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The extremal function of interpolation formulas in $W_2^{(2,0)}$ space

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One of the main problems of computational mathematics is the optimization of computational methods in functional spaces. Optimization of computational methods are well demonstrated in the problems of the theory of interpolation formulas. In this paper, we study the problem of constructing an optimal interpolation formula in a Hilbert space. Here, using the Sobolev method, the first part of the problem is solved, i.e., an explicit expression of the square of the norm of the error functional of the optimal interpolation formulas in the Hilbert space $W_2^{(2,0)}$ is found.

Keywords: optimal interpolation formulas, the error functional, the extremal function, Hilbert space.

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1. Introduction. Statement of the problem

One of the problems of approximation is the interpolation problem. The classical method of its solution consists of construction of a interpolation polynomial. However, it is known that the polynomial approximation is non-practical for approximating functions with finite and small smoothness, which often occurs in applications. Therefore, in practice, splines are used in order to approximate functions. There are algebraic and variational approaches in the theory of splines [1]. In the algebraic approach splines are considered as some smooth piecewise polynomial functions. In the variational approach splines are understood as elements of a Hilbert or a Banach space minimizing certain functionals. Then problems of existence, uniqueness and convergence of splines and

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algorithms for their constructions are studied based on intrinsic properties of splines (see, for example, [11]).

$L-$ (generalized) splines are generalization of polynomial splines. Since the results of this paper are connected with $L-$ splines we give some definitions following Ahlberg, Nilson and Walsh [1].

Let L be a linear differential operator given by formula

$$L \equiv a_m(x)D^m + a_{m-1}(x)D^{m-1} + ... + a_0(x),$$

where $D = d/dx$, each $a_j(x)$ $(j = 0,1,...,m)$ is in $C^m[a,b]$, and $a_m(x) \neq 0$ on $[a,b]$. By L^* is denoted the formal adjoint of L:

$$L^* \equiv (-1)^m D^m\{a_m(x)\} + (-1)^{m-1}D^{m-1}\{a_{m-1}(x)\} + ... - D\{a_1(x)\} + a_0(x),$$

$K^m(a,b)$ is the class of all functions $f(x)$ defined on $[a,b]$ which posses an absolutely continuous $(n-1)$th derivative on $[a,b]$ and whose nth derivative is in $L_2(a,b)$. $K^m(a,b)$ is a Hilbert space with the inner product

$$\langle f,g\rangle = \int\limits_a^b (Lf)(x)\cdot(Lg)(x)\,dx \tag{1}$$

if functions that differ by a solution of the equation $Lf = 0$ are identified.

If $\Delta : a = x_0 < x_1 < ... < x_N = b$ is a mesh on $[a,b]$, then a generalized spline of deficiency k $(0 \leq k \leq m)$ with respect to Δ is a function $S_\Delta(x)$ which is in $K^{2m-k}(a,b)$ and satisfies the differential equation

$$L^*LS_\Delta = 0$$

on each open mesh interval (x_{i-1},x_i) $(i = 1,2,...,N)$ of Δ. The ordinary spline of deficiency 1 allows discontinuities in the $(2m-1)$th derivative, but only at mesh points.

It is known, from the results of Ahlberg, Nilson and Walsh [1], that for generalized spline of deficiency 1 the following is true: Let $\Delta : a = x_0 < x_1 < ... < x_N = b$ and $Y = \{y_i,\ i = 0,1,...,N\}$ be given. Then of all functions $f(x)$ in $K^m(a,b)$ such that $f(x_i) = y_i$ $(i = 0,1,...,N)$ the generalized spline $S_\Delta(Y;x)$, when it exists, minimizes the quantity $\int_a^b (Lf(x))^2\,dx$.

Further, we give some results obtained in the theory of L-splines. First contributions to the theory of splines include the works of Greville, Ahlberg, Nilson and Walsh, as well as Schultz and Varga (see [11, p. 459]). These works are concentrated on natural L-splines, which appear as solutions to the corresponding best interpolation problems and the order of approximation of generalized splines was first studied.

