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On some new estimates for integrals of the square function and analytic Bergman type classes in some domains in $\mathbb{C}^n$

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The purpose of the note is to obtain equivalent quasinorm, sharp estimates for the quasinorm of Hardy's and new Bergman's analytic classes of in the polydisk. We extend some classical onedimensional assertions to the case of several complex variables. Our results more precisely provide direct new extention of some known one variable theorems concerning area integral to the case of simplest product domains namely the unit polydisk in $\mathbb{C}^n$. Let further D be a bounded or unbounded domain in $\mathbb{C}^n$. For example, tubular domain over symmetic cone or bounded pseudoconvex domain with smooth boundary. Our results can be probably extended to the case of products of such type complicated domains, namely even to $D\times\cdots\times D$. This can be probably done based on some approaches we suggested and used in this paper. On the other hand our results in simpler case namely in the unit polydisk may also have various interesting applications in complex function theory in the unit polydisk. We finnaly provide similar type sharp. results in some new Bergman spaces in bounded strongly pseudoconvex domains

Keywords: polydisk, integral operators, analytic functions, analytic spaces, Hardy class, new Bergman space, pseudoconvex domains.

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Introduction

In Euclidean space R^n the problem of finding equivalent quasinorm and exact estimates for the norms of certain function spaces has been solved by many authors (see, for

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example, [19]). Recently, a large number of papers have appeared, where a similar problem was considered for analytic functions in the disk. Different equivalent quasinorms for analytic new Bergman type classes in disk $U = \{z : |z| < 1\}$ of complex plane $\mathbb{C}$ were derived in [10], [11],[20], [21]. The papers [1],[3] gives several characteristics of the analytic classes in bidisk. Finally, at [1], [14] an equivalent quasinorm is indicated for space Lizorkin-Triebel in the polydisk. In the first paragraph well-known estimates for the quasi-norm Hardy classes will be generalized to the case of the polydisk. In the second paragraph, equivalent quasinorms for the new Bergman type classes introduced in [14] are derived. It should be noted that the new functional spaces (see [7]) and their properties modified by us for the case of the polydisk play an essential role in the note.

We finnaly at the end of the paper provide new similar type sharp results in new Bergman type analytic function spaces in bounded pseudoconvex domains with smooth boundary. Note here we again choose the context of the unit disk, then show in details how to pass the same proof to these general domains.

New estimates of the quasinorm of Hardy classes in the polydisk

In this section we will extend some classical onedimensional assertions to Hardy spaces of several variables. We will present the following notation. Let U^n - be the unit polydisk of $\mathbb{C}^n$, $U^n = \Big\{z \in C^n, |z_j| < 1, j = 1,\ldots,n\Big\}$, $T^n = \Big\{z \in C^n, |z_j| = 1, j = 1,\ldots,n\Big\}$ — $m_n(\xi)$ and $m_{2n}(\omega)$ be the normalized Lebesgue measures on T^n and U^n accordingly.

Let further $\widetilde{\Gamma_\alpha}(\xi) = \widetilde{\Gamma_\alpha}(\xi_1,\ldots,\xi_n) = \Gamma_{\alpha_1}(\xi_1)\times\cdots\times\Gamma_{\alpha_n}(\xi_n)$, where $\alpha_i > 0, \xi_i \in T, i = 1,\ldots,n,$

$$\Gamma_{\alpha_i}(\xi_i) = \Big\{z \in U : \Big|1-\overline{\xi}_i z\Big| < \alpha_i\Big(1-|z|^2\Big)\Big\}, i = 1,\ldots,n;$$

and let also

$$I_{\overline{U}^n}(\xi_1,\ldots,\xi_n,t_1,\ldots,t_n) = \Big\{z \in \overline{U}^n : \Big|1-z_1\overline{\xi}_1\Big| < t_1,\ldots,\Big|1-z_n\overline{\xi}_n\Big| < t_n\Big\};$$

$t_j > 0, j = 1,\ldots,n.$

We omit α_j below sometimes dealing with $\Gamma_\alpha(\xi)$. $I_{U^n} = I_{\overline{U}^n}\cap U^n, I_{T^n} = I_{\overline{U}^n}\cap T^n$.

For a measurable f function in U^n we put

$$\Big(A_q(f)\Big)(\xi) = \left(\int\limits_{\widetilde{\Gamma}\alpha(\xi)} \frac{|f(z_1,\ldots,z_n)|^q dm_{2n}(z)}{(1-|z|)^2}\right)^{1/q}, q < \infty;$$

$$\Big(A_\infty(f)\Big)(\xi) = \sup_{z\in U^n}\Big\{|f(z_1,\ldots,z_n)|, z \in \widetilde{\Gamma}_\alpha(\xi)\Big\}, \xi \in T^n.$$

$$\Big(\Big(C_q(f)\Big)(\xi)\Big)^q =$$

$$= \sup_{t_n}\frac{1}{|I_T(\xi_n,t_n)|}\int\limits_{I_U(\xi_n,t_n)}\frac{1}{(1-|z_n|)}\cdots\sup_{t_1}\frac{1}{|I_T(\xi_1,t_1)|}\int\limits_{I_U(\xi_1,t_1)}\frac{|f(z_1,\ldots,z_n)|^q}{(1-|z_1|)}dm_{2n}(z),$$

$\xi \in T^n, \xi = (\xi_1,\ldots,\xi_n), t_j > 0, j = 1,\ldots,n.$

Also let $H(U^n)$ and $H^p(U^n), 0 < p < \infty$ are the space of all holomorphic functions in U^n and Hardy class in the polydisk accordingly.

$$H^p(U^n) = \left\{ f \in H(U^n) : \sup_r \int_{T^n} |f(r\xi)|^p dm_n(\xi) < \infty \right\}, 0 < p \leq \infty.$$

These are Banach spaces for all $p \geq 1$, and quazinormed spaces for other values of parameters.

Remark 1. The values given above in R^{n+1} were first introduced in [7] for the so-called new functional spaces in R^{n+1}.

A well-known statement of the theory of Hardy H^p spaces states that if a function f belongs to the Hardy class $H^p, 0 < p < \infty$, then the quantity $S(f)$ is finite.

$$S(f) = \int_T \left(\int_{\Gamma(\xi)} \left| D^k f(z) \right|^2 (1-|z|)^{2k-2} dm_2(z) \right)^{p/2} dm(\xi) < \infty.$$

D^k is a well-known differential operator of an analytic f function in the unit disk on the complex plane $\mathbb{C}$ (see, for example, [6], [8]) .

S operator is known as the Luzin area integral operator and the above statement about its boundedness in Hardy H^p classes, $0 < p < \infty$ in the disk U was established in [22]. In Theorem 1, relying on the multidimensional maximum theorem established in [22], two generalizations are given - direct polydisk analogues of this statement for $p < 2$.

In Theorem 2 for $p \geq 2$ this result will be generalized in two different ways at once. Let D^α be the fractional derivative of the f function, $\alpha \geq 0$,
$D^\alpha : H(U^n) \to H(U^n)$,

$$(D^\alpha f)(z_1, \ldots, z_n) = \sum_{|k| \geq 0} (k+1)^\alpha \alpha_{k_1 \ldots k_n} z_1^{k_1} \ldots z_1^{k_1}, (z_1, \ldots, z_n) \in U^n,$$

$$(k+1) = \prod_{j=1}^n (k_j + 1), (\text{see } [8],[5],[21]).$$

We define fractional derivative also for negative indexes, namely we put

$$D^{-\alpha}(D^\alpha)(f) = f, for \quad \alpha \geq 0.$$

Let further $C_1, C_2, C(n)$ be various positive constants.

Everywhere below the notation $A < B$ denotes that there is the constant $C > 0$ such that $A \leq CB$, the notation $A = B$ denotes that there are the constants $C_1 > 0$ and $C_2 > 0$ such that $C_2 B \leq A \leq C_1 B$.

Theorem 1.

