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ON SOME NEW ESTIMATES RELATED WITH BERGMAN BALL AND POISSON INTEGRAL IN TUBULAR DOMAIN AND UNIT BALL

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We introduce new Herz type analytic spaces based on Bergman balls in tubular domains over symmetric cones and in products of such type domains. We provide for these Herz type spaces new maximal and embedding theorems extending known results in the unit disk. In addition we define new Poisson-type integral in the unit ball and extend a known classical maximal theorem related with it. Related results for such type integrals will be given.

Key words: tubular domains over symmetric cones, Herz type spaces, Bergman type integral operators, maximal theorems, embedding theorems, Poisson-type integral, unit ball.

Introduction and preliminaries

Let D be bounded domain. We denote by $H(D)$ the space of all analytic functions in D. The analytic Hardy space is as usual

$$H^p_D(D) = \left\{ f \in H(D) : \overline{\lim_{\varepsilon \to 0}} \int_\Gamma |f(\xi - \varepsilon \nu_\xi)|^p d\sigma(\xi) < \infty \right\},\ 0 < p < \infty,$$

where ν_ξ is outer unit normal for tangent plane $T_\xi(\partial D)$, (see [6]) and where $d\sigma$ is a Lebesgues measure on Γ.

Let H^p_D be Hardy space in D and let also $\partial D \in C^2$. Let $0 < p < \infty$, $f \in H^p_D$. Then we have

$$\int_\Gamma \sup_{z \in A_\alpha(\xi)} |f(z)|^p d\sigma(\xi) \le C_{p,\alpha} \|f\|^p_{H^p},$$

where $\Gamma = \partial D$.

We refer to [6] for definition of $A_\alpha(\xi)$ region.

This maximal theorem was proved by E. Stein and L. Hormander, (see [6]). Related results for spaces of harmonic functions can be found in paper of [5]. For similar results for plurisubharmonic functions in bounded strongly pseudoconvex domains in $\mathbb{C}^n$ see [6]. We refer for such type results also to [7] and references there.

For such type results in harmonic functions paces in $\mathbb{R}^{N+1}_+$ and $\mathbb{R}^n$ we refer the reader to [8], [9].

We in this paper find complete analogues of this theorem but in Bergman spaces in some unbounded tube domain in $\mathbb{C}^n$ using rather transparent arguments and ideas related with lattices.

In this paper we also introduce new Herz type analytic spaces in tubular domains over symmetric cones and in products of such type domains. We provide for these Herz type spaces new maximal and sharp embedding theorems extending known classical results in the unit disk. Also, we define Poisson-type integral in the unit ball and extend some known classical maximal theorems related with it.

We introduce some basic definitions, notations for tube (see [10], [12]).

Let $T_\Omega = V + i\Omega$ be the tube domain over an irreducible symmetric cone Ω in the complexification $V^{\mathbb{C}}$ of an n-dimensional Euclidean space V. Following the notation of [2] we denote the rank of the cone Ω by r and by Δ the determinant function on V. Letting $V = \mathbb{R}^n$, we have as an example of a symmetric cone on $\mathbb{R}^n$ the Lorentz cone Λ_n defined for $n \ge 3$ by

$$\Lambda_n = \{y \in \mathbb{R}^n : y_1^2 - \cdots - y_n^2 > 0, y_1 > 0\}.$$

It is equivalent to the forward light cone given by $\left\{y = (y_1, y_2, y') \in \mathbb{R}^n : y_1 y_2 - |y'|^2 > 0\right\}$. Light cones have rank 2. The determinant function in this case is given by the Lorentz form $\Delta(y) = y_1^2 - \cdots - y_n^2$, (see, for example, [1], [2], [3]).

$\mathscr{H}(T_\Omega)$ denotes the space of all holomorphic functions on T_Ω.