In 1963, in papers [9, 10] I.J. Schoenberg showed the interconnection of splines (which are a solution to the minimum norm problem) and optimal quadrature formulas in the sense of Sard. Taking this interconnection into account, in [3, 5, 6, 7] optimal quadrature formulas in the sense of Sard were constructed in different spaces.

It should be noted that the minimum norm problem was studied by many authors in various spaces. The results of [21], which are devoted to abstract splines, supplemented the results of some previous works. In [21], first a theory of splines is presented, in which conditions for the existence and uniqueness of splines with the minimum norm property are studied in Hilbert spaces.

In [12] by Kh.M. Shadimetov and A.R. Hayotov, using S.L. Sobolev's method, interpolation splines minimizing the semi-norm in a Hilbert space are constructed.

Explicit formulas for coefficients of interpolation splines are obtained. The obtained interpolation spline is exact for polynomials of degree $m-2$ and e^{-x}. Also some numerical results are presented.

In papers [8, 13], N.Kh. Mamatova, A.R. Hayotov and Kh.M. Shadimetov considered the problem of constructing optimal lattice interpolation formulas in the Sobolev space $\widetilde{L_2^m}(H)$ of periodic functions of n variables. They found the coefficients of lattice interpolation formulas.

In [4] by A.Cabada, A.R.Hayotov and Kh.M.Shadimetov, interpolation D^m-splines that minimizes the expression $\int\limits_0^1 \left(\varphi^{(m)}(x)\right)^2 dx$ in the $L_2^{(m)}(0,1)$ space are constructed. Explicit formulas for the coefficients of the interpolation splines are obtained. The obtained interpolation spline is exact for polynomials of degree $m-1$. Some numerical experiments are presented. Moreover, the connection between the obtained interpolation splines and the optimal quadrature formulas are shown.

The works [15, 16] of Kh.M.Shadimetov, A.R.Hayotov, F.A.Nuraliev are devoted to construction formulas with derivative for optimal interpolation in the Soboloev space $L_2^{(m)}(0,1)$ using a discrete analogue of the differential operator d^{2m}/dx^{2m}. Authors also obtained explicit formulas for the coefficients of the optimal interpolation formulas.

Recently, in [2], the authors constructed optimal interpolation formulas in the Hilbert space $W_2^{(m,m-1)}$ and, by integrating both sides of this formula, obtained optimal quadrature formulas in the sense of Sard in the same space that were constructed in [14].

Now we consider the problem of optimal interpolation formulas, which was first posed and studied by S.L.Sobolev in 1961 [17].

Assume we are given a table of values $\varphi(x_\beta)$, $\beta=0,1,\ldots,N$ of a function φ at the points $x_\beta\in[0,1]$. It is required to approximate the function φ by another more simple function P_φ, i.e.

$$\varphi(z)\cong P_\varphi(z)=\sum_{\beta=0}^{N} C_\beta(z)\varphi(x_\beta), \tag{2}$$

where $P_\varphi(z)=\sum\limits_{\beta=0}^{N} C_\beta(z)\varphi(x_\beta)$ is an interpolation formula and

$$\ell(x,z)=\delta(x-z)-\sum_{\beta=0}^{N} C_\beta(z)\delta(x-x_\beta) \tag{3}$$

is the error functional of this interpolation formula, $C_\beta(z)$ are the coefficients, and x_β are the nodes of the interpolation formula $P_\varphi(z)$, $x_\beta\in[0,1]$, δ is Dirac's delta-function, the function φ belongs to the Hilbert space $W_2^{(2,0)}(0,1)$. The norm of functions in this space is defined as follows

$$\|\varphi\|_{W_2^{(2,0)}}=\left[\int\limits_0^1 \left(\varphi''(x)+\varphi(x)\right)^2 dx\right]^{1/2}. \tag{4}$$

