MSC 35J15 Research Article

On one boundary value problem for the fourth-order equation in partial derivatives

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The initial-boundary problem for the heat conduction equation inside a bounded domain is considered. It is supposed that on the boundary of this domain the heat exchange takes place according to Newton's law. The control parameter is equal to the magnitude of output of hot air and is defined on a givenmpart of the boundary. Then we determined the dependence $T(\theta)$ on the parameters of the temperature process when θ is close to critical value.

Key words: boundary value problem; Fourier method; the existence of a solution; the uniqueness of a solution.

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Introduction and statement of the Problem

A problem with a high derivative on a part of the domain boundary was studied, for the first time, by A.N. Tikhonov. In [1], he studied the problem of a homogeneous heat equation with the following conditions

$$\sum_{k=0}^{\infty} a_k \frac{\partial^k u}{\partial x^k}(0,t) = f(t), \; u(x,0) = 0,$$

in domain $(0 < x < \infty, \; t > 0)$.

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In [2], in $n-$dimensional bounded domain D A.V. Bitsadze studied the problem

$$\triangle u(x) = 0,\ \frac{d^m u}{d\nu^m} = f(x),\ x \in D,$$

and proved its Fredholm property.

Boundary value problems with boundary conditions, for the Laplace, Poisson and Helmholtz equations in the unit ball, containing high-order derivatives were studied by I.I. Bavrin [3], V.V. Karachik and B.Kh. Turmetov [4], V.V. Karachik [5]-[7], V.B. Sokolovsky [8]. A mixed problem with a high derivative in the initial condition for the heat equation was studied by D. Amanov [9]; for the equation of string vibration, the mixed problem with high derivatives in the initial conditions was studied in [10]-[16]. Authors of [17], [18]-[19] studied boundary value problems for many types of high-order partial differential equations. Mixed problems for fourth-order differential equations were studied in [20]. The difference of this problem from other problems is that in the initial condition the sum of products of higher orders is set.

Consider the following equation

$$\frac{\partial u}{\partial t} + \frac{\partial^4 u}{\partial x^4} = f(x,t) \tag{1}$$

in domain $\Omega = \{(x,t) : 0 < x < p, 0 < t < T\}$, and the boundary value problem with boundary conditions

$$u(0,t) = u(p,t) = 0,\ \ 0 \le t \le T, \tag{2}$$

$$u_{xx}(0,t) = u_{xx}(p,t) = 0,\ \ 0 \le t \le T, \tag{3}$$

and initial conditions

$$\sum_{k=0}^{m} \frac{\partial^k u}{\partial t^k}\bigg|_{t=0} = \varphi(x),\ \ 0 \le x \le p, \tag{4}$$

where $k, m-$ are the fixed natural numbers, $f(x,t)$, $\varphi(x)-$ are the given functions, continuous in $\overline{\Omega}$, and $u(x,t) \in C^{4,k}_{x,t}(\overline{\Omega})-$ is the sought-for function.

Uniqueness of the solution to the problem

Theorem 1. *The solution to problem* (1)-(4) *is unique if it exists.*

Proof. Let $f(x,t) = 0$, $\varphi(x) = 0$ in $\overline{\Omega}$. We show that $u(x,t) = 0$ in $\overline{\Omega}$. Following [21], we consider the following integral

$$\alpha_n(t) = \int_0^p u(x,t) X_n(x)\,dx,\ \ 0 \le t \le T \tag{5}$$

where functions

$$X_n(x) = \sqrt{\frac{2}{p}} \sin\lambda_n x,\ \lambda_n = \frac{n\pi}{p},\ n = 1,2,... \tag{6}$$

form a complete orthonormal system in $L_2(0,p)$ [22]. Differentiating (5) over t, from homogeneous equation $u_t+u_{xxxx}=0$ we find

$$\alpha_n^{'}(t)=\int\limits_0^p u_t(x,t)X_n(x)dx$$

or

$$\alpha_n^{'}(t)=\int\limits_0^p \frac{\partial^4 u}{\partial x^4}(x,t)X_n(x)dx. \quad (7)$$

