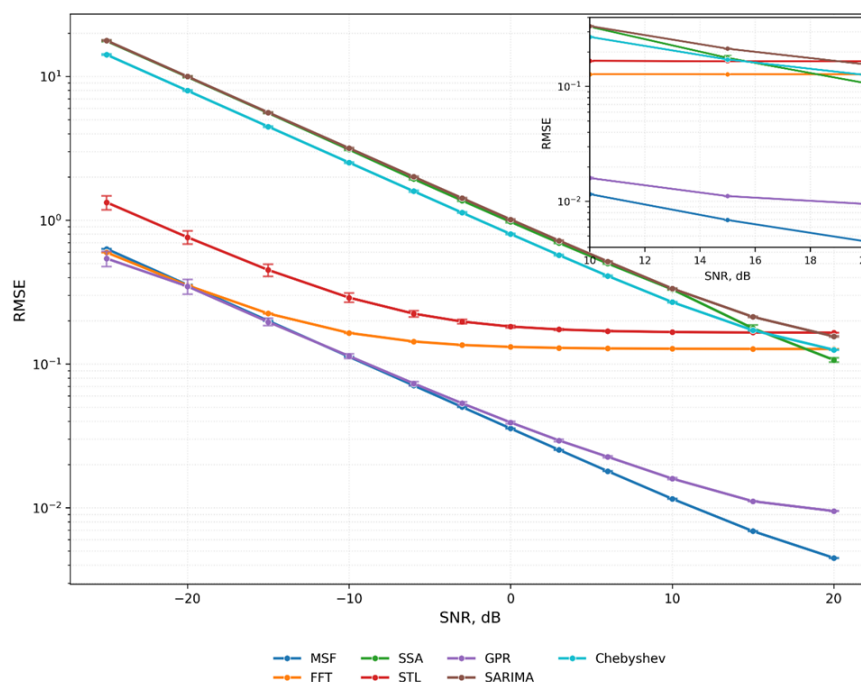


Fig. 7. Comparison of methods in terms of root mean square error (RMSE) at different noise levels

Root mean square error (Table 3, Fig. 7) reflects the absolute deviation of the reconstructed signal from the true meander and is sensitive to distortions in shape, amplitude, and phase.

In the strong signal region ($k \leq 0.3$), MSF demonstrates the lowest error (RMSE = 0.004–0.012 with $\sigma \sim 10^{-4}$ – 10^{-5}). GPR ranks second (0.009–0.016), but its error is 1.5–2 times higher. STL, FFT, SSA, and the Chebyshev filter yield errors of 0.107–0.172, which are 10–40 times higher than MSF's performance. SARIMA shows 0.156–0.335, losing to MSF by a factor of 13–28.

At $k = 1.413$ (lower bound of the real range of conditions for the Kamchatka network), the MSF error is 0.050, GPR — 0.053, FFT — 0.136, STL — 0.197, Chebyshev filter — 1.128, SARIMA and SSA — about 1.42. The difference between the best (MSF) and the other methods reaches 2–28 times.

Beyond the real range ($k \geq 5.6$), the situation changes. MSF, GPR, and FFT maintain errors at the level of 0.20–0.36, whereas STL increases to 0.45–0.76, and the Chebyshev filter, SARIMA, and SSA — to 4.5–17.8. At $k = 10$ (SNR ≈ -20 dB), the lowest errors are shown by MSF (0.355), FFT (0.352), and GPR (0.347). STL (0.761) noticeably loses to them. The remaining methods yield errors 20–30 times higher — 7.96–9.98. At $k = 17.8$, MSF, FFT, and GPR converge in error (0.60–0.54), while STL increases to 1.33.

In terms of estimate stability in the range significant for the original data ($k = 0.3$ – 3.2), MSF shows the best performance (standard deviation 10^{-4} – 10^{-3}). GPR and FFT have a scatter of 10^{-3} – 10^{-2} , which remains acceptable. STL at $k \geq 5.6$ demonstrates a sharp increase in uncertainty (σ reaches 0.39), while SARIMA and SSA are unstable already at $k \geq 1.4$ (σ on the order of 0.04–0.10).