The difference of the interpolation formula (2) is called *the error*

$$
\begin{aligned}
(\ell,\varphi) &= \varphi(z)-P_\varphi(z)=\varphi(z)-\sum_{\beta=0}^{N}C_\beta(z)\varphi(x_\beta)= \\
&= \int\limits_{-\infty}^{\infty}\left(\delta(x-z)-\sum_{\beta=0}^{N}C_\beta(z)\delta(x-x_\beta)\right)\varphi(x)\,dx. \qquad (5)
\end{aligned}
$$

By the Cauchy-Schwarz inequality, the absolute value of the error (4) is estimated as follows

$$|(\ell,\varphi)|\le\|\varphi\|_{W_2^{(2,0)}}\cdot\|\ell\|_{W_2^{(2,0)*}},$$

where

$$\|\ell\|_{W_2^{(2,0)*}}=\sup_{\varphi,\|\varphi\|\neq 0}\frac{|(\ell,\varphi)|}{\|\varphi\|_{W_2^{(2,0)}}}.$$

Therefore, to estimate the error of the interpolation formula (2) over the functions of the space $W_2^{(2,0)}$, we need to find the norm of the error functional ℓ in the conjugate space $W_2^{(2,0)*}$.

From here we get

Problem 1. *Find the norm of the error functional ℓ of the interpolation formula (2) in the space $W_2^{(2,0)}$.*

It is clear that the norm of the error functional ℓ depends on the coefficients $C_\beta(z)$ and the nodes x_β. The problem of minimizing of the quantity $\|\ell\|$ by coefficients $C_\beta(z)$ is a linear problem and by nodes x_β is, in general, a nonlinear and complicated problem. Here we consider the problem of minimizing of the quantity of $\|\ell\|$ by coefficients $C_\beta(z)$ when nodes x_β are fixed.

The coefficients $\mathring{C}_\beta(z)$ (if there exist) satisfying the following equality

$$\left\|\mathring{\ell}\right\|_{W_2^{(2,0)*}}=\inf_{C_\beta(z)}\|\ell\|_{W_2^{(2,0)*}} \qquad (6)$$

are called *the optimal coefficients* and corresponding interpolation formula

$$\mathring{P}_\varphi(z)=\sum_{\beta=0}^{N}\mathring{C}_\beta(z)\varphi(x_\beta)$$

is called *the optimal interpolation formula* in the space $W_2^{(2,0)*}(0,1)$.

Thus, in order to construct the optimal interpolation formula in the space $W_2^{(2,0)}(0,1)$ we need to solve the next problem.

Problem 2. *Find the coefficients $\mathring{C}_\beta(z)$ which satisfy equality (6) when the nodes x_β are fixed.*

The main aim of this work is to solve Problem 2, i.e. to find an explicit expression for square of the norm of the error functional of the optimal interpolation formula in the space $W_2^{(2,0)}$.

2. The extremal function and the representation of the error functional norm

In this section, we solve Problem 1, i.e., we find an explicit expression of the norm of the error functional ℓ. Here we will use the extremal function of this functional.

The function ψ_ℓ for which the equality is performed

$$(\ell,\psi_\ell)=\|\ell\|_{W_2^{(2,0)*}}\cdot\|\psi_\ell\|_{W_2^{(2,0)}} \tag{7}$$

is called the extremal function of the error functional ℓ [18, 19, 20].

