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On a boundary value problem for an odd-order equation with multiple characteristics

O. T. Kurbanov

Tashkent State Economic University, Uzbekistan, 100003, Tashkent, Chilanzar district, Ave. Islam Karimov, 49

E-mail: odil69@inbox.ru

A nonlinear boundary value problem for a third-order nonlinear equation with multiple characteristics is studied in the article in a curvilinear domain. The unique solvability of the problem is proved. The uniqueness of the solution of the boundary value problem is proved by the energy integral method using some elementary inequalities. An auxiliary problem is considered for the existence of a solution, for which the Green function is constructed. By solving an auxiliary problem, the original problem is reduced to a system of Hammerstein integral equations. The solvability of a nonlinear system is proved by the contracting mapping method.

Key words: nonlinearity, uniqueness, existence, system of Hammerstein equations.

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Introduction

The following equation refers to poorly studied third-order equations:

$$u_{xxx} - u_y = f(x, y, u(x, y), u_x(x, y), u_{xx}(x, y)), \quad (M)$$

called an equation with multiple characteristics [4].

Equation (M) is often used in various problems of physics and mechanics, which are of great theoretical and applied importance. This equation contains the well-known Korteweg de Vries equation (KdV)

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$$u_y + uu_x + \beta u_{xxx} = 0 \qquad \text{(KdV)}$$

which is the object of research by many authors and occupies an important place in the nonlinear wave propagation in weakly dispersive media [1, 2, 3].

The Korteweg de Vries equation (KdV) describes the evolution of weakly nonlinear long-wave excitations in a medium with dispersion in the high-frequency region. The KdV equation arises in the study of many physical systems, such as gravitational waves in shallow water, ion-acoustic waves in plasma, Rossby waves in a rotating fluid, waves in electrical circuits containing nonlinear elements, etc. [1].

Some characteristic features of wave propagation in dispersive media can be traced already in the linear approximation [2]

$$u_y + \beta u_{xxx} = 0. \qquad \text{(LKdV)}$$

The (LKdV) equation describes sufficiently long waves in media where limit $\frac{\omega}{k}$ (phase velocity) as $k \to 0$ has a finite value (weakly dispersive waves). The (LKdV) equation is called the linearized Korteweg de Vries equation [2,3].

The study of boundary value problems is also relevant for an odd-order equation with multiple characteristics. Some linear boundary value problems for a linear equation with multiple characteristics of the third order were studied in [4, 5]. In [5], a linear boundary value problem for a nonlinear equation with multiple third-order characteristics was studied by the method of successive approximations. A nonlinear boundary value problem for a linear equation with multiple third-order characteristics was studied in [6]. In [7], a problem was studied for a nonlinear equation with multiple third-order characteristics with nonlinear boundary conditions. In this article, we study a nonlinear boundary value problem for a nonlinear equation with multiple characteristics in a curvilinear domain.

Formulation of the problem

Problem A. It is required to determine function $u(x,y)$ in domain
$D = \{(x,y); h_1(y) < x < h_2(y), 0 < y \leq 1\}$ the function has the following properties:
1)$u(x,y) \in C^{3,1}(D) \cap C^{2,0}(x = h_2(y), 0 < y \leq 1) \cap C^{1,0}(\overline{D})$
2)which is a regular solution to the following equation

$$L(u) \equiv u_{xxx} - u_y = f(x,y,u(x,y)), \qquad (1)$$

in domain D;
3)satisfying the following conditions

$$u(x,0) = u_0(x), h_1(0) \leq x \leq h_2(0), \qquad (2)$$

$$u(h_1(y),y) = \varphi(y), 0 \leq y \leq 1, \qquad (3)$$

$$u_x(h_1(y),y) = \psi(y), 0 \le y \le 1, \tag{4}$$

$$u_{xx}(h_2(y),y) + a u_x(h_2(y),y) = g(u(h_2(y),y),y), 0 \le y \le 1, \tag{5}$$

as well as natural matching conditions at angular points:
$u_0(h_1(0)) = \varphi(0), u\prime_0(h_1(0)) = \psi(0),$
$u\prime\prime_0(h_2(0)) + a u\prime_0(h_2(0)) = g(u(h_2(0),0),0).$