A) Let $f \in H(U^n)$, *Assume that*

$$F(z_1, \ldots, z_n) = \sum_{k_1 \ldots k_n \geq 0}^{+\infty} (k_2+1) \ldots (k_n+1) a_{k_1 \ldots k_n} z_1^{k_1} \ldots z_n^{k_n}, \qquad (1)$$

is in class $H^p(U^n), 0 < p < 2$,

$$S_n(f) = \left(\int_{T^n} \left(\int_{\Gamma(\xi_1)} \cdots \int_{\Gamma(\xi_n)} \left| Df(z_1, \ldots, z_n) \right|^2 dm_2(z) \right)^{p/2} dm_n(\xi) \right)^{1/p}.$$

Then next estimates are correct

$$C_1\left|\left|D^{-\varepsilon}f\right|\right|_{H^p(U^n)} \le S_n(f) \le C_2||f||_{H^p(U^n)}, 0<p<2, \qquad (2)$$

where ε *- arbitrary positive number and* $D(f) = D^1(f)$.

B) Let $0<p<2, f\in H(U^n)$*. Then next estimates are correct*

$$C_1\left|\left|D^{-\varepsilon}f\right|\right|_{H^p(U^n)} \le \widetilde{S}_n(f) \le C_2||f||_{H^p(U^n)},$$

where ε *- arbitrary positive number and*

$$\widetilde{S}_n(f) = \Bigg(\int\limits_T \Big(\int\limits_{\Gamma(\xi_n)} \dots \Big(\int\limits_T \Big(\int\limits_{\Gamma(\xi_1)} \left|Df\right|^2 dm_2(z_1)\Big)^{\frac{p}{2}} \cdot$$

$$\cdot\, dm(\xi_1)\Big)^{\frac{2}{p}} \dots dm_2(z_n)\Big)^{\frac{p}{2}} dm(\xi_n)\Bigg)^{\frac{1}{p}}. (3)$$

Remark 2. For $n=1$ the condition (1) disappears and the statements of the theorem coincide and are well known (see [21],[22]).

Remark 3. It will be seen from the proof that by slightly modifying the reasoning in a similar way a somewhat more general result for Hardy-Sobolev classes can be proved. They are defined as follows:

$$H^p_\alpha = \left\{f\in H(U^n): \left|\left|D^\alpha f\right|\right|_{H^p} < \infty\right\},$$

$\alpha \ge 0, P\in(0,\infty)$.

Proof of Theorem 1.

Lemma 1. Let $f\in H(U^n)$, then next estimates are correct

$$\int\limits_{T^n}\left(\sup_{z\in\widetilde{\Gamma}(\xi)}\left|D^\alpha f(z)\right|(1-|z|)^\alpha\right)^p dm_n(\xi) \le C_1||f||^p_{H^p}, 0<p<+\infty, \alpha\ge 0; \qquad (4)$$

$$\underbrace{\int\limits_T \cdots \int\limits_T}_{n-1}\int\limits_U \left|Df(z_1,\xi_2,\dots,\xi_n)\right|^2 \times$$

$$\times\left|z_1F(z_1,\xi_2,\dots,\xi_n)\right|^{p-2}(1-|z_1|)dm_2(z_1)dm_1(\xi_2)\dots dm_1(\xi_n) \le C_2||F||^p_{H^p(U^n)}, (5)$$

$0<p<\infty$;

$$\int\limits_{U^n}\left|z_1F(z_1,\dots,z_n)\right|^{p-2}\left|Df(z_1,\dots,z_n)\right|^2(1-|z_1|)\dots(1-|z_n|)dm_2(z_1,\dots,z_n) \le$$

$$\le C_3\int\limits_U\cdots\int\limits_U\int\limits_T\left|F(\xi_1,z_2,\dots,z_n)\right|^p(1-|z_2|)\dots(1-|z_n|)dm(\xi)dm_2(z_2),\dots,dm_2(z_n).$$

Here and hereafter F is defined by (1) $0 < p < \infty$. The first inequality is proved in [15] and relies on the multivariable maximal theorem proved in [16], [22]. The second and third inequalities are not difficult to establish directly relying on one consequence of Green's formula (see [16], [22]).

$$\int_{-\pi}^{\pi} W(e^{i\theta})d\theta = \int_U \left(\log\frac{1}{|z|}\right)\Delta W(z)dxdy,$$

where Δ is the Laplace operator, $W \in C^2(U \cup T), W(0) = 0$.

Selecting the function $\left(|zF(\rho z, z_2, \ldots, z_n)|^2 + \varepsilon\right)^{\frac{p}{2}}, 0 < p < 1, \varepsilon > 0$ as W and applying this formula we get

$$\int_{-\pi}^{\pi}\left(|F(\rho e^{i\theta_1}, z_2, \ldots, z_n)|^2 + \varepsilon\right)^{\frac{p}{2}} d\theta_1 =$$

$$= \int_U \log\frac{1}{|z|}\Delta\left(|zF(\rho z, z_2, \ldots, z_n)|^2 + \varepsilon\right)^{\frac{p}{2}} dm_2(z) + 2\pi\varepsilon^{\frac{p}{2}};$$

passing to the limit at $\varepsilon \to 0$ and given that $\Delta F = 4\frac{\partial^2 F}{\partial z \partial \bar{z}}$ we get

$$\int_{-\pi}^{\pi}\left|F\left(\rho e^{i\theta_1}, z_2, \ldots, z_n\right)\right|^p d\theta_1 = \frac{p^2\rho^2}{4}\int_U |zF(\rho z, z_2, \ldots, z_n)|^{p-2}\times$$

$$\times|Df(\rho z, z_2, \ldots, z_n)|^2 \log\frac{1}{|z|}dm_2(z).$$

Now the inequalities (2) and (3) are easily obtained by relying on this proportion. We can integrate both parts either by T^{n-1} with measure $dm(\xi_2)\ldots dm(\xi_n)$ or by U^{n-1} with measure $(1-|z_2|)\ldots(1-|z_n|)dm_2(z_2)\ldots dm_2(z_n)$ appropriately. From here we have for $0 < p < 2$

$$\int_{U^n}\left|F(z_1, \ldots, z_n)\right|^{p-2}\left|Df(z_1, \ldots, z_n)\right|^2(1-|z_1|)\ldots(1-|z_n|)dm_2(z) \le$$

$$\le C_1 \sup_{|z_1|,\ldots,|z_n|}\left(\int_{T^n}\left|F(z_1, \ldots, z_n)\right|^p dm_n(\xi)(1-|z_2|)^p\ldots(1-|z_n|)^p\left(\int_0^1(1-r)^{1-p}dr\right)^{n-1}\right) <$$

$$< C_2||f||^p_{H^p}, 0 < p < 2. \quad (6)$$

Let us first establish that condition (2) is necessary for f function to belong to the Hardy H^p class. Let $0 < p < 2$, then we have

$$S^p_n(f) \le C_1\int_{T^n}\left(\sup_{z\in\Gamma(\xi)}\left|F(z_1, \ldots, z_n)\right|^{\frac{(2-p)p}{2}}\right)\left(\int_{\Gamma(\xi_1)}\cdots\int_{\Gamma(\xi_n)}\frac{|Df|^2 dm_{2n}(z)}{|F(z)|^{2-p}}\right)^{p/2} dm_n(\xi).$$

Recall that F is defined in equality (1). Next we apply the Holder's inequality with the exponent $p' = \frac{2}{2-p}, q' = \frac{2}{p}$ and (7') and we obtain

$$S_n^p(f) \le C_2 \int_{T^n} \left(\sup_{z\in\Gamma(\xi)} \left|F(z)\right|^p dm_n(\xi) \right)^{\frac{2-p}{2}} \times \left(\int_{U_n} |F(z)|^{2-p} |Df(z)|^2 (1-|z|) dm_{2n}(z) \right)^{p/2}.$$

Next we use the condition of the Theorem and estimates (4) and (6) respectively, we get

$$S_n^p(f) \le C ||F||_{H^p(U^n)}^{\frac{(2-p)p}{2}} ||f||_{H^p(U^n)}^{\frac{p^2}{2}}.$$

From here it is easy to get $S_n^p(f) |\le C|f||_{H^p}^p, 0 < p < 2$.

Let us now prove the left inequality.

Let $p \le 1$. Then from known estimate (see [8]) we have

$$\left(\int_{U^n} |f(z)|(1-|z|)^\alpha dm_{2n}(z) \right)^p \le C_3 \int_{U^n} |f(z)|^p (1-|z|)^{\alpha p + 2p - 2} dm_{2n}(z), \quad (6')$$

$p \le 1, (\alpha+2)p > 1, f \in H(U^n)$.