Integrating four times by parts over x in the integral (7) on the right side, we obtain the following equations:

$$\alpha_n^{'}(t)+\lambda_n^4\cdot\alpha_n(t)=0. \quad (8)$$

The general solution to equation (8) is written as

$$\alpha_n(t)=C_n\cdot exp(-\lambda_n^4 t). \quad (9)$$

To find the unknown coefficient C_n , due to conditions (4), we obtain $C_n=0$. Then from (9) it follows that $\alpha_n(t)=0$. Since $X_n(x)$ is the complete orthonormal system in $L_2(0,p)$, it follows from the completeness of function $X_n(x)$ that $u(x,t)=0$ almost everywhere in $\overline{\Omega}$. Considering that $u(x,t)\in C_{x,t}^{4,1}(\overline{\Omega})$, we obtain $u(x,t)\equiv 0$ in $\overline{\Omega}$. Theorem 1 is proved. □

Existence of a solution to the problem

Solutions of the inhomogeneous differential equation (1) are sought in the form of a Fourier series in sines

$$u(x,t)=\sqrt{\frac{2}{p}}\sum_{n=1}^{\infty}u_n(t)\sin\lambda_n x. \quad (10)$$

Let the functions $f(x,t)$, $\varphi(x)$ be expanded in a Fourier series

$$f(x,t)=\sqrt{\frac{2}{p}}\sum_{n=1}^{\infty}f_n(t)\sin\lambda_n x, \quad (11)$$

$$\varphi(x)=\sqrt{\frac{2}{p}}\sum_{n=1}^{\infty}\varphi_n\sin\lambda_n x, \quad (12)$$

where

$$f_n(t)=\sqrt{\frac{2}{p}}\int\limits_0^p f(x,t)\sin\lambda_n x\,dx, \quad (13)$$

$$\varphi_n = \sqrt{\frac{2}{p}} \int\limits_0^p \varphi(x) \sin \lambda_n x \, dx. \tag{14}$$

Substituting the Fourier series (10) and (11) into the given inhomogeneous differential equation (1), we obtain a first-order ordinary differential equation

$$u_n^{'}(t) + \lambda_n^4 \cdot u_n(t) = f_n(t).$$

Its solution satisfies the condition $\sum\limits_{k=0}^{m} u_n^{(k)}(0) = \varphi_n$ and has the following form

$$u_n(t) = w_n(t) + \int\limits_0^t f_n(\tau) e^{-\lambda_n^4 (t-\tau)} d\tau. \tag{15}$$

Here and hereinafter $\sum\limits_{s=0}^{m} (...) = 0$ for $m < 0$,

$$w_n(t) = \frac{e^{-\lambda_n^4 t}}{\sum\limits_{s=0}^{m} \left(-\lambda_n^4\right)^s} \left[\varphi_n - \sum_{s=0}^{m-1} f_n^{(m-1-s)}(0) \left(\sum_{i=0}^{s} \left(-\lambda_n^4\right)^i \right) \right].$$

Substituting solution (15) into series (10), we find

$$u(x,t) = \sqrt{\frac{2}{p}} \sum_{n=1}^{\infty} \left[w_n(t) + \int\limits_0^t f_n(\tau) e^{-\lambda_n^4 (t-\tau)} d\tau \right] \sin \lambda_n x. \tag{16}$$