Uniqueness of the solution

The following theorem holds

Theorem. Let $h_r(y) \in C^1[0,1], r = 1,2$ and $g(u(x,y),y), f(x,y,u(x,y))$ be continuous functions of their arguments $0 \le y \le 1$ for any $|u| < \infty$, that satisfies the following conditions

$$|g(u_1,y) - g(u_2,y)| \le l(y)|u_1 - u_2|, \tag{6}$$

$$|f(x,y,u_1) - f(x,y,u_2)| \le L(x,y)|u_1 - u_2|, \tag{7}$$

$$l(y) + h\prime_2(y) + \alpha^2 \le 0, \tag{8}$$

$$\alpha^3 - \beta + L(x,y) \le 0. \tag{9}$$

Then the solution of problem (1) - (5) is unique.

Proof. The uniqueness of the solution to the problem is proved by the energy integrals method, using some elementary inequalities. Let there be two solutions of the considered problem u_1 and u_2. Consider their difference $\omega = u_1 - u_2$. Forω, we get the following problem:

$$L(\omega) \equiv \omega_{xxx} - \omega_y = f(x,y,u_1(x,y)) - f(x,y,u_2(x,y)), \tag{1_0}$$

$$\omega(x,0) = 0, h_1(0) \le x \le h_2(0), \tag{2_0}$$

$$\omega(h_1(y),y) = 0, 0 \le y \le 1, \tag{3_0}$$

$$\omega_x(h_1(y),y) = 0, 0 \le y \le 1, \tag{4_0}$$

$$u_{xx}(h_2(y),y)+au_x(h_2(y),y)=g(u_1(h_2(y),y),y)-g(u_2(h_2(y),y),y),0\le y\le 1. \quad (5_0)$$

Integrating the identity

$$\nu\omega L(\omega)\equiv\nu\omega(\omega_{xxx}-\omega_y)=\nu\omega(f(x,y,u_1(x,y))-f(x,y,u_2(x,y))) \qquad (10)$$

over domain D, where $\nu=\exp(-\alpha x-\beta y),\alpha>0,\beta>0$, with boundary conditions (2_0) - (5_0), we have

$$\int\limits_0^1[g(u_1(x,y),y)-g(u_2(x,y),y)]\nu\omega|_{x=h_2(y)}dy-\frac{1}{2}\int\limits_0^1\nu\omega_x^2|_{x=h_2(y)}dy+$$

$$+\frac{1}{2}\int\limits_0^1[\alpha^2+h\prime_2(y)]\nu\omega^2|_{x=h_2(y)}dy-\frac{3\alpha}{2}\int\int\limits_D\nu\omega_x^2dxdy+$$

$$+\frac{1}{2}\int\int\limits_D[\nu_y-\nu_{xxx}]\omega^2dxdy-\frac{1}{2}\int\limits_{h_1(y)}^{h_2(y)}\nu\omega^2|_{y=1}dx=$$

$$=\int\int\limits_D[f(x,y,u_1(x,y))-f(x,y,u_2(x,y))]dxdy. \qquad (11)$$

We introduce the following notation:

$$I=\frac{1}{2}\int\limits_0^1\nu\omega_x^2|_{x=h_2(y)}dy+\frac{1}{2}\int\limits_{h_1(y)}^{h_2(y)}\nu\omega^2|_{y=1}dx+\frac{3\alpha}{2}\int\int\limits_D\nu\omega_x^2dxdy\ge 0. \qquad (12)$$

With conditions (6) – (7) from (13), we have

$$I\le\frac{1}{2}\int\limits_0^1[l(y)+\alpha^2+h\prime_2(y)]\nu\omega^2|_{x=h_2(y)}dy+\frac{1}{2}\int\int\limits_D[\alpha^3-\beta-L(x,y)]\nu\omega^2dxdy. \qquad (13)$$

Taking into account the condition of the theorem, the expressions in brackets on the right side of inequality (13) are nonpositive: $I\le 0$. With (12), $0\le I\le 0$, whence it follows that $I=0$.