Hence we deduce

$$\int_{T^n} |f(r_1\xi_1, \dots, r_n\xi_n)|^p dm_n(\xi) <$$

$$\le C_4 \int_{T^n} \left(\int_{U^n} |Df(w)|^p \left| D^\alpha \left(\frac{1}{1-\bar{w}z} \right) \right|^p (1-|w|)^{\alpha p + 2p - 2} dm_{2n}(w) \right) dm_n(\xi) <$$

$$\le C_5 \int_{U^n} |Df(w)|^p (1-|w|)^{p-1} dm_{2n}(w), \quad (6'')$$

$r_j \in (0,1), j = 1, \dots, n$.

For $1 < p \le 2$ the estimate (6") can be obtained similarly using instead (6') estimate

$$\left(\int_{U^n} \frac{|f(z)|(1-|z|)^\alpha}{|1-\bar{w}z|^{\beta+2}} dm_{2n}(z) \right)^p \le C_6 \int_{U^n} \frac{|f(z)|^p (1-|z|)^{\alpha p} (1-|w|)^{-\varepsilon p}}{|1-\bar{w}z|^{(\beta-\varepsilon)p+2}} dm_{2n}(z),$$

where $\alpha > -1, \varepsilon > 0, \beta > 0$, which is easily obtained from the Holder's inequality. Next, we will need the following estimates, they are well-known at $n = 1$ (see [7]),

$$\left| \int_{U^n} \frac{|f(z)| g(\bar{z})}{|1-z|} dm_{2n}(z) \right| \le C_7 \int_{T^n} \Big(A_{q'}(f)(\xi) \Big) \Big(C_q(g)(\xi) \Big) dm_n(\xi) \quad (7)$$

$$\int\limits_{U^n} |\Phi(z)|(1-|z|)^{\alpha} dm_{2n}(z) \le C_8 \int\limits_{T^n} \int\limits_{\Gamma(\xi)} |\Phi(z)|(1-|z|)^{\alpha-1} dm_{2n}(z) dm_n(\xi). \qquad (7')$$

Estimates (7) and (7') are not difficult to obtain from one-dimensional version by sequential application over each variable. Based on (7') and (6") we deduce

$$\left|\left|D^{-\varepsilon} f\right|\right|^p_{H^p} \le C_9 \int\limits_{U^n} \left|Df(w)\right|^p (1-|w|)^{p-1}(1-|w|)^{\varepsilon} dm_{2n}(w) \le$$

$$\le C_{10} \int\limits_{T^n} \left(\int\limits_{\Gamma(\xi_1)} \cdots \int\limits_{\Gamma(\xi_n)} \frac{\left(\left|Df(w)\right|^p (1-|w|)^p\right)^{2/p}}{(1-|w|)^2} dm_{2n}(w) \right)^{p/2} \times$$

$$\times \left(\int\limits_{\Gamma(\xi_1)} \cdots \int\limits_{\Gamma(\xi_n)} (1-|w|)^{-2+\varepsilon(2/p)} dm_{2n}(w) \right)^{1\big/(2/p)'} dm_n(\xi) \le C_{11}(S_n(f))^p.$$

The first part of the theorem is established.

Remark 4.

Note that above we (right inequality) have modified the reasoning applicable to $n=1$, previously by various authors (see, for example, [5], [6], [16], [22]).

An essential role in our reasoning is played by the maximum theorem in U^n, established in [22], the integrals of the squares by Luzin and values $A_q(f)$ and $C_q(f)$ in U^n chosen in a suitable way.

We establish the second statement of the theorem. Applying the estimate (7') for each variable separately with the exponent $2/p > 1$ and repeating the above reasoning for each variable, we obtain

$$\int\limits_{T^n} \left|D^{-\varepsilon} f(r\xi)\right|^p dm_n(\xi) \le C_{12} \int\limits_{U^n} \frac{\left|Df(w)\right|^p (1-|w|)^{p+\varepsilon}}{(1-|w|)} dm_{2n}(w) \le$$

$$\le C_{13} \int\limits_{U^{n-1}} \int\limits_{T} \left(\int\limits_{\Gamma(\xi_1)} |Df(w)|^2 dm_2(w_1) \right)^{p/2} dm(\xi_1) \times$$

$$\times \frac{((1-|w_2|)\dots(1-|w_n|))^{p+\varepsilon}}{(1-|w_2|)\dots(1-|w_n|)} dm_{2n-2}(w) \le C_{14} \left(\widetilde{S}_n(f)\right)^p.$$

Let us now prove the right inequality. For this we modify the arguments given in proving of the corresponding inequality A).

Let

$$F = \frac{\partial^{n-1} w_2 \dots w_n f}{\partial w_2 \dots \partial w_n}.$$

Reasoning similarly we have

$$\widetilde{S}_1 = \int_T \left(\int_{\Gamma(\xi_1)} \left| \frac{\partial^n z_1 \dots z_n f(z)}{\partial z_1 \dots \partial z_n} \right|^2 dm_2(z_1) \right)^{p/2} dm(\xi_1) \leq$$

$$\leq C_{15} \left(\int_T \sup_{z_1 \in \Gamma(\xi_1)} |F(z)|^p \, dm(\xi_1) \right)^{\frac{2-p}{2}} \cdot$$

$$\cdot \left(\int_U |F(z)|^{p-2} \left| \frac{\partial^n (z_1 \dots z_n f)}{\partial z_1 \dots \partial z_n} \right|^2 (1-|z_1|) dm_2(z_1) \right)^{p/2} \leq$$

$$\leq C_{16} \left\| \frac{\partial^{n-1}(z_2 \dots z_n f(z))}{\partial z_2 \dots \partial z_n} \right\|_{H^p_{z_1}(U)}^{p(1-p/2)} \cdot \left\| \frac{\partial^{n-1}(z_2 \dots z_n f(z))}{\partial z_2 \dots \partial z_n} \right\|_{H^p_{z_1}(U)}^{p\cdot(p/2)} =$$

$$= \left\| \frac{\partial^{n-1}(z_2 \dots z_n f(z))}{\partial z_2 \dots \partial z_n} \right\|_{H^p_{z_1}(U)}^{p}$$

We integrate the last inequality by $\Gamma(\xi_2)$ and T by z_2. We apply Minkowski's inequality, Fubini's theorem and repeat the reasoning for $\widetilde{S}_1$ on the variable z_2. Repeating this procedure $n-2$ times will come to the desired result. Theorem 1 is fully proved.

For the f function, $f \in H(U^n)$ denote

$$G(f,\alpha,p,\gamma) = \int_{T^n} \left(\int_{\Gamma(\xi_1)} \cdots \int_{\Gamma(\xi_n)} \left| D^k f(z) \right|^{\alpha+2} \cdot \right.$$

$$\left. \cdot (1-|z|)^{\alpha/p+(\alpha+2)k-2-\gamma} dm_{2n}(z) \right)^{p/2} dm_n(\xi).$$

The following theorem 2 generalizes the known one-dimensional estimates from below for the Hardy class norm $H^p(U)$ for $p \geq 2$ (in two directions at once). This theorem also establishes the $\varepsilon-$ accuracy of this estimate.

Theorem 2. *Let $p \geq 2, f \in H^p(U^n), 0 \leq \alpha < p$. Then*

$$G(f,\alpha,p,0) \leq C_2 ||f||_{H^p}^p \tag{8}$$

This estimate is accurate in the following sense. For all $\varepsilon, \varepsilon > 1-2/p$ there will always be α number, $\alpha \geq 0$, such that the inequality is true

$$||f||_{H^p}^p \leq C_1(G(f,\alpha,p,\varepsilon)),$$

where C_1, C_2 - there are some positive constants.

Proof of Theorem 2.

