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On the control problem associated with the heating process in the bounded domain

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The initial-boundary problem for the heat conduction equation inside a bounded domain is considered. It is supposed that on the boundary of this domain the heat exchange takes place according to Newton's law. The control parameter is equal to the magnitude of output of hot air and is defined on a givenmpart of the boundary. Then we determined the dependence $T(\theta)$ on the parameters of the temperature process when θ is close to critical value.

Key words: heat conduction equation, admissible control, initial-boundary value problem, integral equation.

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1 Introduction

Consider in the bounded domain $\Omega \subset \mathbb{R}^3$ with piecewise smooth boundary $\partial\Omega$ the heat conduction equation

$$u_t(x,t) = \Delta u(x,t), \quad x \in \Omega, \quad t > 0, \tag{1}$$

with boundary conditions

$$\frac{\partial u}{\partial n} + h(x)u(x,t) \; = \; 0, \quad x \in \partial\Omega \setminus \Gamma, \quad t > 0, \tag{2}$$

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$$\frac{\partial u}{\partial n} = a(x)\,\mu(t), \quad x \in \Gamma, \quad t > 0, \tag{3}$$

and initial condition

$$u(x, 0) = 0. \tag{4}$$

Here Γ is some subset of $\partial\Omega$ (heater or air conditioner) with piecewise smooth boundary $\partial\Gamma$ and with $mes\Gamma > 0$ (we denote by $mes\Gamma$ the surface measure of Γ, distinct from Lebesgue measure $|\Gamma|$).

We suppose that $h(x)$ (thermal conductivity of the walls) and $a(x)$ (the density of the power of the heater or air conditioner) are given piecewise smooth non-negative functions, which are not identically zero. The condition (3) means that there is a blast of hot (or cold) air with magnitude of output given by a measurable real-valued function $\mu(t)$, and condition (2) means that on the surface $\partial\Omega$ a heat exchange takes place according to Newton's law (see, e.g. [16], Sect. III.1.4).

We may extend both functions $h(x)$ and $a(x)$ to the whole boundary $\partial\Omega$ by setting $h(x) = 0$ for $x \in \Gamma$, and $a(x) = 0$ for $x \notin \Gamma$. In this case we may write the conditions (2) and (3) in the following form

$$\frac{\partial u(x,t)}{\partial n} + h(x)u(x,t) = a(x)\,\mu(t), \quad x \in \partial\Omega, \quad t > 0. \tag{5}$$

By the solution of the initial boundary value problem (1)-(5), we mean the generalized solution defined in [13] (see Chapter III, Sec. 5).

Let $M > 0$ be some given constant. We say that the function $\mu(t)$ is an *admissible control* if this function is measurable on the half line $t \geq 0$ and satisfies the following constraint

$$|\mu(t)| \leq M, \quad t \geq 0. \tag{6}$$

Let the function $\rho : \overline{\Omega} \to R$ satisfies conditions

$$\int_{\Omega} \rho(x)\,dx = 1, \quad \rho(x) \geq 0.$$

For any $\theta > 0$ consider the condition

$$\int_{\Omega} u(x,t)\rho(x)\,dx = \theta. \tag{7}$$

Note that the weight function $\rho(x)$ is not assumed to be strictly positive. In particular, the value (7) may be the average value over some subdomain of the main region Ω.

Denote by the symbol $T(\theta)$ the minimal time required to reach the given value θ by the average value of the temperature. This means that the equation (7) is fulfilled for $t = T(\theta)$ and is not valid for $t < T(\theta)$.

We present the critical value θ^* such that for any $\theta < \theta^*$ there exists the required admissible control $\mu(t)$ and corresponding value of $T(\theta) < +\infty$, and for $\theta \geq \theta^*$ the equality (7) is impossible.

The purpose of this work is to determine the dependence $T(\theta)$ on the parameters of the temperature process when θ is close to critical value.

We recall that the time-optimal control problem for partial differential equations of parabolic type was first investigated in [6] and [7]. More recent results concerned with this problem were established in [1], [2], [3], [4], [5], [10], [11]. Detailed information on the problems of optimal control for distributed parameter systems is given in [8] and in the monographs [9], [12] and [14].