First we define some known function spaces on T_Ω, (see [1], [2], [3], [13]). For $\tau \in \mathbb{R}_+$ and the associated determinant function $\Delta(x)$ we set

$$A^\infty_\tau(T_\Omega) = \left\{ F \in \mathscr{H}(T_\Omega) : \|F\|_{A^\infty_\tau} = \sup_{x+iy \in T_\Omega} |F(x+iy)| \Delta^\tau(y) < \infty \right\}, \tag{1}$$

(see [2] and references there). It can be checked that this is a Banach space.

For $1 \le p,q < +\infty$, $\nu \in \mathbb{R}$ and $\nu > \frac{n}{r} - 1$ we denote by $A^{p,q}_{\nu}(T_\Omega)$ the mixed-norm weighted Bergman space consisting of analytic functions F in T_Ω that

$$\|F\|_{A^{p,q}_{\nu}} = \left(\int_{\Omega} \left(\int_{V} |F(x+iy)|^p dx \right)^{q/p} \Delta^{\nu}(y) \frac{dy}{\Delta(y)^{n/r}} \right)^{1/q} < \infty.$$

This is a Banach space. Replacing above simply A by L we will get as usual the corresponding larger space of all measurable functions in tube over symmetric cone with the same quazinorm (see, for example, [1], [2], [3], [13]). It is known the $A^{p,q}_{\nu}(T_\Omega)$ space is nontrivial if and only if $\nu > \frac{n}{r} - 1$, (see, for example, [1], [2], [3], [13]) and we will assume this everywhere below. When $p = q$ we write (see [1], [2], [3])

$$A^{p,q}_{\nu}(T_\Omega) = A^{p}_{\nu}(T_\Omega).$$

This is the classical weighted Bergman space with usual modification when $p = \infty$.

The (weighted) Bergman projection P_ν is the orthogonal projection from the Hilbert space $L^2_\nu(T_\Omega)$ onto its closed subspace $A^2_\nu(T_\Omega)$ and it is given by the following integral formula (see [1], [2], [3]).

$$P_\nu f(z) = C_\nu \int_{T_\Omega} B_\nu(z,w) f(w) \Delta^{\nu - \frac{n}{r}}(v) du dv, \tag{2}$$

where $B_\nu(z,w) = C_\nu \Delta^{-(\nu + \frac{n}{r})}((z - \overline{w})/i)$ is the weighted Bergman reproducing kernel, for $A^2_\nu(T_\Omega)$, (see [1], [2], [3]). Below and here we use constantly the following notations $w = u + iv \in T_\Omega$ and $z = x + iy \in T_\Omega$.

We denote $d\nu_\nu(w) = \Delta^\nu(v) du dv$.

Let us first recall the following known basic integrability properties for the determinant function, which appeared already above in definitions. Below we denote by Δ_s the generalized power function, (see [1], [2], [3]).

Lemma 1. *1) The integral*

$$J_\alpha(y) = \int_{\mathbb{R}^n} \left| \Delta^{-\alpha} \left(\frac{x+iy}{i} \right) \right| dx$$

converges if and only if $\alpha > 2\frac{n}{r} - 1$. *In that case*

$$J_\alpha(y) = C_\alpha \Delta^{-\alpha + n/r}(y),$$

$\alpha \in \mathbb{R}$, $y \in \Omega$.

2)Let $\alpha \in \mathbb{C}^r$ *and* $y \in \Omega$. *For any multi-indices* s *and* β *and* $t \in \Omega$ *the function* $y \mapsto \Delta_\beta(y+t)\Delta_s(y)$ *belongs to* $L^1(\Omega, \frac{dy}{\Delta^{n/r}(y)})$ *if and only if* $\Re s > g_0$ *and* $\Re(s+\beta) < -g_0^*$. *In that case we have*

$$\int_{\Omega} \Delta_\beta(y+t) \Delta_s(y) \frac{dy}{\Delta^{n/r}(y)} = C_{\beta,s} \Delta_{s+\beta}(t).$$

We refer to Corollary 2.18 and Corollary 2.19 of [13] for the proof of the above lemma or [2]. As a corollary of one dimensional version of second estimate and first estimate (see, for example, [13]) we obtain the following vital Forelly-Rudin estimate (3) which we will use in proofs of our main results.

$$\int_{T_\Omega} \Delta^{\beta}(y)|B_{\alpha+\beta+\frac{n}{r}}(z,w)|dv(z) \le C\Delta^{-\alpha}(v), \tag{3}$$

(Forelly-Rudin estimate in tube), $\beta > -1$, $\alpha > \frac{n}{r} - 1$, $z = x+iy$, $w = u+iv$, $r > 0$, $z,w \in T_\Omega$, (see [12], [13]).