Thus, in terms of root mean square error, the MSF method consistently leads across the entire range of noise levels. GPR and FFT prove competitive under strong noise, approaching MSF and surpassing STL. The baseline Chebyshev filter, as expected, loses to adaptive methods. STL, which maintained high correlation under weak and moderate noise, noticeably lags behind MSF in absolute reconstruction accuracy and sharply loses stability under strong noise.

The `std_ratio` metric shows how well the reconstructed signal preserves the amplitude of the original meander. The ideal value is 1.0. Values below 1.0 indicate smoothing (amplitude loss), while values above 1.0 indicate outliers or noise amplification.

A quantitative comparison of the reconstructed signals with the original meander in terms of standard deviation ratio is given in Table 4 and Fig. 8.

Table 4

Ratio of standard deviations of the reconstructed signal to the original (`std_ratio`)

k	FFT	GPR	SARIMA	SSA	STL	MSF	Cheb.
0.1	$0.989 \pm 2.8 \times 10^{-3}$	$0.999 \pm 2.9 \times 10^{-3}$	$1.008 \pm 3.1 \times 10^{-3}$	$0.997 \pm 4.4 \times 10^{-3}$	$1.001 \pm 4.1 \times 10^{-3}$	$1.001 \pm 2.7 \times 10^{-3}$	$0.886 \pm 2.7 \times 10^{-3}$
0.178	$0.989 \pm 5.0 \times 10^{-3}$	$0.999 \pm 5.1 \times 10^{-3}$	$1.029 \pm 6.0 \times 10^{-3}$	$1.014 \pm 9.4 \times 10^{-3}$	$1.002 \pm 7.4 \times 10^{-3}$	$1.001 \pm 4.8 \times 10^{-3}$	$0.901 \pm 5.0 \times 10^{-3}$
0.316	$0.990 \pm 8.8 \times 10^{-3}$	$0.999 \pm 9.0 \times 10^{-3}$	$1.089 \pm 1.0 \times 10^{-2}$	$1.073 \pm 1.0 \times 10^{-2}$	$1.003 \pm 1.0 \times 10^{-2}$	$1.002 \pm 8.5 \times 10^{-3}$	$0.945 \pm 9.5 \times 10^{-3}$
0.501	$0.991 \pm 1.0 \times 10^{-2}$	$0.998 \pm 1.0 \times 10^{-2}$	$1.207 \pm 2.0 \times 10^{-2}$	$1.188 \pm 2.0 \times 10^{-2}$	$1.005 \pm 2.0 \times 10^{-2}$	$1.004 \pm 1.0 \times 10^{-2}$	$1.035 \pm 2.0 \times 10^{-2}$
0.708	$0.993 \pm 2.0 \times 10^{-2}$	$0.997 \pm 2.0 \times 10^{-2}$	$1.387 \pm 3.0 \times 10^{-2}$	$1.358 \pm 3.0 \times 10^{-2}$	$1.008 \pm 3.0 \times 10^{-2}$	$1.006 \pm 2.0 \times 10^{-2}$	$1.168 \pm 2.0 \times 10^{-2}$
1	$0.995 \pm 3.0 \times 10^{-2}$	$0.996 \pm 3.0 \times 10^{-2}$	$1.689 \pm 4.0 \times 10^{-2}$	$1.644 \pm 5.0 \times 10^{-2}$	$1.013 \pm 4.0 \times 10^{-2}$	$1.008 \pm 3.0 \times 10^{-2}$	$1.395 \pm 3.0 \times 10^{-2}$
1.413	$0.999 \pm 4.0 \times 10^{-2}$	$0.994 \pm 4.0 \times 10^{-2}$	$2.168 \pm 6.0 \times 10^{-2}$	$2.101 \pm 7.0 \times 10^{-2}$	$1.020 \pm 6.0 \times 10^{-2}$	$1.012 \pm 4.0 \times 10^{-2}$	$1.763 \pm 5.0 \times 10^{-2}$
1.995	$1.004 \pm 6.0 \times 10^{-2}$	$0.990 \pm 6.0 \times 10^{-2}$	$2.892 \pm 8.0 \times 10^{-2}$	$2.796 \pm 1.0 \times 10^{-1}$	$1.034 \pm 9.0 \times 10^{-2}$	$1.018 \pm 5.0 \times 10^{-2}$	$2.328 \pm 7.0 \times 10^{-2}$
3.162	$1.016 \pm 9.0 \times 10^{-2}$	$0.979 \pm 9.0 \times 10^{-2}$	$4.406 \pm 1.2 \times 10^{-1}$	$4.271 \pm 1.4 \times 10^{-1}$	$1.069 \pm 1.4 \times 10^{-1}$	$1.031 \pm 8.0 \times 10^{-2}$	$3.523 \pm 1.1 \times 10^{-1}$
5.623	$1.045 \pm 1.5 \times 10^{-1}$	$0.944 \pm 1.7 \times 10^{-1}$	$7.663 \pm 2.3 \times 10^{-1}$	$7.535 \pm 2.4 \times 10^{-1}$	$1.175 \pm 2.4 \times 10^{-1}$	$1.066 \pm 1.5 \times 10^{-1}$	$6.126 \pm 1.8 \times 10^{-1}$
10	$1.114 \pm 2.7 \times 10^{-1}$	$0.831 \pm 3.4 \times 10^{-1}$	$13.544 \pm 3.9 \times 10^{-1}$	$13.405 \pm 4.1 \times 10^{-1}$	$1.451 \pm 3.9 \times 10^{-1}$	$1.149 \pm 2.5 \times 10^{-1}$	$10.811 \pm 3.2 \times 10^{-1}$
17.783	$1.288 \pm 4.4 \times 10^{-1}$	$0.598 \pm 5.7 \times 10^{-1}$	$24.036 \pm 6.9 \times 10^{-1}$	$23.838 \pm 7.3 \times 10^{-1}$	$2.088 \pm 6.3 \times 10^{-1}$	$1.361 \pm 3.9 \times 10^{-1}$	$19.177 \pm 5.6 \times 10^{-1}$