To formulate the main result we describe some spectral properties of the corresponding self-adjoint extension of Laplace operator.

Consider the following eigenvalue problem for the Laplace operator

$$-\Delta v_k(x) \;=\; \lambda_k v_k(x), \quad x \in \Omega, \tag{8}$$

with boundary condition

$$\frac{\partial v_k(x)}{\partial n} + h(x) v_k(x) = 0, \quad x \in \partial\Omega. \tag{9}$$

Under assumptions made above this problem is self-adjoint in $L_2(\Omega, dx)$ and there exists a sequence of eigenvalues $\{\lambda_k\}$ so that

$$\lambda_1 \le \lambda_2 \le \ldots \le \lambda_k \to +\infty, \quad k \to \infty.$$

The corresponding eigenfunctions form a complete orthonormal system $\{v_k\}_{k\in\mathbb{N}}$ in $L_2(\Omega, dx)$ and these functions belong to $C(\overline{\Omega})$, where $\overline{\Omega} = \Omega \cup \partial\Omega$.

According to (8), we get

$$\lambda_k = -(\Delta v_k, v_k) = \int\limits_{\Omega} |\nabla v_k(x)|^2 dx + \int\limits_{\partial\Omega} |v_k(x)|^2 h(x) d\sigma(x) \ge 0.$$

If $h(x) \ge 0$ and $h(x) \not\equiv 0$ then, $\lambda_1 > 0$. Indeed, assume that $\lambda_1 = 0$. Than the first eigenfunction is an harmonic function

$$\Delta v_1(x) = 0,$$

and, in accordance with the theorem of Giraud and Theorem I.5.II in the book [15], we may state that $v_1 \equiv 0$.

According to the non-negative of the first eigenfunction (see, e.g. [17]) and from the orthogonality of the eigenfunctions v_1 and v_2, we get

$$\lambda_1 < \lambda_2.$$

Recall that we consider the behavior of the function

$$U(t) \;=\; \int\limits_{\Omega} u(x,t)\rho(x)\, dx, \tag{10}$$

where the solution $u(x,t)$ of the problem (1)-(4) depends on the control function $\mu(t)$.

Set

$$\theta^* = M \int_{\Gamma} [(-\Delta)^{-1} \rho(x)] a(x)\, d\sigma(x), \tag{11}$$

and

$$b = \frac{M}{\lambda_1} \cdot (\rho, \nu_1) \int_{\Gamma} \nu_1(y) a(y) d\sigma(y). \tag{12}$$

Theorem 1. *Let* $\theta^* > 0$ *be defined by equation* (11). *Then*
1) for every θ *from the interval* $0 < \theta < \theta^*$ *there exist* $T(\theta)$ *such that*

$$U(t) < \theta, \quad 0 < t < T(\theta),$$

and

$$U(T(\theta)) = \theta.$$

2) for $\theta \to \theta^*$ *the following estimate is valid:*

$$T(\theta) = \ln \frac{1}{\varepsilon(\theta)} + \frac{1}{\lambda_1} \ln b + O(\varepsilon^{\lambda_2 - \lambda_1}),$$

where

$$\varepsilon = |\theta^* - \theta|^{1/\lambda_1}.$$

3) for every $\theta \geq \theta^*$ *the* $T(\theta)$ *does not exists.*

2 The Main Integral Equation

We consider the following Green function:

$$G(x,y,t) = \sum_{k=1}^{\infty} e^{-\lambda_k t} \nu_k(x) \nu_k(y), \quad x \in \Omega, \quad y \in \Omega, \quad t > 0.$$

This function is the solution of the initial-boundary value problem for the equation

$$G_t(x,y,t) = \triangle G(x,y,t), \quad x \in \Omega, \quad t > 0,$$

with boundary condition

$$\frac{\partial G(x,y,t)}{\partial n} + h(x) G(x,y,t) = 0, \quad x \in \partial\Omega, \quad t > 0,$$

and initial condition

$$G(x,y,0) = \delta(x-y).$$

It follows from maximum principle that the Green function is non-negative (see, [1], [3])

$$G(x,y,t) \geq 0, \quad (x,y) \in \bar{\Omega} \times \bar{\Omega}, \quad t > 0.$$