The theorem we prove in bidisk. The General case is exhausted similarly. The proof of the estimate (8) relies on the ratio (7) and (7'). Indeed, using duality arguments and the formula

$$\int\limits_T \left(\int\limits_{\Gamma(\xi)} g(w)dm_2(w) \right) \psi(\xi)dm(\xi) = \int\limits_U g(z) \int\limits_T \lambda_{\Gamma(\xi)}(z_1)\psi(\xi)dm(\xi)dm_2(z)$$

(λ is a characteristic function of $\Gamma(\xi)$) for each variable separately we deduce

$$G(f,\alpha,p,o) = M_2 \le C \int\limits_U \int\limits_U \left| D^k f(z) \right|^{\alpha+2} (1-|z|)^t \times$$

$$\times \int\limits_T \lambda_{\Gamma(\xi_2)}(z_2) \left(\int\limits_T \lambda_{\Gamma(\xi_1)}(z_1)\psi(\xi_1,\xi_2)dm(\xi_1) \right) dm(\xi_2)dm_2(z_1)dm_2(z_2),$$

where $\psi(\xi_1,\xi_2) \in L^q(T^2), q = \left(\frac{p}{2}\right)', t = \alpha\left(\frac{1}{p}+k\right)+2k-2.$

Next given the estimate (7) with the exponent $q=\infty, q'=1$ we deduce the inequality.

$$M_2 \le C_1 \int\limits_T \int\limits_T \sup_{z_1\in\Gamma(\widetilde{\xi}_1)} \sup_{z_2\in\Gamma(\widetilde{\xi}_2)} \left| D^k f(z) \right|^2 (1-|z|)^{2k} \times$$

$$\times \sup_{z_1\in\Gamma(\widetilde{\xi}_1)} \sup_{z_2\in\Gamma(\widetilde{\xi}_2)} \frac{1}{(1-|z_1|)(1-|z_2|)} \int\limits_T \lambda_{\Gamma(\xi_2)}(z_2) \left(\int\limits_T \lambda_{\Gamma(\xi_1)}(z)\psi(\xi_1,\xi_2)dm(\xi_1) \right) \times$$

$$\left(\left| D^k f(z) \right|^{\alpha} (1-|z|)^{\alpha(1/p+k)} \right) dm(\widetilde{\xi}_1), dm(\widetilde{\xi}_2) \le$$

$$\le C_2 \left|\left| \left((1-|z|)^{\alpha(1/p+k)} \left| D^k f(z) \right|^{\alpha} \right) \right|\right|_{L^\infty(T^n)} \times$$

$$\times \int\limits_T \int\limits_T \left\{ \sup_{z_1\in\Gamma(\widetilde{\xi}_1)} \sup_{z_2\in\Gamma(\widetilde{\xi}_2)} \left(\left| D^k f(z) \right|^2 (1-|z|)^{2k} \right) \right\} \times$$

$$\times \left(M(\psi)(\widetilde{\xi}_1,\widetilde{\xi}_2) \right) dm(\widetilde{\xi}_1)dm(\widetilde{\xi}_2), \qquad (8')$$

where $M(f)$ is a maximal function of Hardy-Littlewood.

$$M(f)(\widetilde{\xi}_1,\widetilde{\xi}_2) = \sup_{t_1>0} \frac{1}{|I_{\widetilde{\xi}_1,t}|} \int\limits_{I_{\xi_1,t}} \sup_{t_2>0} \left[\frac{1}{|I_{\widetilde{\xi}_2,t}|} \int\limits_{I_{\xi_2,t}} |\psi(\varphi_1,\varphi_2)|dm(\varphi_1) \right] dm(\varphi_2.) \qquad (9)$$

To estimate the first multiplier, it is sufficient to apply the Holder inequality with the exponent $p/2$ and two maximal theorems (in the polydisk), one of which was mentioned above (see the estimate (4)) and the latter theorem was established in [15]. The second estimate on the action of the Hardy-Littlewood operator is derived by applying a one-dimensional result (see [22]) by each variable.

Now let's estimate the second multiplier. We have

$$A = \left(|D^k f(z)|^{\alpha} (1-|z|)^{k\alpha+\alpha/p} \right)(\xi) \le C_1 \sup_{t_2} \frac{1}{|I_T(\xi_2,t_2)|} \int\limits_{I_U(\xi_2,t_2)} (1-|z_2|)^{k\alpha+\alpha/p-1} \times$$

$$\times \sup_{t_1} \frac{1}{|I_T(\xi_1,t_1)|} \int\limits_{I_U(\xi_1,t_1)} |D^k f(z)|^{\alpha}(1-|z_1|)^{k\alpha+\alpha/p-1} dm_2(z_1)dm_2(z_2). \qquad (10)$$

We will evaluate "the inner sup"(use the Holder inequality with the exponent $(p/\alpha)'$) for $\int\limits_{|1-\bar{\xi}_1\eta_1|<t_1} |D^k f(z)|^{\alpha} dm(\xi_1)$.

$$\sup_{t_1}(t_1^{-1}) \int\limits_{1-t_1}^{1} \int\limits_{|1-\bar{\xi}_1\eta_1|<t_1} |D^k f(z)|^{\alpha}(1-|z_1|)^{k\alpha+\alpha/p-1} dm_2(z_1) \leq$$

$$\leq C_2 \sup_{t_1}(t_1^{-1}) \int\limits_{1-t_1}^{1} M_p^p(D^k f(z_1))^{\alpha/p} t_1^{\frac{1}{(p/\alpha)'}} (1-|z_1|)^{k\alpha+\alpha/p-1} dm|z_1| \leq$$

$$\leq C_3 \left(\int\limits_{T} \left| D^k_{(z_2)} f(\xi_1, |z_2|\xi_2) \right|^p dm(\xi_1) \right)^{\alpha/p}.$$

The second multiplier is evaluated similarly. So,

$$A \leq \widetilde{C_1} ||f||_{H^p}^p. \qquad (11)$$

Remark 4'. The above inequality (8) is well known for $n=1$ (see [22]).

To establish the second inequality, we use the known one-dimensional embedding (see [4]-[7],[21],[22]).

$$A_s^r(U) \subset H^p(U), r < p, s - 1/r = -\frac{1}{p}, p > 2, r < \alpha + 2, \qquad (12)$$

where A_s^r is analytic new Bergman's class in U.

$$A_s^r(U) = \left\{ f \in H(U) : \int\limits_{U} |D^k f(z)|^r (1-|z|)^{(k-s)r-1} dm_2(z) < \infty \right\}.$$

Using embedding (12) (by each variable) and estimate (7') with an exponent $\frac{\alpha+2}{r}$, we deduce

$$||f||_{H^p}^p \leq$$

$$\leq C \int\limits_{T^n} \left(\int\limits_{\Gamma(\xi)} |D^k f(z)|^{\alpha+2} (1-|z|)^{\alpha/p+(\alpha+2)k-2-\varepsilon} dm_2(z) \right)^{\frac{p}{2}} \times$$

$$\times \left| \left| C_{\left(\frac{\alpha+2}{r}\right)} (1-|z|)^{\gamma} \right| \right|_{L^{\infty}(T^n)}, \qquad (13)$$

where

$$\left(\frac{\alpha+2}{r}\right)'\gamma=\left(\frac{\alpha+2}{\alpha+2-r}\right)\left\{(k-s)r-\frac{r}{\alpha+2}(-\varepsilon+\frac{\alpha}{p}+(\alpha+2)k)\right\}. \quad (14)$$

It remains to be noted that from the conditions of the Theorem and the obtained their relation is possible to choose α such that $\gamma>0$, and the latter is sufficient for the finiteness of the second multiplier in (13). Theorem is proved.

New equivalent quasinorm to the Bergman type analytic space in the polydisk

In this section we will find some equivalent norms (quazinorms) for new Bergman type classes of analytic functions. Let's introduce new Bergman type spaces in the polydisk as follows

$$L^{s,\gamma}_{p,q}(U^n)=$$
$$=\left\{f\in H(U^n):||f||^p_{L^{s,\gamma,l}_{p,q}}=\int\limits_{T^n}\left(\int\limits_0^l|R^s f(r\xi)|^q(1-r)^\gamma r^l dr\right)^{p/q}dm_n(\xi)<\infty,\right.$$
$$\left. p,q\in(0,\infty),\gamma>-1,l\geq 0\right\},(15)$$

where R^s - differential operator acting as a bounded operator from $H(U^n)$ to $H(U^n)$

$$(R^s f)(z)=\sum_{k_1,\ldots,k_n\geq 0}(k_1+\cdots+k_n+1)^s a_k z^k, f(z)=\sum_{|k|\geq 0}a_k z^k.$$

Below we establish the theorem for the quasinorm $||\quad||^p_{L^{s,\gamma,l}_{p,q}}$ for $p=q$ using some new space and thier relation to new functional spaces (see [4]-[7])but in U^n.