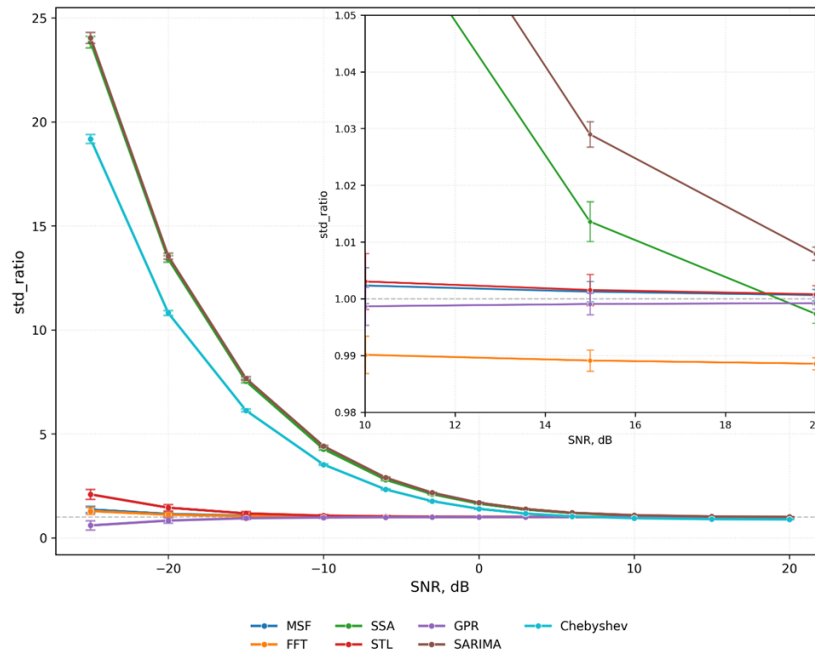


Fig. 8. Standard deviation ratio of the reconstructed seasonal component as a function of the noise level

Ratio of standard deviations of the reconstructed signal to the true one (Table 4, Fig. 8) shows how accurately the method preserves the amplitude of the original meander. The ideal value is 1.0. Values above 1 indicate amplitude overestimation, while values below 1 indicate underestimation.

In the strong signal region ($k \leq 0.3$), all methods except the Chebyshev filter demonstrate std_ratio close to 1 (0.989–1.029). MSF, STL, GPR, and FFT are nearly ideal: their values lie in the range 0.989–1.003 with minimal scatter ($\sigma \approx 10^{-3}$ – 10^{-2}). Chebyshev gives amplitude underestimation (0.886–0.945). This is explained not by noise influence but by the filter design: the passband is limited to frequencies above the third harmonic and below the fifth. The fifth and higher harmonics of the smoothed meander are always suppressed, even at $k \rightarrow 0$, which inevitably leads to loss of part of the signal energy.