We denote by $B(z,r)$ the Bergman ball in T_Ω, (see [12], [13]).

Finally for completeness we provide very vital Whitney decomposition of tubular domain over symmetric cones based on Bergman balls. It was used during many proofs of various assertions (see, for example, [2], [3]).

Lemma 2. *Given $\delta \in (0,1]$ there exist a sequence of points $\{z_j\}$ in T_Ω called δ-lattice such that calling $\{B_j\}$ and $\left\{B'_j\right\}$ the Bergman balls with center z_j and radius δ and $\frac{\delta}{2}$ respectively, then*

1) *the balls (B'_j) are pairwise disjoint;*

2) *the balls (B_j) cover T_Ω with finite overlapping;*

3) $\int\limits_{B_j(z_j,\delta)} \Delta^s(y)dV(z) \asymp \int\limits_{B'_j(z_j,\delta)} \Delta^s(y)dV(z) = \widetilde{C}_\delta \Delta^{2\frac{n}{r}+s}(Im\ z_j)$, $s > \frac{n}{r} - 1$, $J = |B_\delta(z_j)| \asymp \Delta^{2\frac{n}{r}}(Im\ z_j)$, $j = 1,\dots,m$, $J \asymp \Delta^{2\frac{n}{r}}(Im\ w)$, $w \in B_\delta(z_j)$.

We call by $\{z_j\}$ r-lattice of T_Ω below everywhere. This is a vital notation for this note.

We denote m cartesian products of tubes by T_Ω^m, the space of all analytic function on this new product domain which are analytic by each variable separately will be denoted by $\mathscr{H}(T_\Omega^m)$. In this paper we will be interested on properties of certain analytic subspaces of $\mathscr{H}(T_\Omega^m)$. By m here and everywhere below we denote a natural number bigger than 1.

Let further $dv(z_1,\dots,z_m) = \prod\limits_{j=1}^{m} dv(z_j) = dv(\overrightarrow{z})$, $z_j \in T_\Omega$, $j = 1,\dots,m$ be the normalized Lebesgues measure on product domain. We provide now some facts on function spaces on product of tube domains.

We denote as usual by $dv_\gamma(z) = \delta^\gamma(z)dv(z) = \Delta^\gamma(Im\ z)dv(z)$, $\gamma > -1$, the weighted Lebesgues measure on T_Ω domain and similarly on products of such domains using products of Δ functions in a standard way for all $\gamma > -1$. Using dv_α and Δ^τ on product domain we can define $A_\tau^\infty(T_\Omega^m)$ and $A_\alpha^{p,p}(T_\Omega^m) = A_\alpha^p(T_\Omega^m)$ for $1 < p < \infty$, $\alpha > -1$. For example we have

$$A_\tau^\infty(T_\Omega^m) = \left\{ f \in H(T_\Omega) : \sup_{z_j \in T_\Omega} |f(\overrightarrow{z})| \prod_{j=1}^{m} \Delta(Im\ z_j)^\tau < \infty \right\},$$

$\tau \ge 0$, $\overrightarrow{z} = (z_1,\dots,z_m)$, $z_j \in T_\Omega$, $j = 1,\dots,m$. These are Banach spaces.

In our last section we extend the notion of classical Poisson integral in a simple natural way in the unit ball and extend some well known estimates related with it, in particular an extension of a known maximal theorem will be provided in the unit ball. Such type results probably can be proved in context of more general pseudoconvex domains with smooth boundary.