Since $W_2^{(2,0)}$ is a Hilbert space, then by Riesz's theorem on the general form of a linear continuous functional in Hilbert spaces, for the error functional $\ell\in W_2^{(2,0)*}$ there is a unique function $\psi_\ell\in W_2^{(2,0)}$ such that for any $\varphi\in W_2^{(2,0)}$ the following equality is satisfied

$$(\ell,\varphi)=<\psi_\ell,\varphi>_{W_2^{(2,0)}} \tag{8}$$

and $\|\ell\|_{W_2^{(2,0)*}}=\|\psi_\ell\|_{W_2^{(2,0)}}$, where $<\psi_\ell,\varphi>_{W_2^{(2,0)}}$ is the inner product of two functions ψ_ℓ and φ from the space $W_2^{(2,0)}$. Recall that the inner product $<\psi_\ell,\varphi>_{W_2^{(2,0)}}$ is defined as follows

$$<\psi_\ell,\varphi>_{W_2^{(2,0)}}=\int_0^1\left(\psi_\ell''(x)+\psi_\ell(x)\right)\left(\varphi''(x)+\varphi(x)\right)\,dx.$$

In particular, from (9) for $\varphi=\psi_\ell$ we have

$$(\ell,\psi_\ell)=<\psi_\ell,\psi_\ell>_{W_2^{(2,0)}}=\|\psi_\ell\|^2_{W_2^{(2,0)}}=\|\ell\|^2_{W_2^{(2,0)*}}.$$

From this it can be seen that the solution ψ_ℓ of equation (9) satisfies the equation (8) and is an extremal function. Thus, to calculate the norm of the error functional ℓ, at first we need to find the extremal function ψ_ℓ from equation (9), and then calculate the square of the norm of the error functional ℓ as follows

$$\|\ell\|^2_{W_2^{(2,0)*}}=(\ell,\psi_\ell). \tag{9}$$

Let's solve equation (9). Integrating the right-hand side of equation (9) in parts, we have

$$\begin{aligned}(\ell,\varphi)\ &=\ \left(\psi_\ell''(x)+\psi_\ell(x)\right)\varphi'(x)\big|_0^1-\left(\psi_\ell'''(x)+\psi_\ell'(x)\right)\varphi(x)\big|_0^1+\\ &+\ \int_0^1\Big(\psi_\ell^{(4)}(x)+2\psi_\ell^{(2)}(x)+\psi_\ell(x)\Big)\varphi(x)\,dx.\end{aligned} \tag{10}$$

Hence, taking into account the uniqueness of the function ψ_ℓ, we obtain the equation

$$\psi_\ell^{(4)}(x)+2\psi_\ell^{(2)}(x)+\psi_\ell(x)=\ell(x) \tag{11}$$

with boundary conditions

$$\left(\psi_\ell''(x)+\psi_\ell(x)\right)\big|_{x=0}^{x=1}=0, \tag{12}$$

$$\left(\psi_\ell'''(x)+\psi_\ell'(x)\right)\big|_{x=0}^{x=1}=0. \tag{13}$$

For solution of equation (13) with the boundary conditions (14)-(13), the following is true

Theorem 1. *The solution of equation (13) with the boundary conditions (14)-(13) is an extremal function ψ_ℓ of the error functional ℓ of the interpolation formula (2) and has the form:*

$$\psi_\ell(x) = (\ell * G_2)(x) + d_1 \sin(x) + d_2 \cos(x),$$

where d_1 and d_2 are arbitrary real numbers, and

$$G_2(x) = \frac{\mathrm{sgn} x}{4}\left[\sin(x) - x \cdot \cos(x)\right] \tag{14}$$

is the solution of the equation

$$\psi_\ell^{(4)}(x) + 2\psi_\ell^{(2)}(x) + \psi_\ell(x) = \delta(x).$$

Proof. As it is known from the theory of linear differential equations, the general solution of an inhomogeneous differential equation is the sum of the general solution of the corresponding homogeneous equation and the partial solution of this equation. Therefore, first we find the general solution of the following homogeneous equation corresponding to equation (13):

$$\psi_\ell^h(x) = d_1 \sin(x) + d_2 \cos(x) + d_3 x \ \sin(x) + d_4 x \ \cos(x),$$

where d_k, $k = 1,2,3,4$, are arbitrary constants. It is not difficult to verify that a particular solution of the differential equation (13) can be expressed as a convolution of the functions $\ell(x)$ and $G_2(x)$. The function $G_2(x)$ is a fundamental solution of the equation (13), and it is determined by (14).