At $k = 1.4$ (lower bound of the real range), MSF preserves almost exact amplitude (1.012). STL and GPR are also close to 1 (1.020 and 0.994). FFT slightly underestimates (0.999), while SARIMA and SSA already noticeably overestimate (2.168 and 2.101). The Chebyshev filter gives strong overestimation (1.763), indicating loss of stability.

Beyond the real range ($k \geq 5.6$), the scatter increases sharply. MSF remains the most stable: at $k = 10$ its std_ratio = 1.149, at $k = 17.8$ – 1.361. STL overestimates amplitude moderately (1.451 and 2.088). FFT is also acceptable (1.114–1.288). GPR at $k = 10$ already underestimates (0.831), and at $k = 17.8$ drops to 0.598 with catastrophic scatter ($\sigma = 0.57$). SARIMA and SSA at $k \geq 5.6$ give huge overestimation (7.5–24.0), making them unsuitable for amplitude extraction. The Chebyshev filter also overestimates by a factor of 10–19.

In terms of stability in the range significant for the original data ($k = 0.3\text{--}3.2$), MSF, STL, and FFT show the best performance (standard deviation on the order of 10^{-2}). GPR and the Chebyshev filter have somewhat larger scatter but remain acceptable. SARIMA and SSA already at $k \geq 1.4$ demonstrate strong instability (σ reaches 0.07–0.10), and at $k \geq 10$ the scatter exceeds 0.4.

Thus, in terms of amplitude preservation, MSF confidently leads across the entire range of noise levels. STL and FFT also show good results, lagging behind MSF only in the strong noise region. GPR, despite good behavior under weak and moderate noise, loses stability at $k \geq 10$. SARIMA and SSA are not recommended for tasks where accurate seasonal component amplitude is important, already at $k \geq 1.4$. The baseline Chebyshev filter, as expected, underestimates amplitude under weak noise (due to passband limitation) and sharply overestimates under strong noise.

The Q metric (Table 5, Fig. 9) characterizes the degree of suppression of the seasonal component in the residuals.

Table 5

Ratio of the power spectral density of the residual to the power spectral density of the original noisy series at frequency $1/365$