Calculating the products of the solution of equation (1), we obtain

$$\frac{\partial u}{\partial t} = \sqrt{\frac{2}{p}} \sum_{n=1}^{\infty} \lambda_n^4 \left[-w_n(t) + \lambda_n^{-4} f_n(t) - \int\limits_0^t f_n(\tau) e^{-\lambda_n^4 (t-\tau)} d\tau \right] \sin \lambda_n x, \tag{17}$$

$$\frac{\partial^4 u}{\partial x^4} = \sqrt{\frac{2}{p}} \sum_{n=1}^{\infty} \lambda_n^4 \left[w_n(t) + \int\limits_0^t f_n(\tau) e^{-\lambda_n^4 (t-\tau)} d\tau \right] \sin \lambda_n x, \tag{18}$$

$$\frac{\partial^k u}{\partial t^k} = \sqrt{\frac{2}{p}} \sum_{n=1}^{\infty} \left(-\lambda_n^4\right)^k \left[w_n(t) + \sum_{s=0}^{k-1} \left(-\lambda_n^4\right)^{s-k} f_n^{(k-1-s)}(t) + \int\limits_0^t f_n(\tau) e^{-\lambda_n^4 (t-\tau)} d\tau \right] \cdot$$
$$\cdot \sin \lambda_n x. \tag{19}$$

Let us show that series (16) and (17)-(19) converge absolutely and uniformly. If the series (19) converges then the series (16)-(18) whose terms are less than the corresponding terms of the series (19) converge absolutely and uniformly. Let us show the absolute and uniform convergence of series (19).

Lemma 1. *If* $\varphi(x) \in W_2^1(\Omega)$, $\varphi(0) = \varphi(p) = 0$, *then the series*

$$\sqrt{\frac{2}{p}} \sum_{n=1}^{\infty} \sin \lambda_n x \cdot \frac{\left(-\lambda_n^4\right)^k \cdot \varphi_n \cdot e^{-\lambda_n^4 t}}{\sum\limits_{s=0}^{m} \left(-\lambda_n^4\right)^s} \tag{20}$$

converges absolutely and uniformly in $\overline{\Omega}$.

Proof. Simplifying series (20), we obtain the following series

$$\sum_{n=1}^{\infty} \left| \sin \lambda_n x \cdot \frac{\left(-\lambda_n^4\right)^k \cdot \varphi_n \cdot e^{-\lambda_n^4 t}}{\sum\limits_{s=0}^{m} \left(-\lambda_n^4\right)^s} \right| \leq \sum_{n=1}^{\infty} \left| \frac{\left(-\lambda_n^4\right)^{k+1} - \left(-\lambda_n^4\right)^k}{\left(-\lambda_n^4\right)^{m+1} - 1} \cdot \varphi_n \right|.$$

If we choose the largest value of k ($k \leq m$), we have

$$\sum_{n=1}^{\infty} \left| \frac{\left(-\lambda_n^4\right)^{m+1} - \left(-\lambda_n^4\right)^m}{\left(-\lambda_n^4\right)^{m+1} - 1} \cdot \varphi_n \right| \leq \sum_{n=1}^{\infty} |\varphi_n|.$$

Integrating (14) by parts, we obtain $|\varphi_n| = \frac{1}{\lambda_n} \cdot \left|\varphi_n^{'}\right|$, where

$$\varphi_n^{(1)} = \int_0^p \varphi^{'}(x) \sqrt{\frac{2}{p}} \cos \lambda_n x \, dx.$$

Applying the Cauchy-Bunyakovsky inequality, we find

$$\sum_{n=1}^{\infty} |\varphi_n| = \sum_{n=1}^{\infty} \frac{1}{\lambda_n} \left|\varphi_n^{(1)}\right| \leq \frac{p}{\pi} \left(\sum_{n=1}^{\infty} \frac{1}{n^2} \right)^{1/2} \left(\sum_{n=1}^{\infty} \left|\varphi_n^{(1)}\right|^2 \right)^{1/2} \leq \frac{p}{\sqrt{6}} \left(\sum_{n=1}^{\infty} \left|\varphi_n^{(1)}\right|^2 \right)^{1/2}.$$