Then from (12) we obtain the following conditions:

$$\omega(h_2(y),y)=0,$$

$$\omega(x,1)=0,$$

$$\omega_x(x,y)=0.$$

Hence we have $\omega(x,y)=p(y),(x,y)\in D$.

Since $\omega(h_2(y),y)=0,0\le y\le 1$, then $p(y)=0$.

Due to the continuity of $\omega(x,y)$ in $\overline{D}$, we have $\omega(x,y)=0$.

Existence of solution

Before proceeding to the proof of the problem of solution existence, it is necessary to study the following auxiliary problem.

Problem B. It is required to determine in domain D a regular solution

$$u(x,y) \in C^{3,1}(D) \cap C^{2,0}(x = h_2(y), 0 < y \leq 1) \cap C^{1,0}(\overline{D})$$

of equation

$$L(u) \equiv u_{xxx} - u_y = f(x,y), \tag{1_1}$$

satisfying the following conditions

$$u(x,0) = u_0(x), h_1(0) \leq x \leq h_2(0), \tag{2}$$

$$u(h_1(y),y) = \varphi(y), 0 \leq y \leq 1, \tag{3}$$

$$u_x(h_1(y),y) = \psi(y), 0 \leq y \leq 1, \tag{4}$$

$$u_{xx}(h_2(y),y) = \psi_1(y), 0 \leq y \leq 1 \tag{5_1}$$

The solution to this problem has the form:

$$u(x,y) = \frac{1}{\pi}\int_0^y G u_{\xi\xi}|_{\xi=h_2(\eta)} d\eta - \frac{1}{\pi}\int_0^y [G_{\xi\xi} + h\prime_1(y)G]u|_{\xi=h_1(\eta)} d\eta + \frac{1}{\pi}\int_0^y G_\xi u_\xi|_{\xi=h_1(\eta)} d\eta +$$

$$+\frac{1}{\pi}\int_{h_1(0)}^{h_2(0)} G u|_{\eta=0} d\xi - \frac{1}{\pi}\iint_D G(x,y;\xi,\eta) f(\xi,\eta) d\xi d\eta. \tag{14}$$

here $G(x,y;\xi,\eta) = U(x,y;\xi,\eta) - W(x,y;\xi,\eta)$, where $W(x,y;\xi,\eta)$ - be any regular solution of equation

$$-\varpi_{\xi\xi\xi} + \varpi_\eta = 0,$$

$U(x,y;\xi,\eta)$ is the fundamental solution of equation [4]

$$\varpi_{\xi\xi\xi} - \varpi_\eta = 0$$

.

$$U(x,y;\xi,\eta) = \frac{1}{(y-\eta)^{1/3}} f\left(\frac{x-\xi}{(y-\eta)^{1/3}}\right), y > \eta, x \neq \xi,$$

$$V(x,y;\xi,\eta) = \frac{1}{(y-\eta)^{1/3}} \varphi\left(\frac{x-\xi}{(y-\eta)^{1/3}}\right), y > \eta, x \neq \xi,$$

where

$$f(t)=\int\limits_0^\infty \cos(\lambda^3-\lambda t)d\lambda,$$

$$\varphi(t)=\int\limits_0^\infty (exp(-\lambda^3-\lambda t)+sin(\lambda^3-\lambda t))d\lambda,$$

$$t=\frac{x-\xi}{(y-\eta)^{1/3}}.$$

Functions $f(t)$ and $\varphi(t)$, called the Airy functions, satisfy the following equation

$$z\prime\prime(t)+\frac{t}{3}z(t)=0.$$

$$\int\limits_0^\infty f(t)=\frac{2\pi}{3},\int\limits_{-\infty}^0 f(t)=\frac{\pi}{3},\int\limits_{-\infty}^\infty f(t)=\pi,\int\limits_0^\infty \varphi(t)=0.$$