k	FFT	GPR	SARIMA	SSA	STL	MSF	Cheb.
0.1	$0.003 \pm 2.7 \times 10^{-3}$	$0.002 \pm 2.3 \times 10^{-3}$	$0.378 \pm 1.5 \times 10^{-1}$	$0.004 \pm 3.9 \times 10^{-3}$	$0.006 \pm 6.0 \times 10^{-3}$	$0.005 \pm 5.7 \times 10^{-3}$	$0.345 \pm 1.6 \times 10^{-1}$
0.178	$0.004 \pm 4.4 \times 10^{-3}$	$0.002 \pm 2.4 \times 10^{-3}$	$0.190 \pm 1.4 \times 10^{-1}$	$0.004 \pm 4.5 \times 10^{-3}$	$0.006 \pm 6.2 \times 10^{-3}$	$0.005 \pm 6.0 \times 10^{-3}$	$0.163 \pm 1.3 \times 10^{-1}$
0.316	$0.005 \pm 6.0 \times 10^{-3}$	$0.003 \pm 2.7 \times 10^{-3}$	$0.091 \pm 9.0 \times 10^{-2}$	$0.005 \pm 7.1 \times 10^{-3}$	$0.007 \pm 6.9 \times 10^{-3}$	$0.006 \pm 6.6 \times 10^{-3}$	$0.074 \pm 8.0 \times 10^{-2}$
0.501	$0.007 \pm 7.8 \times 10^{-3}$	$0.003 \pm 3.3 \times 10^{-3}$	$0.047 \pm 6.0 \times 10^{-2}$	$0.007 \pm 9.9 \times 10^{-3}$	$0.008 \pm 8.4 \times 10^{-3}$	$0.007 \pm 8.1 \times 10^{-3}$	$0.044 \pm 6.0 \times 10^{-2}$
0.708	$0.009 \pm 1.0 \times 10^{-2}$	$0.004 \pm 4.2 \times 10^{-3}$	$0.036 \pm 5.0 \times 10^{-2}$	$0.009 \pm 1.0 \times 10^{-2}$	$0.011 \pm 1.0 \times 10^{-2}$	$0.009 \pm 1.0 \times 10^{-2}$	$0.034 \pm 5.0 \times 10^{-2}$
1	$0.013 \pm 1.0 \times 10^{-2}$	$0.006 \pm 6.1 \times 10^{-3}$	$0.032 \pm 5.0 \times 10^{-2}$	$0.013 \pm 2.0 \times 10^{-2}$	$0.016 \pm 2.0 \times 10^{-2}$	$0.013 \pm 2.0 \times 10^{-2}$	$0.032 \pm 5.0 \times 10^{-2}$
1.413	$0.022 \pm 2.0 \times 10^{-2}$	$0.010 \pm 9.7 \times 10^{-3}$	$0.035 \pm 6.0 \times 10^{-2}$	$0.022 \pm 3.0 \times 10^{-2}$	$0.026 \pm 3.0 \times 10^{-2}$	$0.022 \pm 2.0 \times 10^{-2}$	$0.036 \pm 5.0 \times 10^{-2}$
1.995	$0.038 \pm 4.0 \times 10^{-2}$	$0.018 \pm 2.0 \times 10^{-2}$	$0.046 \pm 8.0 \times 10^{-2}$	$0.039 \pm 6.0 \times 10^{-2}$	$0.047 \pm 5.0 \times 10^{-2}$	$0.038 \pm 4.0 \times 10^{-2}$	$0.049 \pm 7.0 \times 10^{-2}$
3.162	$0.086 \pm 9.0 \times 10^{-2}$	$0.041 \pm 4.0 \times 10^{-2}$	$0.086 \pm 1.4 \times 10^{-1}$	$0.134 \pm 2.6 \times 10^{-1}$	$0.107 \pm 1.1 \times 10^{-1}$	$0.086 \pm 9.0 \times 10^{-2}$	$0.089 \pm 1.2 \times 10^{-1}$
5.623	$0.239 \pm 2.5 \times 10^{-1}$	$0.112 \pm 1.0 \times 10^{-1}$	$0.235 \pm 4.1 \times 10^{-1}$	$0.395 \pm 6.5 \times 10^{-1}$	$0.307 \pm 3.4 \times 10^{-1}$	$0.239 \pm 2.5 \times 10^{-1}$	$0.225 \pm 3.3 \times 10^{-1}$
10	$0.587 \pm 5.8 \times 10^{-1}$	$0.289 \pm 2.5 \times 10^{-1}$	0.639 ± 1.33	0.717 ± 1.13	0.841 ± 1.15	$0.585 \pm 5.8 \times 10^{-1}$	0.569 ± 1.02
17.783	1.465 ± 2.55	$0.620 \pm 3.6 \times 10^{-1}$	1.813 ± 4.98	1.338 ± 2.87	2.357 ± 4.80	1.450 ± 2.49	1.519 ± 3.75