Similar maximal theorems we proved in section 2 in harmonic function spaces in the unit ball and $\mathbb{R}^{n+1}_{+}$, were proved by T. Flett, (see [5]). They have many applications (see [17]). Various nice results related to various maximal theorems and Poisson integrals in various domains in $\mathbb{C}^n$ and their applications can be seen in [17].

We denote in this paper, as usual, by $C, C_1, C_2, ..., C_\alpha$ various positive constants.

Maximal and embedding theorems in Herz spaces

The intention of this section is to provide new maximal and embedding theorems for Herz type spaces in tubular domains over symmetric cones. We alert the reader some arguments are sketchy since they can be easily recovered by readers based on simpler cases and remarks we make. All results of this section are known in particular case of simplest one domain namely the unit disk.

This topic is well-developed in the unit disk and other simple domains like unit ball and polydisk (see, for example [17], [18]). In [10], [12] this type embedding theorems can be seen in context of Bergman type harmonic spaces and tubular domains over symmetric cones T_Ω. In this section we add some new results in this direction in same tubular domains over symmetric cones but in new Herz-type analytic spaces.

Maximal theorems are vital classical topic in complex function theory in the unit disk. We provide such type theorems in Bergman type (Herz) spaces in the unit disk and then using same arguments in the different domains, namely, in tubular domain over symmetric cones in $\mathbb{C}^n$ which where under attention in recent decades (see [3], [12], [13]).

Our maximal theorems then will be used to get some new embedding theorems in tube domains.

Some related new results in analytic Herz type spaces in product domains will be also provided in this section. We start with the case of unit disk, then pass easily same arguments to more complicated domains as tubular domain in $\mathbb{C}^n$.

Let U be the unit disk. Let $H(U)$ be the class of all analytic functions in U. Let

$$D^\gamma f(z) = \sum_{k\geq 0} (k+1)^\gamma a_k z^k, \ \gamma \in \mathbb{R}$$

be the fractional derivative of $f \in H(U)$. We denote by dm_2 the normalized Lebesgues measure on U. We note the following simple arguments are valid in U. Let $f \in H(U)$, $\alpha > -1$, then

$$f(z) = c(\beta) \int_U \frac{f(w)(1-|w|)^\beta}{(1-\bar{w}z)^{\beta+2}} dm_2(w), \ \beta > \beta_0,$$

β_0 is large enough, $z \in U$, (see [18]). This representation also is valid in tube, for Bergman spaces and $A^\infty_\alpha(T_\Omega)$ spaces, (see [1], [3], [12] and references there).

We will need Forelly-Rudin estimate in the unit disk

$$\int_U \frac{(1-|z|)^\alpha}{|1-\bar{w}z|^\beta} dm_2(z) \leq C(1-|w|)^{\alpha-\beta-2}, \ \beta > -1, \ \beta > \alpha+2, \ w \in U.$$

Then we have based on this estimates for Bergman disk $D(z,R)$

$$I(f)=\int_U \sup_{z\in D(v,R)}(|D^\gamma f(z)|(1-|z|)^\tau)(1-|v|)^s dm_2(v)$$

$$I(f)\le c_1\int_U\int_U \frac{f(w)(1-|w|)^\beta}{|1-\bar{w}z|^{\beta+2}}dm_2(w)(1-|v|)^\tau(1-|v|)^s dm_2(v)$$
$$\le c_2\int_U |f(w)|(1-|w|)^{\tau+s-\gamma}dm_2(w),$$

where $\tau\ge 0$, $s>-1$, $\gamma\ge 0$.

We used simple properties of lattice in U (see [18]): $(1-|v|)\asymp(1-|z|)\asymp(1-|\bar{w}|)$ and $|1-\bar{w}v|\asymp|1-\bar{w}z|$, $v,z\in D(\tilde{w},r)$, $\tilde{w}\in U$, $w\in U$.

So $I(f)\le c\|f\|_{A^1_{\tau+s-\gamma}}$, $\tau+s-\gamma=\alpha>-1$. So we have that $I(f)\le c\|f\|_{A^1_\alpha}$, $\alpha>-1$. The $p>1$ case for A^p_α needs only small modification.