The following estimates hold for the functions $U(x,y;\xi,\eta)$, $V(x,y;\xi,\eta)$

$$|U(x,y;\xi,\eta)|<\frac{K}{(y-\eta)^{1/3}},$$

$$|\frac{\partial^{i+j}U(x,y;\xi,\eta)}{\partial x^i\partial y^j}|<C_1\frac{|x-\xi|^{\frac{2i+6j-1}{4}}}{|y-\eta|^{\frac{2i+6j-1}{4}}},$$

$$|\frac{\partial^{i+j}V(x,y;\xi,\eta)}{\partial x^i\partial y^j}|<C_2\frac{|x-\xi|^{\frac{2i+6j-1}{4}}}{|y-\eta|^{\frac{2i+6j-1}{4}}},$$

for
$\frac{x-\xi}{(y-\eta)^{1/3}}\to+\infty, i+j\geq 1, C_1>0, C_2>0,$

$$|\frac{\partial^{i+j}U(x,y;\xi,\eta)}{\partial x^i\partial y^j}|<\frac{C_3}{|y-\eta|^{\frac{i+3j+1}{4}}}e^{C_4\frac{|x-\xi|^{\frac{3}{2}}}{y-\eta|^{\frac{1}{2}}}},$$

for
$\frac{x-\xi}{(y-\eta)^{1/3}}\to-\infty, i+j\geq 1, C_3>0, C_4>0.$

Note that the same estimates are valid for function $G(x,y;\xi,\eta)$ as for function $U(x,y;\xi,\eta)$.

Function $G(x,y;\xi,\eta)$ we call the Green function of problem B.

We now turn to problem A.

Theorem. Let, along with the conditions of the uniqueness, the following conditions be satisfied

$$u_0(x)\in[h_1(0),h_2(0)],\varphi(y)\in C^2[0,1],\varphi(0)=0,\psi(y)\in C^1[0,1],$$

$$\frac{\partial f(x,y,u)}{\partial y}\in C(\overline{D}),f(x,0,u(x,0))=0$$

and $y\in[0,1]$ for any $|u|<\infty$, the inequalities hold

$$|g(u,y)|<M_1,|g_y(u,y)|<M_2,|g_u(u,y)|<M_3,$$

And for $(x,y)\in D$ and $|u|<\infty$

$$|f(x,y,u)|<M_4,|f_x(x,y,u)|<M_5,|f_y(x,y,u)|<M_5,|f_u(x,y,u)|<M_7.$$

Then the solution to problem (1) - (5) exists.

Proof. According to (14), we find the solution to problem A in the following form

$$u(x,y)=\frac{1}{\pi}\int_0^y G(x,y;h_2(\eta),\eta)g(u(h_2(\eta),\eta))d\eta-\frac{1}{\pi}\int_0^y G(x,y;h_2(\eta),\eta)au_x(h_2(\eta),\eta)d\eta-$$

$$-\frac{1}{\pi}\int_0^y[G_{\xi\xi}+h\prime_1(y)G]u|_{\xi=h_1(\eta)}d\eta+\frac{1}{\pi}\int_0^y G_\xi u_\xi|_{\xi=h_1(\eta)}d\eta+$$

$$+\frac{1}{\pi}\int_{h_1(0)}^{h_2(0)} Gu|_{\eta=0}d\xi-\frac{1}{\pi}\iint_D G(x,y;\xi,\eta)f(\xi,\eta,u(\xi,\eta))d\xi d\eta. \tag{15}$$