Set

$$H(x,t) = \int_{\Omega} \rho(y)\, G(x,y,t) dy, \quad x \in \Omega, \quad t > 0. \tag{13}$$

It is clear that the function (13) is a solution of the following initial-boundary value problem:

$$H_t(x,t) - \triangle H(x,t) = 0, \quad x \in \Omega, \quad t > 0$$

$$\frac{\partial H(x,t)}{\partial n} + h(x)u(x,t) = 0, \quad x \in \partial\Omega, \quad t > 0,$$

and

$$H(x,0) = \rho(x), \quad x \in \Omega.$$

In this using the spectral theorem in $L_2(\Omega, dx)$ we may write

$$H(x,t) \;=\; \int_0^\infty e^{-\lambda t} dE_\lambda \rho(x)$$

Obviously,

$$H(x,t) \;=\; (\rho, \nu_1) e^{-\lambda_1 t} \nu_1(x) \;+\; H_1(x,t), \quad t \geq 0, \tag{14}$$

where

$$H_1(x,t) \;=\; \int_{\lambda_2}^\infty e^{-\lambda t} dE_\lambda \rho(x). \tag{15}$$

Set

$$A_k \;=\; \int_\Gamma \nu_k(y) a(y) d\sigma(y).$$

Proposition 1. The following estimate is valid:

$$A_1 \;=\; \int_\Gamma \nu_1(y) a(y) d\sigma(y) > 0. \tag{16}$$

Proof. Assume that this integral is equal to 0. Then on some surface $\Gamma_1 \subset \Gamma$ ν_1 equals 0:

$$\nu(s) = 0, \quad s \in \Gamma_1.$$

It follows from (9) that

$$\frac{\partial \nu(s)}{\partial n} = 0, \quad s \in \Gamma_1.$$

Hence, $\nu_1(x)$ is a solution to homogeneous Cauchy problem and from the uniqueness of the solution $\nu_1(x) \equiv 0$, and this contradicts the assumption that $\nu_1(x)$ is an eigenfunction. □

Set

$$G_2(x,y) \;=\; \sum_{k=2}^\infty \frac{\nu_k(x)\nu_k(y)}{\lambda_k^2}. \tag{17}$$

Proposition 2. The function $H_1(x,t)$ satisfies the following estimate

$$\left|H_1(x,t)\right| \;\leq \|\Delta\rho\| \cdot \sqrt{G_2(x,x)} e^{-\lambda_2 t}, \quad t \geq 0,$$

uniformly in $x \in \overline{\Omega}$.

Proof. From (15), we can write

$$H_1(x,t) = \int_{\lambda_2}^{\infty} e^{-\lambda t} dE_\lambda \rho(x) = \sum_{k=2}^{\infty} (\rho, \nu_k) e^{-\lambda_k t} \nu_k(x), \quad t \geq 0.$$

Then, we have

$$\left|H_1(x,t)\right|^2 = \left|\sum_{k=2}^{\infty} (\rho, \nu_k) e^{-\lambda_k t} \nu_k(x)\right|^2 \leq$$

$$\leq \left(\sum_{k=2}^{\infty} |(\rho, \nu_k)|^2 \lambda_k^2\right) \left(\sum_{k=2}^{\infty} e^{-2\lambda_k t} |\nu_k(x)|^2 \lambda_k^{-2}\right), \quad t \geq 0.$$

Then, we get the following estimate

$$\left|H_1(x,t)\right| \leq \|\Delta\rho\| \cdot \sqrt{G_2(x,x)} e^{-\lambda_2 t}.$$

□

Now we introduce the kernel of a main integral operator:

$$K(t) = \int_{\Gamma} H(y,t) a(y) d\sigma(y). \tag{18}$$

According to (14), we may write

$$K(t) = (\rho, \nu_1) e^{-\lambda_1 t} \int_{\Gamma} \nu_1(y) a(y) d\sigma(y) + \int_{\Gamma} H_1(y,t) a(y) d\sigma(y) =$$

$$= A_1 \cdot (\rho, \nu_1) e^{-\lambda_1 t} + \beta(t) e^{-\lambda_2 t}, \tag{19}$$

where

$$|\beta(t)| \leq B = \|\Delta\rho\| \int_{\Gamma} \sqrt{G_2(y,y)}\, a(y) d\sigma(y).$$

The proof of the following Proposition 3 can be seen [1].