By virtue of $\left|\varphi_n^{(1)}\right| \leq \left\|\varphi_n^{'}\right\|_{L_2(0,p)}^2$, we have

$$\sqrt{\frac{2}{p}} \sum_{n=1}^{\infty} \sin \lambda_n x \cdot \frac{\left(-\lambda_n^4\right)^k \cdot \varphi_n \cdot e^{-\lambda_n^4 t}}{\sum\limits_{s=0}^{m} \left(-\lambda_n^4\right)^s} \leq \frac{p}{\sqrt{6}} \left\|\varphi_n^{'}\right\|_{L_2(0,p)}.$$

Lemma 1 is proved. □

Lemma 2. *If* $f(x,t) \in W_2^{(4k-3,k-1)}(\Omega)$, $\frac{\partial^{2l} f(0,t)}{\partial x^{2l}} = \frac{\partial^{2l} f(p,t)}{\partial x^{2l}} = 0$, $l = \overline{0,(2k-2)}$, *then the series*

$$\sqrt{\frac{2}{p}} \sum_{n=1}^{\infty} \sin \lambda_n x \cdot \frac{\left(-\lambda_n^4\right)^k \cdot e^{-\lambda_n^4 t}}{\sum\limits_{s=0}^{m} \left(-\lambda_n^4\right)^s} \sum_{s=0}^{m-1} f_n^{(m-1-s)}(0) \cdot \left(\sum_{i=0}^{s} \left(-\lambda_n^4\right)^i \right), \tag{21}$$

$$\sqrt{\frac{2}{p}} \sum_{n=1}^{\infty} \sin \lambda_n x \cdot \sum_{s=0}^{k-1} \left(-\lambda_n^4\right)^s f_n^{(k-1-s)}(t) \tag{22}$$

converge absolutely and uniformly in $\overline{\Omega}$.

Proof. Let us expand the series (21) and (22); it is easy to see that the term

$$\sum_{n=1}^{\infty} \lambda_n^{4k-4} |f_n(t)| \tag{23}$$

is the largest of the two series.

If series (23) converges, then series (21) and (22) also converge. Integrating (13) by parts, we obtain

$$|f_n(t)| = \frac{1}{\lambda_n^{4k-3}} \left| f_n^{(4k-3,0)}(t) \right| \tag{24}$$

where

$$f_n^{(4k-3,0)}(t) = \int_0^p \frac{\partial^{4k-3} f(x,t)}{\partial x^{4k-3}} \sqrt{\frac{2}{p}} \sin\left((4k-3)\frac{\pi}{2} + \lambda_n x \right) dx.$$

If to apply (24) to (23), due to the Bunyakovsky and Bessel inequalities [23], the series is

$$\sqrt{\frac{2}{p}} \sum_{n=1}^{\infty} \sin \lambda_n x \sum_{s=0}^{k-1} \left(-\lambda_n^4\right)^s f_n^{(k-1-s)}(t) \le \frac{p}{\sqrt{6}} \left\| \frac{\partial^{4k-3} f}{\partial x^{4k-3}} \right\|_{L_2(\Omega)},$$

$$\sqrt{\frac{2}{p}} \sum_{n=1}^{\infty} \sin \lambda_n x \frac{\left(-\lambda_n^4\right)^k e^{-\lambda_n^4 t}}{\sum\limits_{s=0}^{m} \left(-\lambda_n^4\right)^s} \sum_{s=0}^{m-1} f_n^{(m-1-s)}(0) \left(\sum_{i=0}^{s} \left(-\lambda_n^4\right)^i \right) \le \frac{p}{\sqrt{6}} \left\| \frac{\partial^{4k-3} f}{\partial x^{4k-3}} \right\|_{L_2(\Omega)}.$$

Lemma 2 is proved. □

Lemma 3. *If* $f(x,t) \in W_2^{(4k-3,0)}(\Omega)$, $\frac{\partial^{2l} f(0,t)}{\partial x^{2l}} = \frac{\partial^{2l} f(p,t)}{\partial x^{2l}} = 0$, $l = 0,1,...,(2k-2)$, *then the series*

$$\sqrt{\frac{2}{p}} \sum_{n=1}^{\infty} \sin \lambda_n x \cdot \left(-\lambda_n^4\right)^k \cdot \int_0^t f_n(\tau) e^{-\lambda_n^4 (t-\tau)} d\tau \tag{25}$$

converges absolutely and uniformly in $\overline{\Omega}$.