Let

$$u(h_2(\eta),\eta)=\tau(\eta),u_x(h_2(\eta),\eta)=\nu(\eta). \tag{16}$$

Then, from (16) we have

$$u(x,y)=\frac{1}{\pi}\int_0^y G(x,y;h_2(\eta),\eta)g(\tau(\eta),\eta)d\eta-\frac{a}{\pi}\int_0^y G(x,y;h_2(\eta),\eta)\nu(\eta)d\eta-$$

$$-\frac{1}{\pi}\int\limits_0^y [G_{\xi\xi}+h\nu_1(y)G]|_{\xi=h_1(\eta)}\varphi(\eta)d\eta+\frac{1}{\pi}\int\limits_0^y G_\xi|_{\xi=h_1(\eta)}\psi(\eta)d\eta+$$

$$+\frac{1}{\pi}\int\limits_{h_1(0)}^{h_2(0)} G|_{\eta=0}u_0(\xi)d\xi-\frac{1}{\pi}\iint\limits_D G(x,y;\xi,\eta)f(\xi,\eta,u(\xi,\eta))d\xi d\eta, \quad (17)$$

or

$$u(x,y)=\frac{1}{\pi}\int\limits_0^y G(x,y;h_2(\eta),\eta)g(\tau(\eta),\eta)d\eta-\frac{a}{\pi}\int\limits_0^y G(x,y;h_2(\eta),\eta)\nu(\eta)d\eta-$$

$$-\frac{1}{\pi}\iint\limits_D G(x,y;\xi,\eta)f(\xi,\eta,u(\xi,\eta))d\xi d\eta+H(x,y), \quad (18)$$

where

$$H(x,y)=\frac{1}{\pi}\int\limits_0^y [G_{\xi\xi}+h\nu_1(y)G]|_{\xi=h_1(\eta)}\varphi(\eta)d\eta+\frac{1}{\pi}\int\limits_0^y G_\xi|_{\xi=h_1(\eta)}\psi(\eta)d\eta+\frac{1}{\pi}\int\limits_{h_1(0)}^{h_2(0)} G|_{\eta=0}u_0(\xi)d\xi.$$

Now passing to limit $x\to h_2(y)$, according to notation in (16), from (18) we have

$$\tau(y)=\frac{1}{\pi}\int\limits_0^y G(h_2(y),y;h_2(\eta),\eta)g(\tau(\eta),\eta)d\eta-\frac{a}{\pi}\int\limits_0^y G(h_2(y),y;h_2(\eta),\eta)\nu(\eta)d\eta-$$

$$-\frac{1}{\pi}\iint\limits_D G(h_2(y),y;\xi,\eta)f(\xi,\eta,u(\xi,\eta))d\xi d\eta+H(h_2(y),y). \quad (19)$$

Further, differentiating with respect to x (18), passing to limit $x\to h_2(y)$, according to notation in (16), we have

$$\nu(y)=\frac{1}{\pi}\int\limits_0^y G_x(h_2(y),y;h_2(\eta),\eta)g(\tau(\eta),\eta)d\eta-\frac{a}{\pi}\int\limits_0^y G_x(h_2(y),y;h_2(\eta),\eta)\nu(\eta)d\eta-$$

$$-\frac{1}{\pi}\iint\limits_D G_x(h_2(y),y;\xi,\eta)f(\xi,\eta,u(\xi,\eta))d\xi d\eta+H_x(h_2(y),y). \quad (20)$$

System (18) - (20) is a system of Hammerstein's nonlinear integral equations with respect to $u(x,y),\tau(y)$ and $\nu(y)$. We will prove the unique solvability of this system using the contracting mapping principle.

Let $G_\theta -$ be a set of functions $F=\{u(x,y),\tau(y),\nu(y)\}$, that are continuous in domain $D_\theta=\{(x,y);h_1(y)<x<h_2(y),0<y\le\theta\}$ and have on the interval $0<y\le\theta$, bounded norm $\|F\|=\|u\|+\|\tau\|+\|\nu\|$, where $\|u\|=\max\limits_{(x,y)\in D}|u|,\|\tau\|=\max\limits_{0\le y\le\theta}|\tau|,\|\nu\|=\max\limits_{0\le y\le\theta}|\nu|$.

Let $G_{\theta,N}$ denote subset $\{F:F\in G_\theta,|F|\le N\}$ of set G_θ.

Denoting the right-hand sides of (19), (20), (21) by $A_i(u,\tau,\nu),i=\overline{1,3}$, , respectively, we define the mapping $A=\{A_1(.),A_2(.),A_3(.)\}$.