Theorem 4.

1). Let $R^s f(z,\ldots,z)\in L^{0,r+n-1}_{p,p}(U), 0<p<\infty, r,s>0, n\in N,\ \gamma=\frac{r+1}{n}-1$ *and assume all conditions imposed on* γ,β *in 2) are valid. Then we have*

$$||f||^p_{L^{s,r}_{p,p}}\equiv\int\limits_{T^n}\left(\int\limits_{\Gamma_\alpha(\xi)}|\widetilde{D}^\beta f(z_1,\ldots,z_n)|^p(1-|z|)^{\frac{1+r}{n}+p(\beta-\frac{s}{n})-2}dm_{2n}(z)\right)dm_n(\xi);(16)$$

2). Let $0<p<\infty, \beta>\frac{s}{n}, \gamma,s>0, n\in N, \frac{s}{n}>\max\left(1+\frac{\gamma+1}{p},\frac{\gamma+2}{p}-1\right)$. *Then*

$$||f||^p_{\widetilde{L}^{s,r}_{p,p}}\equiv\int\limits_{T^n}\left(\int\limits_{\Gamma_\alpha(\xi)}|\widetilde{D}^\beta f(z_1,\ldots,z_n)|^p(1-|z|)^{\gamma+p(\beta-\frac{s}{n})-1}dm_{2n}(z)\right)dm_n(\xi),(17)$$

where $\widetilde{D}^\beta: H(U^n)\to H(U^n)$ *is a differential operator*

$$\left(\widetilde{D}^{\beta} f\right)(z)=\sum_{|k|\geq 0}\frac{\Gamma(k+\beta+1)}{\Gamma(k+1)\Gamma(\beta+1)}a_k z^k, \beta\geq 0; \Gamma(k+\beta+1)=\prod_{j=1}^{m}\Gamma(k_j+\beta+1).$$

Mixed norm analogues of these results by similar methods can be also obtained (the case when p is not equal to q) but there our results are not sharp.

Remark 5. The Theorem 3 was announced at [3].

Remark 6. For $n=1$ operators R^s and D^s, classes $L^{s,\gamma}_{p,p}$ and $\widetilde{L}^{s,\gamma}_{p,p}$ relations (16) and (17) coincide and Theorem 3 is not difficult to deduce from the well-known embedding theorems and the Hardy-Littlewood theorem (see [22], [3], [21]) .

The proof of Theorem 3. First, we prove the second part of the theorem. Note that

$$G_{\xi}(f_{R^2})=\int\limits_{\Gamma_{\alpha}(\xi_1,\dots,\xi_n)}|\widetilde{D}^{\beta} f_{R^2}(z)|^q(1-|z|)^{\alpha-2}dm_{2n}(z)=$$

$$\int\limits_{\Gamma_{\alpha}(\xi_1,\dots,\xi_n)}\left|\int\limits_{T^n}R^s f_R(tz)\left(R^{-s}\frac{1}{(1-\bar{t}R)^{\beta+1}}\right)dm_n(t)\right|^q(1-|z|)^{\alpha-2}|z|dm_{2n}(z), \quad (17')$$

$s>0, \alpha>0, f_R(z)=f(Rz), R\in I$, where α will be chosen below. Next we will use the following two relations

$$\left|\int\limits_{T^n}f(r_1t_1,\dots,r_nt_n)g(r_1\bar{t}_1,\dots,r_n\bar{t}_n)dm_n(t)\right|=$$

$$=\left|\int\limits_{T^n}\int\limits_0^1 f(rw)(g(r\bar{w}))\left(\log\frac{1}{\rho^2}\right)^{s-1}\rho d\rho dm_n(\varphi)\right|;(18)$$

where $s>0, f,g\in H(U^n), w=(\rho_1\varphi_1,\dots,\rho_n\varphi_n), \rho d\rho=\prod\limits_{k=1}^{n}\rho_k d\rho_k$.

$$\int\limits_{T^n}\int\limits_0^1|\Phi(w|\xi|)|^q(1-|w|)^{\alpha+n-1}d|w|dm_n(\xi)\leq\int\limits_{U^n}|\Phi(w)|^q(1-|w|)^{\alpha/n}dm_{2n}(w), (19)$$

where $\alpha>-1, q\in(0,\infty), \Phi\in H(U^n)$.

The first estimate is easily deduced from the following equality

$$C\sum_{k_1\geq 0}\dots\sum_{k_n\geq 0}\hat{g}(k_1,\dots,k_n)\hat{f}(k_1,\dots,k_n)\left(r_1^{2k_1}\dots r_n^{2k_n}\right)=$$

$$\sum_{|k|\geq 0}\int\limits_0^1(k_1+\dots+k_n+1)^s\hat{g}(k_1,\dots,k_n)\hat{f}(k_1,\dots,k_n)\left(r_1^{2k_1}\dots r_n^{2k_n}\right)\times$$

$$\left(\log\frac{1}{|w|^2}\right)^{s-1}|w|^{2(k_1+\cdots+k_n)}|w|d|w|=\int\limits_{T^n}\int\limits_0^1 R^s f(rw)g(r\bar{w})\left(\log\frac{1}{|w|}\right)^{s-1}d\rho dm_n(\varphi),$$

where $\rho=|w|$.

The second estimate we see in [1], [8]. Using successively (19), (6') and estimate (6"), we deduce the following inequalities

$$\left(\int\limits_{T^n}\int\limits_0^1\frac{|f(z)|(1-|z|)^\gamma}{|1-\bar{w}z|^\beta}dm_n(\varphi)d|z|\right)^q\leq$$
$$\leq C_1\int\limits_{T^n}\int\limits_{I^n}\frac{|f(z)|^q(1-|z|)^{q-2+(\frac{r+1}{n})q}}{|1-\bar{w}z|^{\beta q}}dm_{2n}(z),\quad (20)$$

where $w\in U^n, z=(|z|\varphi_1,\ldots,|z|\varphi_n), \beta>0, \gamma>-1$ for $q\leq 1, \gamma>-\frac{n}{q'}-1$ and

$$\left(\int\limits_{T^n}\int\limits_0^1\frac{|f(z)|(1-|z|)^\gamma}{|1-\bar{w}z|^\beta}dm_n(\varphi)d|z|\right)^q\leq$$
$$\leq C_2\int\limits_{T^n}\int\limits_{I^n}\frac{|f(z)|^q(1-|z|)^{\frac{\gamma+1-n}{n}q}(1-|w|)^{\varepsilon q}}{|1-\bar{w}z|^{\beta q-(2+\varepsilon)q+2}}dm_{2n}(z),\quad (21)$$

where $\beta>0, \gamma>-1$ for $q\geq 1, \gamma>-\frac{n}{q'}-1$.

Next from (17') given (18), (20), (21) we get

$$G_\xi(f_{R^2})\leq$$
$$\leq C_3\int\limits_{\widetilde{\Gamma}(\xi_1,\ldots,\xi_n)}(1-|z|)^{\alpha-2}\int\limits_{T^n}\int\limits_0^1\cdots\int\limits_0^1\frac{|R^s f_R(w)|^p(1-|w|)^t d|w|dm_n(\varphi)dm_{2n}(w)}{|1-\bar{w}Rz|^{(\beta+1)p}}\quad (21')$$

for $p\leq 1$, where $t=\frac{sp}{n}+p-2$ and

$$G_\xi(f_{R^2})\leq$$
$$\leq C_4\int\limits_{\widetilde{\Gamma}(\xi_1,\ldots,\xi_n)}(1-|z|)^{\alpha-2}\int\limits_{T^n}\int\limits_0^1\cdots\int\limits_0^1\frac{|R^s f_R(w)|^p(1-|w|)^t(1-|z|)^{-\varepsilon p}dm_n(\varphi)dm_{2n}(w)}{|1-Rwz|^{(\beta+1)p+2-(2+\varepsilon)p}}\quad (21'')$$

for $p\geq 1$, where $t=\left(\frac{s}{n}-1\right)p$.

Let $\alpha=\gamma+p\left(\beta-\frac{s}{n}\right)+1$.