Proof. Introducing representation (24) into series (25), we have

$$\sum_{n=1}^{\infty} \left| \lambda_n^{4k} \cdot \int_0^t f_n(\tau) e^{-\lambda_n^4 (t-\tau)} d\tau \right| \le \sum_{n=1}^{\infty} \lambda_n^3 \cdot \left| \int_0^t f_n^{(4k-3,0)}(\tau) \cdot e^{-\lambda_n^4 (t-\tau)} d\tau \right| \le$$

$$\le \sum_{n=1}^{\infty} \lambda_n^3 \cdot \left(\int_0^t \left| f_n^{(4k-3,0)}(\tau) \right|^2 d\tau \right)^{\frac{1}{2}} \cdot \left(\int_0^t e^{-2\lambda_n^4 (t-\tau)} d\tau \right)^{\frac{1}{2}} \le$$

$$\le \frac{p}{2\pi} \sum_{n=1}^{\infty} \frac{1}{n} \cdot \left(\int_0^T \left| f_n^{(4k-3,0)}(\tau) \right|^2 d\tau \right)^{\frac{1}{2}}.$$

$$\sum_{n=1}^{\infty}\left|\lambda_n^{4k}\cdot\int_0^t f_n(\tau)e^{-\lambda_n^4(t-\tau)}d\tau\right| \le \frac{p}{2\sqrt{6}}\left\|\frac{\partial^{4k-3}f}{\partial x^{4k-3}}\right\|_{L_2(\Omega)}.$$

This implies the uniform and absolute convergence of series (25) in $\overline{\Omega}$.

Lemma 3 is proved. □

Theorem 2. *Let*

i) $\varphi(x) \in W_2^1(\Omega)$, $\varphi(0) = \varphi(p) = 0$

ii) $f(x,t) \in W_2^{(4k-3,k-1)}(\Omega)$, $\frac{\partial^{2l}f(0,t)}{\partial x^{2l}} = \frac{\partial^{2l}f(p,t)}{\partial x^{2l}} = 0$, $l = \overline{0,(2k-2)}$,

then the series (16)-(19) *converge absolutely and uniformly in* $\overline{\Omega}$. *Solution* (16) *satisfies equation* (1) *and conditions* (2) - (4).

Proof. By virtue of the proved lemmas, it is easy to show that the series (16)-(19) converge absolutely and uniformly. Adding (17) and (18) we make sure that solution (16) satisfies equation (1) in Ω . Conditions (2) and (3) are satisfied due to the properties of function $X_n(x)$. Theorem 2 is proved. □

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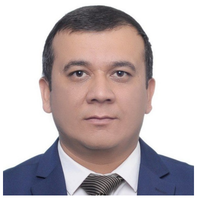

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УДК 517.95 Научная статья

Об одной краевой задаче для уравнения четвертого порядка в частных производных

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Рассмотрена начально-краевая задача для уравнения теплопроводности внутри ограниченной области. Предполагается, что на границе этой области происходит теплообмен по закону Ньютона. Параметр управления равен величине выхода горячего воздуха и определяется на заданном участке границы. Затем была определена зависимость $T(\theta)$ от параметров температурного процесса, когда θ близко к критическому значению.

Key words: краевая задача; метод Фурье; существование решения; единственность решения.

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Конкурирующие интересы. Авторы заявляют, что конфликтов интересов в отношении авторства и публикации нет.

Авторский вклад и ответственность. Все авторы учавствовали в написании статьи и полностью несут ответственность за предоставление ококнчательной версии статьи в печать. Окончательная форма рукописи была одобрена всеми авторами.

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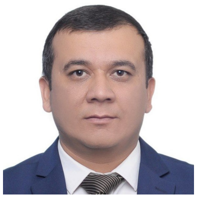

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