The only new ingredient is an estimate which follows directly from well known Forelly-Rudin and Holder's inequality for each $\varepsilon>0$, $\beta>-1$, $1<p<\infty$, $\tau>0$

$$\left(\int_U \frac{|f(w)|(1-|w|)^\beta}{|1-\bar{w}z|^\tau}dm_2(w)\right)^p\le c\int_U \frac{|f(w)|^p(1-|w|)^{\beta p}}{|1-\bar{w}z|^{\tau p+2-\varepsilon p}}dm_2(w)(1-|z|)^{-\varepsilon p},\ z\in U.$$

This estimate, with same proof, also is valid in tube, see estimate (3) (and see, for example [1], [3], [12] and references there).

As a result, we have for $p>1$

$$\int_U \sup_{z\in D(v,R)}|D^\gamma f(z)|^p(1-|z|)^\tau(1-|v|)^s dm_2(v)\le c\int_U |f(w)|^p(1-|w|)^{\tau+s-\alpha}dm_2(w).$$

The repetition of these arguments leads to the same estimate, but on product domains (polydisk)

$$\int_U\cdots\int_U \sup_{z_1\in D(\nu_1,R)}\cdots\sup_{z_m\in D(\nu_m,R)} D^\alpha_{z_1\dots z_m}|f(z_1,\dots,z_m)|^p\times$$
$$\times\prod_{j=1}^m(1-|z_j|)^{\tau_j}(1-|\nu_j|)^{s_j}dm_2(\nu_1)\cdots dm_2(\nu_m)\le$$
$$\le C\int_U\cdots\int_U |f(\overrightarrow{w})|^p\prod_{j=1}^m(1-|w_j|)^{\tau_j+s_j-\alpha_j}dm_2(w_j),$$

with the same restrictions of parameters. (This remark will be used by as below to get same type result in different unbounded tube domains.)

These arguments under one condition on Bergman kernel leads to new maximal and embedding theorems in very general tube domains over symmetric cones and for Herz type spaces on them even on product of such domains.

We assume for Bergman kernel B_ν, $\nu>\nu_0$, the following condition is valid for Bergman ball $B(z,R)$ in tube.

$$\sup_{w\in B(z,R)}|B_\nu(w,\tilde{w})|\le c|B_\nu(z,\tilde{w})|,\ \tilde{w}\in T_\Omega,$$

where $\nu>\nu_0$, ν_0 is large enough.

Note the reverse is valid for c=1 obviously and this is valid in ball and polydisk (see, for example, [18]).

As usual we shall denote by $\Box_z$ the natural extension to the complex space $\mathbb{C}^n$ of the generalized wave operator $\Box_x$ of the cone Ω:

$$\Box_z = \Delta\left(\frac{1}{i}\frac{\partial}{\partial z}\right),$$

which is the differential operator of degree r defined by the equality:

$$\Delta\left(\frac{1}{i}\frac{\partial}{\partial z}\right)[e^{i(z|\zeta)}] = \Delta(\zeta)e^{i(z|\zeta)},\ \zeta \in \mathbb{R}^n.$$

Repeating arguments of the unit disk case step by step and using preliminaries of previous section about tubular domains and properties of analytic functions on them, we have the following:

Theorem 1. *(a maximal theorem) Let $f \in A^p_{\vec{\alpha}}(T^m_\Omega)$, $\alpha_j > \frac{n}{r} - 1$, $j = 1,\ldots,m$, $\alpha_j = \tau_j + s_j - \gamma_j > \frac{n}{r} - 1$, $p > 1$. Then we have that*

$$\|f\|^p_{S^{\gamma,p}_{\tau,s}} = \int_{T_\Omega} \sup_{z_1 \in B(w_1,r)} \cdots \int_{T_\Omega} \sup_{z_m \in B(w_m,r)} |\Box^{\vec{\gamma}}_{z_1,\ldots,z_m} f|^p dv(w_m) \times$$

$$\times \prod_{j=1}^m \Delta^{\tau_j}(Im\ z_j)\Delta^{s_j}(Im\ w_j)dv(w_m)\ldots dv(w_1) \le c\|f\|_{A^p_{\vec{\alpha}}(T^m_\Omega)}$$

for all $\gamma_j > \gamma_0$, $j = 1,\ldots,m$ where γ_0 is large enough and the reverse is also true if $\gamma_j = 0$, $j = 1,\ldots,m$.