Given that

$$\int\limits_{\widetilde{\Gamma}(\xi_1,\dots,\xi_n)} \frac{(1-|z|)^{\alpha-2}dm_{2n}(z)}{|1-zR_0\bar{w}|^{(\beta+1)p}} \leq C_5 \prod_{k=1}^{n} \frac{1}{|1-R_0\bar{w}\xi_k|^{p(\beta+1)-\alpha}},$$

$w_k \in U, \xi_k \in T, R_0 \in I, I=(0,1)$, (see [8], [22]),
and passing to the limit for $R_0 \to 1$ from (21') we get

$$G_\xi(f) \leq C_6 \int\limits_{T^n}\int\limits_{I^n} \frac{|R^s f(w)|^p (1-|w|)^t}{|1-\bar{w}\xi|^{p(\beta+1)-\alpha}} d|w|dm_n(\varphi), t=\frac{sp}{n}+p-2.$$

To obtain the first half of the statement of the second part of the Theorem for $p \leq 1$ it remains to integrate both parts of the last inequality in T^n and to take into account the following known estimate (see [8])

$$\int\limits_{T^n} \left(\prod_{k=1}^{n} \frac{1}{\left|1-\bar{\xi}_k w_k\right|^{\nu}}\right) dm_n(\xi) \leq C \prod_{k=1}^{n} \frac{1}{(1-|w_k|)^{\nu-1}}, \nu>1. \qquad (22)$$

To obtain the first half of the statement of the second part of the theorem for $p \geq 1$ from (21”), it is necessary to conduct similar argument by putting
$\alpha=\gamma+\beta p+2-\frac{sp}{n}$. And given the (22) and conditions of the Theorem it is not difficult to deduce that

$$\int\limits_{T^n} G_\xi(f)dm_n(\xi) \leq C||f||^p_{\widetilde{L}^{s,\gamma}_{p,p}}.$$

We now will prove the inverse estimates. Thus we will complete the proof of the second part of the second half of Theorem 3 completely. Based on formula (7') it is easy to see that it is enough to set the estimate

$$||f||^p_{\widetilde{L}^{s,\gamma}_{p,p}} \leq C \int\limits_{U^n} \left|D^\beta f(z)\right|^p (1-|z|)^{\gamma+p(\beta-s/n)} dm_{2n}(z).$$

Note that from (18) it follows that

$$|D^\alpha R^s f(z)| \leq C(\gamma,n) \int\limits_0^1 \int\limits_{T^n} \left|D^\beta f(rw)\right| \times$$
$$\times \left|R^{s+t} D^\alpha D^{-\beta}\left(\frac{1}{1-r\bar{w}\xi}\right)\right| (1-|w|)^{t-1} d|w|d\varphi, \quad (22')$$

where $z=r^2\xi, w=(|w|\varphi_1,\dots,|w|\varphi_n), t$ - relatively large.

Let $p \leq 1$. We will need an analogue of the estimate (6') for the subframe of $\widetilde{U}^n$. We have

$$\left(\int\limits_{T^n}\int\limits_0^1 |f(z)|(1-|z|)^{\alpha} d|z| dm_n(\varphi)\right)^p \leq$$

$$\leq C(p)\int\limits_{T^n}\int\limits_0^1 |f(z)|^p(1-|z|)^{\alpha p+(n+1)(p-1)} d|z| dm_n(\varphi), \quad (23)$$

$\alpha > \frac{n(1-p)}{p} - 1$. It is easy to deduce, given that

$$M_1(f,r_1,\dots,r_n) \leq C_2 \prod_{k=1}^{n}(1-r_k)^{1-1/p} M_p(f,r_1,\dots,r_n), p \leq 1,$$

and from the elementary inequality

$$\left(\int\limits_0^1 G(\rho)(1-\rho)^t d\rho\right)^p \leq C\int\limits_0^1 (G(\rho))^p(1-\rho)^{tp+p-1} d\rho,$$

where $p \leq 1, t > -1$, where G is an increasing function. Next we note that

$$\int\limits_{T^n}\left| R^{s+t}\left(\frac{1}{1-r\bar{w}\xi}\right)^{\alpha-\beta+1}\right|^p dm_n(\xi) \leq$$

$$\leq C_1 \sum_{\substack{\alpha_j \geq 0 \\ \sum\limits_{j\geq 0}\alpha_j = s+t}} \prod_{k=1}^{n} \frac{1}{(1-|w||z_k|)^{p(\alpha_k+\alpha-\beta+1)-1}}, z = (r_1\xi_1,\dots,r_n\xi_n); \quad (24)$$

where α and t - are sufficiently large positive numbers.

Estimates (24) are not difficult to deduce directly from Newton's binomial formula and (22). Given (23) and (24) we get

$$\int\limits_{U^n} |D^{\alpha}R^s f(z)|^p (1-|z|)^{\gamma+\alpha p} dm_{2n}(z) \leq$$

$$\leq C_1 \int\limits_0^1\int\limits_{T^n} \left|D^{\beta}f(w)\right|^p (1-|w|)^{(t-1)p+(n+1)(p-1)} \times$$

$$\times \sum_{\substack{\alpha_j \geq 0 \\ \sum\alpha_j = s+t}} \int\limits_0^1 \cdots \int\limits_0^1 \frac{(1-|z|)^{\gamma+\alpha p} d|w| d|z| dm_n(\varphi)}{(1-|w||z_1|)^{p(\alpha_1+\alpha-\beta+1)-1}\dots(1-|w||z_n|)^{p(\alpha_n+\alpha-\beta+1)-1}} \leq$$

$$\leq C_2 \int\limits_0^1\int\limits_{T^n} \left|D^{\beta}f(w)\right|^p (1-|w|)^{\gamma n+\beta\rho n-ps+n-1} dm_n(\varphi) d|w| \leq$$

$$\leq C_3 \int\limits_{U^n} \left|D^{\beta}f(w)\right|^p (1-|w|)^{\gamma+p(\beta-s/n)} dm_{2n}(w),$$

where in the last inequality we used the estimate (19).

For $p \geq 1$ instead of estimating (23), we should use the analogue (6“) for subostov (subpolydisk) (which easily follows from the Holder inequality)

$$J = \left(\int_0^1 \int_{T^n} \frac{|R^s f(w)| \, (1-|w|)^{s-1}}{|1-\bar{w}z|^{\gamma+1}} dm_n(w) d|w| \right)^p \leq$$

$$\leq C_4 \int_0^1 \int_{T^n} \frac{|R^s f(w)|^p \, (1-|w|)^{(s-1)p} \, ((1-|z_1|)\dots(1-|z_n|))^{-1+1/n+\varepsilon p}}{|1-\bar{w}z|^{(\gamma-1)p+2+\varepsilon p}} d|w| d\xi,$$

where ε is any number, $\varepsilon > 0$ and to carry out similar above to the reasoning. The second part of the theorem is proved.

To prove the first part of the theorem, it is necessary to use the already established second part of Theorem 3 and the following Lemma A.

Lemma A.

1). The following estimate is valid

$$S_1 < C\widetilde{S}_1, \qquad (25) \qquad (1)$$

where

$$S_1 = \int_{T^n} \int_0^1 |f(z)|^q (1-|z|)^{\alpha} dm_n(\varphi) |z| d|z|$$

and

$$\widetilde{S}_1 = \int_{U^n} |f(w)|^q \prod_{k=1}^{n} (1-|w_k|)^{\frac{\alpha+1}{n}-1} dm_{2n}(w),$$

here $z = (|z|\varphi_1, \dots, |z|\varphi_n), f \in H(U^n), 0 < q < \infty, \alpha > -1.$

2). Let $S_1 < \infty$, then if

$$S_2 = \int_U |f(z,\dots,z)|^q (1-|z|)^{\alpha+n-1} dm_2(z) < \infty, q \in (0,\infty).$$

Then the inverse of (25) is also true. The proof of Lemma A.

The first inequality is actually established in [8]. (It is a theorem on mapping from polydisk to diagonal in A^p_α classes). It is necessary to use the dyadic division of the parallelepiped

$$I^n = \bigcup_{k_1,\dots,k_n \geq 0} [1-2^{-k_1}; 1-2^{-k_1-1}) \times \dots \times [1-2^{-k_n}; 1-2^{-k_n-1})$$

and to limit the summation for n-dimensional sums on the diagonal (see [8]).