Proposition 3. The derivative of the kernel (18) satisfies the following estimates:

$$K'(t) = \frac{O(1)}{\sqrt{t}}, \quad 0 < t < 1,$$

and

$$K'(t) = -\lambda_1 A_1 e^{-\lambda_1 t} + O(1)\, e^{-\lambda_2 t}, \quad t \geq 1.$$

where A_1 is defined by the equality (16).

It is well-known (see, e.g. [13]) that the solution of the initial-boundary value problem (1) + (4) + (5) may be represented by the Green function:

$$u(x,t) = \int_0^t \mu(s)\, ds \int_{\Gamma} G(x,y,t-s)\, a(y)\, d\sigma(y).$$

According to condition (10) we can write

$$\int_\Omega \rho(x)u(x,t)dx = \int_0^t \mu(s)ds \int_{\partial\Omega} a(y)d\sigma(y) \int_\Omega \rho(x)G(x,y,t-s)dx \;=\; U(t)$$

Then, from (13) and (18), we get the following integral equation

$$\int_\Omega \rho(x)u(x,t)dx \;=\; \int_0^t K(t-s)\,\mu(s)\,ds \;=\; U(t). \tag{20}$$

3 Proof of the Theorem 1

Set

$$L(x,t) \;=\; \int_0^t H(x,s)\,ds. \tag{21}$$

Then we can write

$$L(x,t) \;=\; \sum_{k=1}^{\infty} (\rho,v_k)v_k(x) \int_0^t e^{-\lambda_k s}\,ds \;=$$

$$= \sum_{k=1}^{\infty} \frac{1-e^{-\lambda_k t}}{\lambda_k}(\rho,v_k)v_k(x) \;=\; (-\Delta)^{-1}\rho(x) \;-\; \frac{e^{-\lambda_1 t}}{\lambda_1}(\rho,v_1)v_1(x) \;-\; L_1(x,t),$$

where

$$L_1(x,t) \;=\; \sum_{k=2}^{\infty} \frac{e^{-\lambda_k t}}{\lambda_k}(\rho,v_k)v_k(x).$$

We have the following estimate

$$|L_1(x,t)| \;\le\; e^{-\lambda_2 t}\left(\sum_{k=2}^{\infty} |(\rho,v_k)|^2\right)^{1/2} \cdot \left(\sum_{k=2}^{\infty} \frac{|v_k(x)|^2}{\lambda_k^2}\right)^{1/2}.$$

Hence,

$$|L_1(x,t)| \;\le\; e^{-\lambda_2 t}\sqrt{G_2(x,x)}\;\|\rho\|. \tag{22}$$

Further,

$$\int_\Gamma L(x,t)a(x)\,d\sigma(x) \;=\; \int_\Gamma [(-\Delta)^{-1}\rho(x)]a(x)\,d\sigma(x) \;-$$

$$-\;\frac{A_1}{\lambda_1}(\rho,v_1)e^{-\lambda_1 t} \;-\; \int_\Gamma L_1(x,t)a(x)\,d\sigma(x). \tag{23}$$

We introduce a specific heating as

$$Q(t) = \int_0^t K(t-s)ds = \int_0^t K(s)ds. \tag{24}$$

The physical meaning of this function is evident: $Q(t)$ equals the average temperature of Ω in case where the heater is acting unit load.

It is clear that $Q(0) = 0$ and $Q'(t) = K(t) \geq 0$.