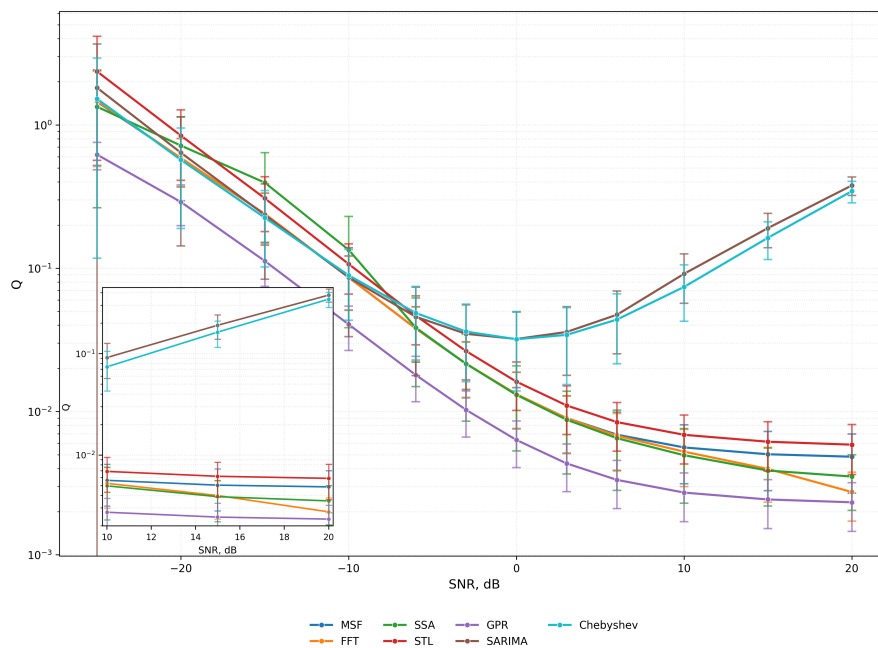


Fig. 9. The Q metric

Normalization by the power spectral density of the original noise allows correct comparison of results at different SNR levels. Values $Q \leq 0.05$ are considered excellent, $0.05 < Q \leq 0.2$ — satisfactory, and $Q > 0.5$ — unsatisfactory.

In the operating range $k = 0.3$ – 1.4 , all methods except SARIMA and the Chebyshev filter under weak noise demonstrate Q values ≤ 0.05 . At $k = 0.316$, Q is 0.005 – 0.007 for FFT, GPR, SSA, STL, and MSF; for the Chebyshev filter — 0.074 ; for SARIMA — 0.091 . At $k = 0.708$, values decrease to 0.008 – 0.011 for the same group, to 0.034 for the Chebyshev filter, and to 0.036 for SARIMA. At $k = 1.4$, Q practically equalizes: 0.020 – 0.026 for all methods, including the Chebyshev filter (0.036) and SARIMA (0.035). Thus, in the operating range, FFT, GPR, SSA, STL, MSF, and the Chebyshev filter almost completely remove the periodic component from the residual.

Under weak noise ($k < 0.3$), all methods except SARIMA and the Chebyshev filter yield $Q < 0.01$. SARIMA gives a significantly higher value — 0.378 at $k = 0.1$, and the Chebyshev filter — 0.345 , which is almost two orders of magnitude higher than for the others. At $k = 0.178$, SARIMA and the Chebyshev filter decrease to 0.190 and 0.163 , respectively, but still remain an order of magnitude higher than the other methods.

Under strong noise ($k > 1.4$), Q increases for all methods: at $k = 5.6$ values reach 0.23 – 0.40 , at $k = 10$ — 0.57 – 0.84 , at $k = 17.8$ — 1.34 – 2.36 . The standard deviation becomes comparable to the Q value itself (σ reaches 2.5 – 5.0), which limits interpretation of the metric in this region. An exception is GPR, for which at $k = 17.8$, $Q = 0.62 \pm 0.36$ — the scatter remains moderate.

In terms of noise robustness in the operating range, the smallest scatter is demonstrated by FFT, MSF, and GPR ($\sigma = 0.006$ – 0.01). The largest scatter is shown by SARIMA ($\sigma = 0.09$ – 0.15) and the Chebyshev filter under weak noise ($\sigma = 0.13$ – 0.16), which further confirms their instability in this region. Under strong noise ($k \geq 10$), the scatter of all methods increases sharply, making the Q metric of little informative value.

Thus, under conditions corresponding to real geophysical data ($k = 0.3$ – 1.4), all methods, including the baseline Chebyshev filter and GPR, effectively remove the seasonal component from the residual. SARIMA and the Chebyshev filter under weak noise give anomalously high Q values, which appears to be associated with distortions at the edges of the series. Under strong noise, interpretation of Q is complicated due to high variance; however, GPR maintains relative stability.

5 Discussion

Real tiltmeter observations in Kamchatka are characterized by a signal-to-noise ratio close to 0 dB (equal powers of the seasonal component and the residual). In this work, the level $k = 1$ (0 dB) was chosen as the reference, and the real range was defined as $k = 0.3$ – 1.4 (from -10 to $+3$ dB). Based on the analysis of four metrics (RMSE, correlation, std_ratio, Q) and estimate stability in this range, the following ranking of methods was obtained.

MSF confidently takes first place. Under conditions of a strictly periodic signal, averaging over periods introduces no waveform distortions and maximally suppresses

noise. MSF results are minimal in RMSE (0.012–0.050), maximal in correlation (0.999–1.000), practically ideal in `std_ratio` (1.002–1.012) and `Q` (0.006–0.022), and the scatter of estimates is the smallest (10^{-4} – 10^{-3}). The weak point is inapplicability under non-stationary seasonality or unknown period; however, within the framework of this problem, these limitations do not manifest.