We prove the second assertion of the Lemma. In [8] it is fixed that for the function $f(z_1,\dots,z_n)$:

$$f(z_1,\dots,z_n) = C(\gamma,n) \int_U \frac{f(z,\dots,z)(1-|z|)^{\gamma} dm_2(z)}{(1-z\bar{z}_1)^{\frac{\gamma+2}{n}} \dots (1-z\bar{z}_n)^{\frac{\gamma+2}{n}}}, z_j \in U, j = 0,\dots,m;$$

the value $\widetilde{S}_1$ is finite if only $S_2 < \infty$, and the inequality $\widetilde{S}_1 < const S_2$ is also true, so

$$\int\limits_{U^n} |f(z_1,\dots,z_n)|^q (1-|z|)^{\frac{\alpha+1}{n}-1} dm_{2n}(z) \le C \int\limits_{U} |f(z,\dots,z)|^q (1-|z|)^{\alpha+n-1} dm_2(z),$$

$0<q<\infty, \alpha>-1, n\ge 1.$

It remains to use once again the dyadic partition of the subostov (polydisk). $\widetilde{U}^n=\{z=(r\xi_1,\dots,r\xi_n),\xi_i\in T, r\in I\}$ and the summation of the n- dimensional sum along the diagonal to establish the estimate (see [1], [3], [8]).

$$\int\limits_{U^n} |f(z,\dots,z)|^q (1-|z|)^{\alpha+n-1} dm_2(z) \le \tilde{\tilde{C}} \int\limits_0^1 \int\limits_{T^n} |f(z_1,\dots,z_n)|^q (1-|z|)^{\alpha} dm_n(\varphi) d|z|,$$

$z=(|z|\varphi_1,\dots,|z|\varphi_n), 0<q<\infty, \alpha>-1$ (see [1], [3], [8]). At the same time it is necessary to take into account also that the function defined on subostov $\widetilde{U}^n$ can be extended uniqily to arbitrary point of the unit polydisk U^n (see [8], [18]) .

The Lemma A is proved.

On some new equivalent norms for analytic Bergman type spaces in bounded strongly pseudoconvex domains

In this final section we prove similar type result for Bergman type spaces in bounded pseudoconvex domains.

Let D be a bounded strictly pseudoconvex domain in $\mathbb{C}^n$ with smooth boundary, let $d(z)=dist(z,\partial D)$.

Then there is a neighborhood U of $\bar{D}$ and $\rho\in C^\infty(U)$ such that $D=\{z\in U:\rho(z)>0\}, |\bigtriangledown\rho(z)|\ge c>0$ for $z\in\partial D, 0<\rho(z)<1$ for $z\in D$ and $-\rho$ is strictly plurisubharmonic in a neighborhood U_0 of ∂D. Note that $d(z)\asymp\rho(z), z\in D$. Then there is an $r_0>0$ such that the domains $D_r=\{z\in D:\rho(z)>r\}$ are also smoothly bounded strictly pseudoconvex domains for all $0\ge r\ge r_0$. Let $d\sigma_r$ be the normalized surface measure on ∂D_r and dv the Lebesgue measure on D. The following mixed norm spaces were investigated in [20]. For $0<p<\infty$, $0<q\le\infty, \delta>0$ and $k=0,1,2,\dots$ set

$$||f||_{p,q,\delta;k} = \left(\sum_{|\alpha|\le k} \int_0^{r_0} \left(r^\delta \int_{\partial D_r} |D^\alpha f|^p d\sigma_r\right)^{q/p} \frac{dr}{r}\right)^{1/q}, 0<q<\infty$$

and weighted Hardy space $(A_0^{p,\infty}=H^p)$

$$||f||_{p,\infty,\delta;k} = \sup_{0<r<r_0} \sum_{|\alpha|\le k} \left(r^\delta \int_{\partial D_r} |D^\alpha f|^p d\sigma_r\right)^{1/q}, 0<q<\infty,$$

where D^α is a derivative off (see [20]) The corresponding spaces $A^{p,q}_{\delta;k}=A^{p,q}_{\delta;k}(D)=\{f\in H(D):||f||_{p,q,\delta;k<\infty}\}$ are complete quasi normed spaces, for $p,q\ge 1$ they are Banach spaces. We mostly deal with the case $k=0$, when we write simply $A^{p,q}_\delta$ and $||f||_{p,q,\delta}$. We also consider this spaces for $p=\infty$ and $k=0$, the corresponding space is denoted by $A^{\infty,p}_\delta=A^{\infty,p}_\delta(D)$ and consists of all $f\in H(D)$ such that

$$||f||_{\infty,p,\delta} = \left(\int_0^{r_0} (\sup_{\partial D_r}|f|)^p r^{\delta p-1} dr\right)^{1/p} < \infty.$$

Also, for $\delta > -1$, the space $A^\infty_\delta = A^\infty_\delta(D)$ consist of all $f \in H(D)$ such that

$$||f||_{A^\infty_\delta} = \sup_{z\in D}|f(z)|\rho(z)^\delta < \infty,$$

and the weighted Bergman space $A^p_\delta = A^p_\delta(D) = A^{p,p}_{\delta+1}(D)$ consists of all $f \in H(D)$ such that

$$||f||_{A^p_\delta} = \left(\int_D |f(z)|^p \rho^\delta(z) dv(z)\right)^{1/p} < \infty.$$

We denote by K_β the weighted Bergman kernel on D (see [4], [3]).

Since $|f(z)|^p$ is subharmonic (even plurisubharmonic) for a holomorphic f, we have $A^p_s(D) \subset A^\infty_t(D)$ for $0 < p < \infty, sp > n$ and $t = s$. Also, $A^p_s(D) \subset A^1_s(D)$ for $0 < p \leq 1$ and $A^p_s(D) \subset A^1_t(D)$ for $p > 1$ and t sufficiently large. Therefore we have an integral representation

$$f(z) = C_\beta \int_D f(\xi)K(z,\xi)\rho^t(\xi)dv(\xi), \qquad f \in A^1_t(D), z \in D, \qquad (*)$$

where $K(z,\xi)$ is a kernel of type t, that is a smooth function on $D \times D$ such that $|K(z,\xi)| \leq C|\tilde\Phi(z,\xi)|^{-(n+1+t)}$, where $\tilde\Phi(z,\xi)$ is so called Henkin-Ramirez function for D. Note that (*) holds for functions in any space X that embeds into A^1_t.

The following is the main result of this section.

Theorem 4. *Let $p,q \in (0,\infty), \gamma > -1, \beta > -1$. Then for $f \in H(D)$ we have*

$$\int_0^\rho \left(\int_{\partial D_\varepsilon} |f|^p\right)^{q/p} \varepsilon^{\beta+(\gamma+1)q/p} d\varepsilon \asymp \int_0^\rho \left(\int_{D_\varepsilon} |f(z)|^p d^\gamma(z) dv(z)\right)^{q/p} r^\beta dr.$$

Remark. This theorem almost with similar proof probably can be extended also to $A^{p,\infty}_\delta$ and $A^{\infty,p}_\delta, p \in (0,\infty), \delta > 0$.

Note in the Unit disk we see this result in [14].

We will need the following simple lemmas for the proof of our main result.

Lemma D. (see [14]). Let $0 < p < \infty$ and $\beta > 0$. Let (α_n) be a sequence of nonnegative real numbers such that $\sum_{n=1}^\infty 2^{-n\beta}\alpha_n^p < \infty$. Then there is a constant $C > 0$, depending only on p and β, such that

$$\sum_{n=1}^\infty 2^{-n\beta}\alpha_n^p \leq C(\alpha_0^p + \sum_{n=1}^\infty 2^{-n\beta}|\alpha_n - \alpha_{n-1}|^p).$$

Lemma B. (see [18], [20]).

1. Let $G \in L^1(D_\varepsilon), w \in C^\infty(D)$.

$$\int\limits_{D_\varepsilon} G(z)d\varepsilon \asymp \int\limits_0^\varepsilon \int\limits_{\partial D_\varepsilon} G(z)w(z)d\varepsilon.$$

2. Let

$$h_p^P(r) = \int\limits_{\partial D_r} |f|^p d\sigma_r.$$

Then function is non icreasing, and

$$(t^{\delta\beta})h_p(2t^\beta) \le C \int\limits_{t^\beta} 2t^\beta h_p(r) r^{\delta-1} dr,$$

$\beta > 0, 0 < p < \infty, \delta > 0$.