According to (18), we have

$$\int_\Gamma L(x,t)a(x)\, d\sigma(x) = \int_0^t ds \int_\Gamma H(x,s)\, a(x) d\sigma(x) =$$

$$= \int_0^t K(s)ds = Q(t). \tag{25}$$

Set

$$Q^* = \lim_{t\to\infty} Q(t) = \int_0^\infty K(s)ds. \tag{26}$$

Obviously, the average temperature of Ω in the case where the heater is acting with unit load cannot exceed Q^*.

Set

$$\theta^* = MQ^*. \tag{27}$$

Then, according to (22) and (23)

$$\theta(t) = M Q(t) = \theta^* - be^{-\lambda_1 t} + O(e^{-\lambda_2 t}), \tag{28}$$

where b defined by (12).

According to (26)-(28), for every θ from the interval $0 < \theta < \theta^*$ there exist $T(\theta)$ such that

$$u(t) < \theta, \quad 0 < t < T(\theta),$$

and

$$u(T(\theta)) = \theta.$$

Proposition 4. There exist $T(\theta) > 0$ and a real-valued measurable function $\mu(t)$ so that $|\mu(t)| \leq M$ and the following equality

$$\int_0^T K(T-s)\mu(s)ds = u(T), \tag{29}$$

is valid.

Proof. This follows from the properties of the function Q. Indeed, if we set $\mu(t) = M$, then we have

$$\int_0^t K(t-s)\mu(s)ds = M\int_0^t K(t-s)ds = MQ(t),$$

and because of (29) there exists $T(\theta) > 0$ so that $M\,Q(T) = U(T)$. □

Remark. It is clear that the value $T(\theta)$, which was found in Proposition 4, gives a solution to the problem. Namely, $T(\theta)$ is the root of the equation

$$Q(T) = \frac{U(T)}{M} = \frac{\theta}{M}.$$

Proposition 5. Let $f(r)$ be increasing on the interval $(0,1]$ and for some b, $\beta > 0$

$$f(r) \;=\; br + O(r^{1+\beta}). \tag{30}$$

Then for inverse function $r = f^{-1}(s)$ the following estimate is valid:

$$\ln\frac{1}{r} \;=\; \ln\frac{1}{s} + \ln b + O(s^{\beta}).$$

Proof. According to (30),

$$s \;=\; br\big[1 + \alpha(r)\big], \tag{31}$$

where

$$\alpha(r) \;=\; O(r^{\beta}).$$

Note that $f(r) > 0$ on the interval $0 < r \le 1$. Hence,

$$s \;\ge\; Cr, \quad 0 < r \le 1.$$

Then

$$r(s) \;=\; f^{-1}(s) \;\le\; \frac{1}{C}\cdot s,$$

and

$$r(s) \;=\; O(s).$$

Hence,

$$\alpha\big(r(s)\big) \;=\; O(s^{\beta}).$$

Then, according to (31),

$$\ln\frac{1}{s} \;=\; \ln\frac{1}{br} + \ln\frac{1}{1+\alpha(r)} \;=\; \ln\frac{1}{br} - \ln[1+\alpha(r)] \;=$$

$$=\; \ln\frac{1}{r} + \ln\frac{1}{b} + O\big(|\alpha(r)|\big) \;=\; \ln\frac{1}{r} - \ln b + O(s^{\beta}).$$

□

Corollary. *The following equality is true:*

$$t \;=\; \ln\frac{1}{|\theta^* - \theta(t)|^{1/\lambda_1}} \;+\; \frac{1}{\lambda_1}\ln b \;+\; O\Big(|\theta^* - \theta(t)|^{(\lambda_2-\lambda_1)/\lambda_1}\Big).$$

Indeed, according to (28),

$$\theta^* - \theta(t) \;=\; be^{-\lambda_1 t} \;+\; O(e^{-\lambda_2 t}).$$

Set

$$r = e^{-\lambda_1 t}, \quad s = \theta^* - \theta(t), \quad \beta = \frac{\lambda_2}{\lambda_1} - 1.$$

Then, we get

$$e^{-\lambda_2 t} = e^{-\lambda_1 t(1+\beta)} = r^{1+\beta}.$$

We can apply Proposition 5 and get

$$t = \frac{1}{\lambda_1} \ln \frac{1}{\theta^* - \theta(t)} + \frac{1}{\lambda_1} \ln b + O\left(|\theta^* - \theta(t)|^{\beta}\right).$$

Then, for $\theta \to \theta^*$, we have the following estimate

$$T(\theta) = \ln \frac{1}{\varepsilon(\theta)} + \frac{1}{\lambda_1} \ln b + O(\varepsilon^{\lambda_2 - \lambda_1}),$$

where

$$\varepsilon = |\theta^* - \theta|^{1/\lambda_1}.$$

The proof of Theorem 1 follows from Propositions 4 and 5.