The mapping A is well defined (since all integrals on the right-hand sides of (19)-(21) exist).

We show that for some θ and $N>0$, , for $0\le y\le\theta$, , operator A transforms $G_{\theta,N}$ into itself. That is, the inequalities $|A_i|\le\frac{N}{3},i=\overline{1,3}$, are valid when $(u,\tau,\nu)\in G_{\theta,N}$ . . To do this, assume that $A_i(u,\tau,\nu),i=\overline{1,3}$ are defined in $G_{\theta,N}$, respectively.

From relation (19) we obtain

$$|A_1(u,\tau,\nu)|\le\{[\frac{3KM_1}{2\pi}+\frac{3aK_1}{2\pi}\|\nu\|+\frac{3KM_4}{2\pi}(h_2(y)-h_1(y))+\frac{3K}{2\pi}\|\varphi\|\|h\prime_1(y)\|]\theta_1^{5/12}+$$

$$+[\frac{\|\varphi\|\|h\prime_1(y)\|}{3\pi}K\theta_1^{1/3}+\frac{\|\varphi\|}{3\pi}(x-h_1(y))C_5]\theta_1^{1/12}+\frac{4C_1}{\pi}\|\psi\|\}\theta_1^{1/4}+\|u_0\|+\|\varphi\|.$$

ForN_1 we take $N_1=3(\|u_0\|+\|\varphi\|+1)$, a θ_1 , and θ_1 is chosen so that the following inequality holds

$$\{[\frac{3KM_1}{2\pi}+\frac{3aK_1}{2\pi}\|\nu\|+\frac{3KM_4}{2\pi}(h_2(y)-h_1(y))+\frac{3K}{2\pi}\|\varphi\|\|h\prime_1(y)\|]\theta_1^{5/12}+$$

$$+[\frac{\|\varphi\|\|h\prime_1(y)\|}{3\pi}K\theta_1^{1/3}+\frac{\|\varphi\|}{3\pi}(x-h_1(y))C_5]\theta_1^{1/12}+\frac{4C_1}{\pi}\|\psi\|\}\theta_1^{1/4}\le 1.$$

Then the relation $|A_1|\le\frac{N_1}{3}$ holds

Likewise, from (20) and (21) we have

$$|A_2|\le\frac{N_2}{3},|A_3|\le\frac{N_3}{3}.$$

$$|A_2|\le\frac{N_2}{3},|A_3|\le\frac{N_3}{3}.$$

Setting $N=maxN_i$ and $\theta=min\theta_i,i=\overline{1,3}$, for $0\le y\le\theta$ we prove that the operator A maps the set $G_{\theta,N}$ into itself. Let us show that with an appropriate choice of θ, operator A is contractive. We have

$$|A_1(u,\tau,\nu)-A_2(u^*,\tau^*,\nu^*)|\le\frac{1}{\pi}\int\limits_0^y|g(\tau(y),y)-g(\tau^*(y),y)|G(x,y;h_2(\eta),\eta)d\eta+$$

$$+\frac{a}{\pi}\int\limits_0^y|\nu(y)-\nu^*(y)|G(x,y;h_2(\eta),\eta)d\eta+\frac{1}{\pi}\int\int\limits_D|f(\xi,\eta,u(\xi,\eta))-f(\xi,\eta,u^*(\xi,\eta))|G(x,y;\xi,\eta)d\xi d\eta\le$$

$$\leq \{\|\tau-\tau^*\|+\|\nu-\nu^*\|+\|u-u^*\}l_1\theta^{2/3},$$
$$l_1=max\{\frac{3K}{2\pi}l, \frac{3aK}{2\pi}, \frac{3K(h_2(y)-h_1(y))}{2\pi}L.\}$$

We choose θ_1 so that the inequalities $l_1\theta^{2/3}<1$ hold.