3. Let $0 < p < \infty$,

$$\alpha_n = \int\limits_0^{r_n} \int\limits_{\partial D_\varepsilon} |f_t(z)|^p \delta^\gamma(z) w(z) dv(z), w \in \mathbb{C}^\infty(D).$$

$r_n = 2^{-n}, n = 0, 1, \ldots, t > 0$.

Then the assertion of Lemma D is valid.

For any Lebesgue measurable function f in U, we define

$$M_p(r,f) = \left(\int\limits_T |f(r\xi)|^p dm(\xi) \right)^{1/p},$$

where $0 < p < \infty$.

If $0 < p < \infty, 0 < q < \infty$, and $\alpha > -1$, let

$$||f||_{p,q,\alpha}^q = \int\limits_I (1-r^2)^\alpha M_p(r,f)^q dr,$$

where $I = [0,1)$. The mixed norm space $H^{p,q,\alpha} = H^{p,q,\alpha}(U)$ is then defined to be the space of function f holomorphic in U, $(f \in H(U))$ such that $||f||_{p,q,\alpha} < \infty$.

The result of this section in the unit disk can be expanded to this classes in the polydisk from the unit disk (see below).

First we have a new characterization of the mixed norm spaces $H^{p,q,\alpha(U)}$.

Let $0 < p, q < \infty$ and $-1 < \beta, \gamma < \infty$. A function $f \in H(U)$ belongs to $H^{p,q,\beta+p/q(\gamma+1)}(U)$ if and only if

$$\int\limits_0^1 \left(\int\limits_{|z|<r} |f(z)|^p (1-|z|)^\gamma dm_2(z) \right)^{q/p} (1-r)^\beta dr < \infty.$$

Note that, it is our theorem in the unit disk. We follow arguments from [14] first assume that $||f||_{p,q,\beta+q/p(\gamma+1)} < \infty$. For $0 < t < 1$ define $f_t(z) = f(tz), z \in U$. Let $r_n = 1 - 2^{-n}, n = 0, 1, \ldots$. Then

$$\int_0^1 (1-r)^\beta \left(\int_{|z|<r} |f_t(z)|^p (1-|z|)^\gamma dm_2(z) \right)^{q/p} dr =$$

$$= \sum_{n=1}^{\infty} \int_{r_{n-1}}^{r_n} (1-r)^\beta \left(\int_{|z|<r} |f_t(z)|^p (1-|z|)^\gamma dm_2(z) \right)^{q/p} dr \le C_1 \sum_{n=1}^{\infty} 2^{-n(\beta+1)} A_n^{q/p},$$

where

$$A_n = \int_{|z|<r_n} |f_t(z)|^p (1-|z|)^\gamma dm_2(z).$$

Now by using Lemma D we find that

$$\int_0^1 (1-r)^\beta \left(\int_{|z|<r} |f_t(z)|^p (1-|z|)^\gamma dm_2(z) \right)^{q/p} dr \le$$

$$\le C_2 \sum_{n=1}^{\infty} 2^{-n(\beta+1)} \left(\int_{r_{n-1}\le|z|<r_n} |f_t(z)|^p (1-|z|)^\gamma dm_2(z) \right)^{q/p} \le$$

$$\le C_3 \sum_{n=1}^{\infty} M_p(r_n, f_t)^q 2^{-n(\gamma+1)p/q} \le$$

$$\le C_4 \sum_{n=1}^{\infty} \int_{r_n}^{r_{n+1}} M_p(r_n, f_t)^q (1-r)^{\beta+q/p(\gamma+1)} dr \le C_5 \int_0^1 (1-r)^{\beta+q/p(\gamma+1)} M_p(r_n, f_t)^q dr.$$

Letting $t \to 1$, we get

$$\int_0^1 (1-r)^\beta \left(\int_{|z|<r} |f(z)|^p (1-|z|)^\gamma dm_2(z) \right)^{q/p} dr \le C_6 ||f||^q_{p,q,\beta+q/p(\gamma+1)}.$$

Conversely

$$||f||^q_{p,q,\beta+q/p(\gamma+1)} = \int_0^1 (1-r)^{\beta+q/p(\gamma+1)} M_p(r,f)^q dr =$$

$$= \sum_{n=1}^{\infty} \int_{r_{n-1}}^{r_n} (1-r)^{\beta+q/p(\gamma+1)} M_p(r,f)^q dr \le C_7 \sum_{n=1}^{\infty} 2^{-n(\beta+q/p(\gamma+1)+1)} M_p(r_n,f)^q \le$$

$$\le C_8 \sum_{n=1}^{\infty} \left(\int_{r_n \le |z| < r_{n+1}} |f(z)|^p (1-|z|)^\gamma dm_2(z) \right)^{q/p} 2^{-n(\beta+1)} \le$$

$$\le C_9 \sum_{n=1}^{\infty} \left(\int_{r_n \le |z| < r_{n+1}} |f(z)|^p (1-|z|)^\gamma dm_2(z) \right)^{q/p} \int_{r_{n+1}}^{r_{n+2}} (1-r)^\beta dr \le$$

$$\le C_{10} \sum_{n=1}^{\infty} \int_{r_{n+1}}^{r_{n+2}} (1-r)^\beta \left(\int_{|z|<r} |f(z)|^p (1-|z|)^\gamma dm_2(z) \right)^{q/p} dr \le$$

$$\le C_{11} \int_0^1 (1-r)^\beta \left(\int_{|z|<r} |f(z)|^p (1-|z|)^\gamma dm_2(z) \right)^{q/p} dr.$$

Note we used Lemma D in our arguments above. The carefull analysis of the unit disk proof we just provided shows that the repetition of arguments provided in the unit disk and applications of two lemmas, Lemma D and Lemma B, easily lead us also to the proof of the main theorem of this final section.

We leave these easy details to interested readers. For any Lebesgue measurable function f in U^n, we define

$$M_p(r,f) = \left(\int_{T^n} |f(r\xi)|^p dm_n(\xi) \right)^{1/p},$$

where $0 < p < \infty$ and $r\xi = (r_1\xi_1, \dots, r_n\xi_n)$.

If $0 < p < \infty, 0 < q < \infty$, and $\alpha = (\alpha_1, \dots, \alpha_n), \alpha_j > -1, j = 0, \dots, n$, let

$$||f||^q_{p,q,\alpha} = \int_{I^n} \left(\prod_{j=1}^n (1-r_j^2)^{\alpha_j} M_p(r,f)^q \right) dr,$$

where $I^n = [0,1)^n$ and $dr = dr_1 \dots dr_n$. The mixed norm space $H^{p,q,\alpha} = H^{p,q,\alpha}(U^n)$ is then defined to be the space of function f holomorphic in U^n, $(f \in H(U^n))$ such that $||f||_{p,q,\alpha} < \infty$.

The result of this section in the unit disk can be expanded to these classes in the polydisk from the unit disk.

It will be nice to obtain some direct analogues of our results of first sections in more general bounded pseudoconvex domains or in unbounded tube domains over symmetric cones.

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УДК 517.55+517.33 Научная статья

О некоторых новых оценках интегралов функции площадей и аналитических классов типа Бергмана в некоторых областях в $\mathbb{C}^n$

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В работе приведены новые эквивалентные квазинормы для некоторых новых пространств типа Бергмана в полидиске и в ограниченных псевдовыпуклых областях. Подобные оценки установлены также для классов типа Харди в полидиске. Эти результаты обобщают некоторые известные одномерные неравенства для пространств типа Харди и классов типа Бергмана в единичном круге. на случай полидиска и ограниченной псевдовыпуклой области. Оценки такого типа могут иметь также различные приложения. Пусть D ограниченная или неограниченная область в $\mathbb{C}^n$ (ограниченная псевдовыпуклая или неограниченная трубчатая область над симметрическим конусом). Подходы, примененные в данной работе при доказательстве утверждений в полидиске могут быть, по-видимому, также использованы для доказательства подобных приведенных в данной работе оценок, но в полиобластях $D \times \cdots \times D$ существенно более общего типа, чем единичный полидиск.

Ключевые слова: интегральные операторы, аналитические функции, псевдовыпуклые области, полидиск, классы типа Бергмана, классы Харди.

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