The same proof can be given for very similar another maximal theorem.

Theorem 2. *Let $f \in A^p_{\vec{\alpha}}(T^m_\Omega)$, $\alpha_j > \frac{n}{r} - 1$, $j = 1,\ldots,m$, $\alpha_j = \tau_j + s_j - \gamma_j > \frac{n}{r} - 1$, $p > 1$. Then we have that*

$$\|f\|^p_{\tilde{S}^{\gamma,p}_{\tau,s}} = \int_{T_\Omega} \ldots \cdots \int_{T_\Omega} \sup_{z_1 \in B(w_m,r)} \sup_{z_m \in B(w_1,r)} |\Box^{\vec{\gamma}}_{z_1,\ldots,z_m} f|^p dv(w_m) \times$$

$$\times \prod_{j=1}^m \Delta^{\tau_j}(Im\ z_j)\Delta^{s_j}(Im\ w_j)dv(w_1)\ldots dv(w_m) \le c\|f\|_{A^p_{\vec{\alpha}}(T^m_\Omega)}$$

for all $\gamma_j > \gamma_0$, $j = 1,\ldots,m$ where γ_0 is large enough and the reverse is also true if $\gamma_j = 0$, $j = 1,\ldots,m$.

Remark 1. Note, T. Flett proved some very similar to our maximal theorem results in Bergman harmonic spaces in $\mathbb{R}^{n+1}_+$ in [5].

Note for H^p Hardy space in the unit disk we have similar type classical result

$$\int_T \sup_{z \in \Gamma_\alpha(\xi)} |f(z)|^p d\xi \le C\|f\|^p_{H^p},\ 0 < p < \infty,$$

where $0 < p < \infty$, $\Gamma_\alpha(\xi)$ is a classical Lusin cone, and where $T = \{|z| = 1\}$, see [17].

The short proof in the unit disk was provided above. The T_Ω case is the same, properties of r-lattices for T_Ω must be used (see [12] and our first section). Note, for $\gamma_j = 0$, $j = 1,\ldots,m$ this result is sharp.

The crucial ingredient is the next lemma (see [10], [12]).

Lemma 3. *Let* $1 \le p < \infty$, $\nu > \frac{n}{r} - 1$, $f \in A^p_\nu$, *then for* $l > l_0$, $m \ge 0$, *where* l_0 *is large enough, we have that*

$$\Box^l f(z) = c \int_{T_\Omega} B_{\nu+l}(z,w)\Box^m f(w)\Delta^m(Im\, w)dv(w),\ z \in T_\Omega;$$

We have to use also short comments in tube we provided in the proof of the unit disk case by us. The rest is the repetition of arguments we gave above.

This theorem provided a way for various embedding theorems for Herz-type spaces. Our proof use also some arguments of [10], [12]. Namelly, combining results from [10], [12] and our maximal theorem we have:

Theorem 3.

1) *Let* $\gamma > \gamma_0$, γ_0 *is large enough. Let* $p \ge 1$, *let* $\mu = \mu_1 \times \cdots \times \mu_m$ *be positive Borel measure on* T^m_Ω. *Let* $p \le q$, *if*

$$\left(\int_{T^m_\Omega} |f(\overrightarrow{z})|^q d\mu(\overrightarrow{z})\right)^{\frac{1}{q}} \le c\|f\|_{S^{\gamma,p}_{\tau,s}}$$

then $\mu_j(B(a_k,r)) \le c\Delta^{q_i}(Im\, a_k)$ *for some* $q_j = q_j(p,\gamma,\vec{\tau},\vec{s})$, $\alpha_j = \tau_j + s_j - \gamma > \frac{n}{r} - 1$, $j = 1,\dots,m$, $\tau_j \ge 0$, $s_j > -1$, $\tau_j = s_j - \frac{n}{r}$, $j = 1,\dots,m$, *where* $\{a_k\}$ *is r-lattice. For* $\gamma = 0$ *this condition on measure is sufficient for this embedding.*