It should be noted that the following rule for finding a fundamental solution of a linear differential operator

$$P_m\left(\frac{d}{dx}\right) := \frac{d^m}{dx^m} + a_1 \frac{d^{m-1}}{dx^{m-1}} + \cdots + a_m,$$

where a_j are real numbers, is given in [22, p. 88-89].

The rule: replacing $\frac{d}{dx}$ by p instead of the operator $P_m(\frac{d}{dx})$ we get the polynomial $P_m(p)$. We expand the expression $\frac{1}{P_m(p)}$ to partial fractions

$$\frac{1}{P_m(p)} = \prod_j (p-\lambda)^{-k_j} = \sum_j \left[c_{j,k_j}(p-\lambda_j)^{-k_j} + \ldots + c_{j,1}(p-\lambda_j)^{-1}\right]$$

and to every partial fraction $(p-\lambda)^{-k}$ we correspond the expression $\frac{x^{k-1}\mathrm{sgn} x}{2(k-1)!} \cdot e^{\lambda x}$.

Using this rule, it is found the function $G_2(x)$ which is the fundamental solution of the operator $\frac{d^4}{dx^4} + 2\frac{d^2}{dx^2} + 1$ and has the form (14).

Further, the partial solution of equation (13) is $\ell(x) * G_2(x)$, where $G_2(x)$ satisfies equation (14). Really,

$$(\ell(x) * G_2(x))^{(4)} + (\ell(x) * G_2(x))^{(2)} + (\ell(x) * G_2(x)) =$$

$$= \ell(x) * \left(G_2^{(4)}(x) + G_2^{(2)}(x) + G_2(x)\right) = \ell(x) * \delta(x) = \ell(x).$$

Thus, the general solution of equation (13) has the following form

$$\psi_\ell(x) = (\ell * G_2)(x) + d_1\sin(x) + d_2\cos(x) + d_3x\ \sin(x) + d_4x\ \cos(x). \quad (15)$$

In order for the function $\psi_\ell(x)$ to be unique in the space $W_2^{(2,0)}(0,1)$, it must satisfy (14)-(13). Here, derivatives are understood in a generalized sense. In calculations, we need the first three derivatives of the function $G_2(x)$:

$$\begin{aligned} G_2'(x) &= \frac{\mathrm{sgn}x}{4}\cdot x\cdot\sin(x),\\ G_2''(x) &= \frac{\mathrm{sgn}x}{4}\cdot[\sin(x)+x\cdot\cos(x)],\\ G_2'''(x) &= \frac{\mathrm{sgn}x}{4}\cdot[2\cos(x)-x\cdot\sin(x)]. \end{aligned} \quad (16)$$

The following well-known formulas from the theory of generalized functions [22] are used

$$(\mathrm{sgn}x)' = 2\delta(x), \qquad \delta(x)f(x) = \delta(x)f(0).$$

Next, using the well-known formula

$$(f(x)*g(x))' = f'(x)*g(x) = f(x)*g'(x),$$

from (15), taking into account (16), we get

$$\begin{aligned} \psi_\ell(x) &= \ell(x)*G_2(x)+d_1\sin(x)+d_2\cos(x)+d_3x\cdot\sin(x)+d_4x\cdot\cos(x),\\ \psi_\ell'(x) &= \ell(x)*G_2'(x)+d_1\cos(x)-d_2\sin(x)+d_3[\sin(x)+x\cdot\cos(x)]+\\ &\quad +d_4[\cos(x)-x\cdot\sin(x)],\\ \psi_\ell''(x) &= \ell(x)*G_2''(x)-d_1\sin(x)-d_2\cos(x)+d_3[2\cos(x)-x\cdot\sin(x)]-\\ &\quad -d_4[2\sin(x)+x\cdot\cos(x)],\\ \psi_\ell'''(x) &= \ell(x)*G_2'''(x)-d_1\cos(x)+d_2\sin(x)-d_3[3\sin(x)+x\cdot\cos(x)]-\\ &\quad -d_4[3\cos(x)-x\cdot\sin(x)]. \end{aligned} \quad (17)$$