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References

1. Albeverio S., Alimov Sh. A. On a time-optimal control problem associated with the heat exchange process, *Applied Mathematics and Optimization*, 2008. vol. 57, no. 1, pp. 58–68.
2. Alimov Sh. A., Dekhkonov F. N. On a control problem associated with fast heating of a thin rod, *Bulletin of National University of Uzbekistan: Mathematics and Natural Sciences*, 2019. vol. 2, no. 1, pp. 1–14.
3. Alimov Sh. A., Dekhkonov F. N. On the time-optimal control of the heat exchange process, *Uzbek Mathematical Journal*, 2019. no. 2, pp. 4–17.
4. Altmüller A., Grüne L. Distributed and boundary model predictive control for the heat equation, *Technical report, University of Bayreuth, Department of Mathematics*, 2012.
5. Chen N., Wang Y., Yang D. Time–varying bang–bang property of time optimal controls for heat equation and its applications, *Syst. Control Lett*, 2018. vol. 112, pp. 18–23.
6. Egorov Yu. V. Optimal control in Banach spaces, *Dokl. Akad. Nauk SSSR*, 1963. vol. 150, pp. 241–244 (In Russian).
7. Fattorini H. O. Time-optimal control of solutions of operational differential equations, *SIAM J. Control*, 1964. vol. 2, pp. 49–65.
8. Fattorini H. O. Time and norm optimal controls: a survey of recent results and open problems, *Acta Math. Sci. Ser. B Engl. Ed.*, 2011. vol. 31, pp. 2203–2218.
9. Friedman A. Optimal control for parabolic equations, *J. Math. Anal. Appl.*, 1967. vol. 18, pp. 479–491.
10. Dekhkonov F. N. On time-optimal control problem associated with parabolic equation, *Bulletin of National University of Uzbekistan*, 2021. vol. 4, no. 1, pp. 54–63.
11. Dekhkonov F. N. On a time-optimal control of thermal processes in a boundary value problem, *Lobachevskii journal of mathematics*, 2022. vol. 43, no. 1, pp. 192–198 DOI:10.1134/S1995080222040096.

12. Fursikov A. V. *Optimal Control of Distributed Systems*: Math. Soc., Providence, Rhode Island, 2000.
13. Ladyzhenskaya O. A., Solonnikov V. A., Uraltseva N. N. *Linear and Quasi-Linear Equations of Parabolic Type*. Moscow: Nauka, 1967 (In Russian).
14. Lions J. L. *Contróle optimal de systèmes gouvernés par deséquations aux dérivées partielles*. Dunod Gauthier-Villars: Paris, 1968.
15. Miranda C. *Equazioni alle derivate parziali di tipo ellittico*. Springer Verlag: Berlin-Göttingen-Heidelberg, 1955.
16. Tikhonov A. N., Samarsky A. A. *Equations of Mathematical Physics*. N. Chelmsford: Courier Corporation, 2013. 800 pp.
17. Vladimirov V. S. *Equations of Mathematical Physics*. Marcel Dekker: New York, 1971.

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К задаче управления, связанной с процессом нагрева в ограниченной области

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Рассмотрена начально-краевая задача для уравнения теплопроводности внутри ограниченной области. Предполагается, что на границе этой области происходит теплообмен по закону Ньютона. Параметр управления равен величине выхода горячего воздуха и определяется на заданном участке границы. Затем определяли зависимость $T(\Theta)$ от параметров температурного процесса, когда Θ близко к критическому значению.

Ключевые слова: уравнение теплопроводности, допустимое управление, начально-краевая задача, интегральное уравнение.

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