2) *Let* $\gamma > \gamma_0$, γ_0 *is large enough. Let* $p \ge 1$, *let* μ *be positive Borel measure on* T_Ω. *Let* $q < p$, *if*

$$\left(\int_{T_\Omega} |f(z)|^q d\mu(z)\right)^{\frac{1}{q}} \le c\|f\|_{S^{\gamma,p}_{\tau,s}}$$

then $\dfrac{\mu(B_\delta(z))}{\Delta^{\alpha+\frac{n}{r}}(Im\, z)} \in L^s_\alpha(T_\Omega)$ *for some* q, $\alpha = \tau + s - \gamma > \frac{n}{r} - 1$, $\tau \ge 0$, $s > -1$, $\tau = s - \frac{n}{r}$, $s = \frac{p}{p-q}$, $v \in (0,1)$, *if* P_ν *is bounded on* L^p_α, $\nu > \nu_0$, ν_0 *is large enough. For* $\gamma = 0$ *this condition on measure is sufficient for this embedding.*

Sketch of proof of Theorem . Indeed, in [10] we can see the complete description of μ Borel positive measures defined in T_Ω, so that the following embeddings are valid

$$\left(\int_{T_\Omega} |f(z)|^q d\mu(z)\right)^{\frac{1}{q}} \le C\|f\|_{A^p_\alpha(T_\Omega)},$$

where $q \ge p$, $\alpha > \dfrac{n}{r} - 1$, and

$$\left(\int_{T_\Omega} |f(z)|^q d\mu(z)\right)^{\frac{1}{q}} \le C_1\|f\|_{A^p_\alpha(T_\Omega)},$$

where $q < p$, $\alpha > \dfrac{n}{r} - 1$.

Note, also, the same result with same proof is valid for $q \ge p$ case for Bergman $A^p_\alpha(T^m_\Omega)$ spaces on product T^m_Ω domains. It remains to combine this result with our maximal theorems to get easily what we need.

We define $S^{q,p}_{\tau,s}(d\mu)$ replacing dv by $d\mu$ in quazinorms, where $d\mu_j,\ j=1,\dots,m$ is a Borel measure on T_Ω.

Remark 2. For $\gamma=0$ similar result is valid for embedding of type $\|f\|_{S^{0,p}_{\tau,s}(d\vec{\mu})}\le c\|f\|_{A^q_\alpha(T^m_\Omega)},\ p\ge q\ge 1$.

Methods used in this paper also allows to find for $0<p,q<\infty,\ \alpha>-1$ necessary conditions on measures $(\mu_1,\dots,\mu_m)$ on T_Ω, so that the following embedding is valid for Herz type spaces

$$\int_{T_\Omega}\cdots\int_{T_\Omega}\sup_{z_1\in B(w_1,r)}\cdots\sup_{z_m\in B(w_m,r)}|f(\overrightarrow{w})|^q\prod_{j=1}^m d\mu(w_j)\le C_1\|f\|^p_{A^p_\alpha}.$$

To obtain necessary condition for embeddings

$$\|f\|_{S^{0,p}_{\vec{\gamma},\vec{\tau}}(d\vec{\mu})}\le C\|f\|_{A^q_\alpha(T^m_\Omega)}$$

or

$$\|f\|_{\widetilde{S}^{0,p}_{\vec{\gamma},\vec{\tau}}(d\vec{\mu})}\le C_1\|f\|_{A^q_\alpha(T^m_\Omega)},$$

for all $p,q\in(0,\infty)$ and all $\alpha_j>-1,\ \gamma_j,\ \tau_j\ge 0,\ j=1,\dots,m$ and for positive Borel measures μ_j on $T_\Omega,\ j=1,\dots,m$ we have to use an elementary estimate