Then, using (17), meaning (16), from (14)-(13), substituting instead of x, respectively, $x=0, x=1$ and the resulting expressions, equating to zero, we obtain the following system of linear equations:

$$\begin{cases} (\ell(y),\cos(y)) - 2d_4 = 0,\\ (\ell(y),\sin(y)) + 2d_3 = 0,\\ (\ell(y),\cos(1-y)) - 2d_3\sin(1) - 2d_4\cos(1) = 0,\\ (\ell(y),\sin(1-y)) + 2d_3\cos(1) - 2d_4\sin(1) = 0. \end{cases}$$

Hence we have $d_3 = 0$, $d_4 = 0$, and therefore

$$(\ell(y),\sin(y)) = 0, \quad (\ell(y),\cos(y)) = 0. \quad (18)$$

Substituting these values into equality (15) we get the assertion of this theorem. Thus, Theorem 1 is proved □.

The equalities in (18) obtained in the proof of Theorem 1 mean that our interpolation formula will be exact on the functions $\sin(x)$ and $\cos(x)$.

Now, taking the equality (11) and using Theorem 1, we calculate the norm of the error functional. For the square of the norm of the error functional ℓ, taking into account (18), from equality (11) we have

$$\|\ell\|^2_{W_2^{(2,0)*}} = (\ell,\psi_\ell) = \int_{-\infty}^{\infty} \ell(x)\psi_\ell(x)\,dx \int_{-\infty}^{\infty} \ell(x)\,(G_2(x-z)-$$

$$-\sum_{\beta=0}^{N} C_\beta(z)G_2(x-x_\beta)+d_1\sin(x)+d_2\cos(x)\Bigg)\,dx = \int_{-\infty}^{\infty}(\delta(x-z)-$$

$$-\sum_{\beta=0}^{N} C_\beta(z)\delta(x-x_\beta)\Bigg)\left(G_2(x-z)-\sum_{\beta=0}^{N} C_\beta(z)G_2(x-x_\beta)\right)\,dx.$$

Since $G_2(x)$ is an even function,

$$G_2(x_\beta-x)=G_2(x-x_\beta),$$

we get the equality

$$\begin{aligned}\left\|\ell|W_2^{(2,0)*}\right\|^2 &= \sum_{\beta=0}^{N}\sum_{\gamma=0}^{N} C_\beta(z)C_\gamma(z)\,G_2(x_\beta-x_\gamma)-\\ &\quad -2\sum_{\beta=0}^{N} C_\beta(z)G_2(z-x_\beta).\end{aligned} \tag{19}$$

Thus, Problem 1 is solved.

3. Conclusion

Thus, in this paper, we solved the first part of the problem, i.e. we found an explicit expression for square of the norm of the error functional of the optimal interpolation formula in the Hilbert space $W_2^{(2,0)}(0,1)$.

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ИНФОРМАЦИОННЫЕ И ВЫЧИСЛИТЕЛЬНЫЕ ТЕХНОЛОГИИ

УДК 519.64 Научная статья

Экстремальная функция интерполяционных формул в пространстве $W_2^{(2,0)}$

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Одна из основных проблем вычислительной математики — оптимизация вычислительных методов в функциональных пространствах. Оптимизация вычислительных методов хорошо проявляется в задачах теории интерполяционных формул. В данной статье исследуется проблема построения оптимальной интерполяционной формулы в гильбертовом пространстве. Здесь с помощью метода Соболева решается первая часть задачи — явное выражение квадрата нормы функционала погрешности оптимальных интерполяционных формул в гильбертовом пространстве $W_2^{(2,0)}$.

Ключевые слова: оптимальные интерполяционные формулы, функционал погрешности, экстремальная функция, гильбертово пространство.

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