GPR ranks second. Under weak and moderate noise ($k \leq 3.2$), it demonstrates high correlation (0.999–1.000), low `Q` (≤ 0.01), and low RMSE (0.009–0.114). This is explained by the flexibility of nonparametric approximation and the ability to handle missing data. However, at $k \geq 5.6$, GPR loses stability: the Gaussian kernel poorly approximates the steep edges of the meander, and the Gaussian assumption for flicker noise is only approximately valid. As a result, `std_ratio` drops to 0.6–0.8, and the scatter increases sharply (σ up to 0.57). Disadvantages include labor-intensive tuning and high computational cost.

FFT takes third place due to stability across the entire noise range. The spectrum of the smoothed meander is concentrated in a few odd harmonics, which are easily identifiable even at low SNR. At $k = 10$, FFT RMSE (0.352) is practically the same as MSF RMSE (0.355), `std_ratio` remains acceptable (1.114), and at $k = 17.8$, RMSE = 0.597, correlation = 0.81. Weaknesses: with an unknown or drifting period, accuracy decreases; at extremely low SNR, random phase shifts are possible. Simplicity of implementation and the absence of complex tuning make FFT a reasonable compromise, but it assumes precise knowledge of the period.

STL ranks fourth. Under weak noise, it maintains high correlation (0.996–1.000) and low `Q` (0.007–0.026). However, LOESS smoothing is not designed for signals with steep edges: local polynomials smooth transitions, underestimating amplitude and distorting shape. Even at high SNR, STL RMSE (0.165–0.197) is 10–40 times higher than that of MSF. In the real range, `std_ratio` is overestimated already at $k > 3.2$, and at $k > 10$ the scatter becomes unacceptable (σ up to 0.4–0.6). STL is not recommended for signals with steep edges and strong noise.

SARIMA and SSA share fifth–sixth places, showing the worst results. SARIMA assumes linear dynamics with short-term memory, whereas flicker noise with $\alpha = 2.9$ has long-term memory and is non-stationary. Aggregation of data to a 10-day step reduced resolution and smoothed the edges, and the moving average component leads to phase lag. SSA is oriented toward extracting smooth modes, and a meander with steep edges cannot be decomposed into singular vectors without loss of quality. Already at $k = 1.4$, `std_ratio` for both methods exceeds 2.1, and RMSE — 1.4. At $k > 5.6$, the error becomes unacceptably high (RMSE > 7.5 , `std_ratio` > 7.5), and the scatter of estimates at $k > 10$ exceeds 0.4. These methods are unsuitable for seasonal extraction under strong flicker noise and steep edges.

The Chebyshev filter ranks last, which is expected for a baseline non-adaptive method. Under weak noise, its passband is limited to frequencies between the third and fifth harmonics, so the fifth and higher harmonics are always suppressed, even at k near 0, leading to amplitude underestimation (`std_ratio` = 0.886–0.945). In the real range, RMSE reaches 1.128, correlation drops to 0.505, and `std_ratio` = 1.763. Under

strong noise, the filter completely loses stability ($k = 10$: RMSE = 7.96, std_ratio = 10.81). Non-adaptivity and fixed passband make it unsuitable for variable noise levels, although as a simple baseline it fulfills its role.

Thus, under conditions of a strictly periodic signal, known period, and flicker noise with $\alpha = 2.9$, MSF is the undisputed leader. GPR and FFT can serve as alternatives, but they are inferior to MSF in stability or accuracy. STL, SARIMA, and SSA are not recommended, and the Chebyshev filter confirms its status as a baseline method.

For quantitative comparison, a set of four metrics (RMSE, correlation, std_ratio, Q) was used, covering different aspects of quality. Their complementarity allows identifying both shape errors, amplitude distortions, and residual periodicity. Limitations of the set are related to the absence of a single integrated metric and possible contradictions between indicators. For example, at $k = 1.4$, MSF RMSE = 0.05, while FFT RMSE = 0.136, but Corr for both is high (0.998 and 0.984) — the shape is well reconstructed, but the absolute error for FFT is higher, which is not reflected in correlation. No single metric by itself provides a complete objective assessment.