Likewise, for $A_2(u,\tau,\nu)$ and $A_3(u,\tau,\nu)$ we have

$$|A_2(u,\tau,\nu)-A_2(u^*,\tau^*,\nu^*)|\leq \{\|\tau-\tau^*\|+\|\nu-\nu^*\|+\|u-u^*\}l_2\theta_2^{2/3},$$

$$|A_3(u,\tau,\nu)-A_3(u^*,\tau^*,\nu^*)|\leq \{\|\tau-\tau^*\|+\|\nu-\nu^*\|+\|u-u^*\}l_3\theta_3^{1/4}.$$

For $\theta=min\theta_i, i=\overline{1,3}$, operator $A(u,\tau,\nu)$ is a contraction mapping. Then, by virtue of the contraction mapping principle, it has a single fixed point $(u,\tau,\nu)\in G_{\theta,N}$. We assume that θ is chosen so as to ensure the compressibility of operator $A(u,\tau,\nu)$ and that operator $A(u,\tau,\nu)$ maps $G_{\theta,N}$ to itself.

Therefore, $(u,\tau,\nu)\in G_{\theta,N}$ is a solution of system (19) - (21) for $0\leq y\leq\theta$.

Let us establish estimate $u_x(x,y)$ в $\overline{D}$ in , which ensures the global solvability of the problem posed [9].

We have

$$u_x(x,y)=\frac{1}{\pi}\int_0^y G_x(x,y;h_2(\eta),\eta)g(u(h_2(\eta),\eta)d\eta-\frac{1}{\pi}\int_0^y G_x(x,y;h_2(\eta),\eta)au_x(h_2(\eta),\eta)d\eta-$$

$$-\frac{1}{\pi}\int_0^y [G_{\xi\xi x}+h\prime_1(y)G_x]u|_{\xi=h_1(\eta)}d\eta+\frac{1}{\pi}\int_0^y G_{\xi x}u_\xi|_{\xi=h_1(\eta)}d\eta+$$

$$+\frac{1}{\pi}\int_{h_1(0)}^{h_2(0)} G_x(x,y;\xi,0)u_0\xi d\xi-\frac{1}{\pi}\iint_D G_x(x,y;\xi,\eta)f(\xi,\eta,u(\xi,\eta))d\xi d\eta=I_1+I_2+I_3+I_4+I_5+I_6.$$

We have

$$|I_1|\leq\frac{CM_1}{\pi}\int_0^y\frac{d\eta}{(y-\eta)^{3/4}}\leq\frac{4CM_1}{\pi}\sqrt[4]{y}\leq K_1,$$

$$|I_2|\leq\frac{a\|\nu\|C_1}{\pi}\int_0^y\frac{d\eta}{(y-\eta)^{3/4}}\leq\frac{a\|\nu\|C_1}{\pi}\sqrt[4]{y}\leq K_2.$$

The remaining integrals are estimated in a similar way.

Let $maxK_i=\frac{K}{6}, i=\overline{1,6}$, then we have

$$\|u_x\|\leq K.$$

Thus, we conclude that the solution of problem (1)-(5) can be extended to [0, 1] in y.

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Kurbanov Odilzhan Tukhtamuradovich – PhD (Phys. & Math.), Associate Professor, Associate Professor, Dept. Apl. Math., Tashkent State University of Economics, Tashkent, Uzbekistan, ORCID 0000-0003-1360-325x.

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Об одной краевой задаче для уравнения нечетного порядка с кратными характеристиками

О. Т. Курбанов

Ташкентский государственный экономический университет, Узбекистан, 100003, Ташкент, ул. Ислама Каримова, 49

E-mail: odil69@inbox.ru

Для нелинейного уравнения с кратными характеристиками в криволинейной области исследована однозначная разрешимость одной краевой задачи.

Ключевые слова: нелинейность, единственность, существование, система уравнений Гаммерштейна.

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Курбанов Одилжан Тухтамурадович – кандидат физико-математических наук, доцент, доцент кафедры прикладной математики, Ташкенский государственный экономический университет, Ташкент, Узбекистан, ORCID 0000-0003-1360-325x.

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