The Q metric, developed specifically for this work, should be noted separately. It is normalized by the power spectral density of the original noise, which allows correct comparison at different SNR levels. Low Q values (≤ 0.05) confirm that the method has indeed removed seasonality rather than merely distorted it. In the operating range $k = 0.3$ – 1.4 , all methods except SARIMA and the Chebyshev filter under weak noise yield $Q \leq 0.05$. Under strong noise ($k \geq 10$), the scatter of Q becomes comparable to the mean value (σ up to 0.6–2.5), which limits interpretation of the metric, but in the real range ($k \leq 3.2$), Q remains informative and reliable. The behavior of the Q metric under small phase shifts was not considered in this work and requires separate investigation; therefore, the proposed metric should not be regarded as universal for problems with non-stationary seasonality.

Statistical significance of differences between methods is confirmed by the fact that the confidence intervals of each metric for different methods at $k > 0.3$ do not overlap in most cases. The sample size (30 realizations) is sufficient to suppress random fluctuations. Thus, the set of metrics used allows objective comparison of methods and substantiated conclusions.

Wavelet transform methods, which are also used for seasonal component extraction, were not considered in this work. Their use requires the choice of a mother wavelet — a parameter determining the shape of the basis function, which introduces an element of subjectivity and requires additional justification. In the case of tiltmeter observations, under certain conditions, the auxiliary temperature channel, which is typically used in such measurements, could be used as such a wavelet. However, this radically changes the problem statement: wavelet methods receive a priori information about the signal shape, while the other approaches work only with the original series. In real geophysical data processing practice, additional channels are not always available, and the problem of seasonal extraction is often solved precisely from a single time series. Therefore, wavelet methods are not included in the comparison.

6 Conclusions

This work presents a systematic comparison of seven approaches to seasonal component extraction (MSF, FFT, GPR, STL, SARIMA, SSA, Chebyshev filter) on model data simulating real tiltmeter observations in Kamchatka. The main features of the experiment were: the seasonal signal shape in the form of a smoothed meander, flicker noise with exponent $\alpha = 2.9$, absence of trend, known period (365 days), and variation of the signal-to-noise ratio over the range from -25 to $+20$ dB. The analysis was performed using four metrics (RMSE, correlation, std_ratio, Q) based on 30 realizations for each of the 12 noise levels.

Across all metrics and over the entire SNR range, the mean seasonal function (MSF) method showed the best results, combining high accuracy, noise robustness, and minimal computational cost. Its effectiveness is due to the strict periodicity of the signal, known period, and absence of trend — precisely the conditions embedded in the model. Among the adaptive methods, only Gaussian process regression (GPR) approached MSF in quality; however, under strong noise ($k \geq 5.6$), its stability dropped sharply. The STL, SARIMA, and SSA methods, as well as the Chebyshev filter, demonstrated systematic distortions (amplitude underestimation, phase lag) or extremely high error growth, making them unsuitable for seasonal extraction from signals with steep edges under flicker noise conditions.

The residual periodicity metric Q developed in this work allowed quantitative assessment of the degree of suppression of the seasonal component in the residual. In the real SNR range ($k \leq 3.2$), it reliably complements the standard metrics; however, under strong noise ($k \geq 10$), its variance becomes too large for meaningful interpretation. The lowest computational time was shown by MSF and FFT (0.01–0.1 s per series), while GPR and SARIMA required orders of magnitude more time (up to several minutes per series), which is critical for processing large volumes of monitoring data.

The conclusions of this work are strictly limited to the experimental conditions: smoothed meander, known period, absence of trend, fixed flicker noise exponent $\alpha = 2.9$, and series length of 10 periods. For signals of different shapes, other noise types, or the presence of trend, the ranking of methods may change. For practical processing of Kamchatka tiltmeter data (smoothed meander, $\alpha = 2.9$, SNR around 0 dB), the MSF method is optimal. GPR and FFT can be considered as alternatives under special requirements (uncertainty estimation or implementation simplicity), but with due regard to their limitations. STL, SARIMA, SSA, and the Chebyshev filter are not recommended for standalone seasonal extraction.

All computations and plotting were performed using the Python programming language with the NumPy and SciPy libraries (filtering, spectral analysis, noise generation), Pandas (data handling), Matplotlib (visualization), Statsmodels (SARIMA, STL), and Scikit-learn (GPR). The WinABD package (developed at the Schmidt Institute of Physics of the Earth, Russian Academy of Sciences) [23] was also used.

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