$$\sup_{z\in B(w,r)}|\Phi(z,\widetilde{w})|\ge|\Phi(w,\widetilde{w})|$$

for every measurable function and every $\tilde{w},\ \tilde{w}\in T_\Omega$, and every Bergman ball $B(w,r)\subset T_\Omega$, $r>0,\ w\in T_\Omega$ and standard arguments, see [10], based on a estimates from below of Bergman kernel on Bergman ball and Forelly-Rudin type estimate for Bergman kernel. For same type condition for μ Borel measure on T_Ω embeddings $\|f\|_{A^q(d\mu)}\le c\|f\|_{S^{0,p}_{\vec{\gamma},\vec{\tau}}}$, $0<p,q<\infty$ or embeddings $\|f\|_{A^q(d\mu)}\le C_1\|f\|_{\widetilde{S}^{0,p}_{\vec{\tau},\vec{\gamma}}}$ we must use same type arguments with condition discussed them partially below.

Let now

$$A^{\vec{p}}_{\vec{\alpha}}(T^m_\Omega)=\{f\in H(T^m_\Omega):\left(\int_{T_\Omega}\left(\dots\left(\int_{T_\Omega}|f(w_1,\dots,w_m)|^{p_1}d\nu_{\alpha_1}(w_1)\right)^{\frac{p_2}{p_1}}\dots\right)d\nu_{\alpha_m}(w_m)\right)^{\frac{1}{p_m}}<\infty\}$$

$1<p_j<\infty,\ j=1,\dots,m,\ \nu_{\alpha_j}(w)=\Delta^{\alpha_j}(Im\ w)d\nu(w),\ \alpha_j>-1,\ j=1,\dots,m.$

These spaces are direct extensions of Bergman $A^p_\alpha(T^m_\Omega)$ function classes. In our embeddings relating $S^{0,p}_{\vec{\gamma},\vec{\tau}}(d\vec{\mu})$ and $A^p_\alpha(T^m_\Omega)$ the Bergman space can be replaced by $A^{\vec{p}}_{\vec{\alpha}}(T^m_\Omega)$ easily also.

And we obtain again a necessary condition on $\vec{\mu}=(\mu_1,\dots,\mu_m)$ similarly. (Carleson type condition on measure). We here omit easy details.

Next in our maximal theorem for $S^{0,p}_{\vec{\gamma},\vec{\tau}}$ classes probably can be extended to $S^{0,\vec{p}}_{\tau,\gamma}$ mixed norm spaces where

$$S^{0,\vec{p}}_{\tau,\gamma}=\{f\in H(T^m_\Omega):$$

$$\left(\int_{T_\Omega}\sup_{z_1\in B(w_1,r)}\cdots\left(\int_{T_\Omega}\sup_{z_m\in B(w_m,r)}(|f(z_1,\dots,z_m)|)^{p_1}d\nu_{\alpha_1}(z_1)\right)^{\frac{p_2}{p_1}}d\nu_{\alpha_m}(z_m)\right)^{\frac{1}{p_m}}<\infty\}.$$

We will study this problem in our next paper of this topic.

Based on discussion above we get the following estimates which at the same time provide proofs of last assertions on embeddings. We have for standard test function (see [10]):

$$f_{\tilde{z}}(\vec{w}) = \prod_{j=1}^{m} \frac{\delta^{\alpha}(\tilde{z}_j)}{\Delta^{\beta}(\frac{\tilde{z}_j - \bar{w}_j}{i})}, \ w_j \in T_{\Omega}, \ j = 1, \ldots, m,$$

where $\delta(z) = \Delta(Im\ z)$, $z \in T_{\Omega}$, $\tilde{z} \in T_{\Omega}^m$, $\tilde{z} = (\tilde{z}_1, \ldots, \tilde{z}_m)$, for some α and $\beta > \beta_0, \beta_0$ is large enough,

$$\|f_{\tilde{z}}\|_{A^p_{\gamma}(T^m_{\Omega})} \le c \prod_{j=1}^{m} \delta^{\tau}(z_j), \ z_j \in T_{\Omega}, \ j = 1, \ldots, m, \ \tau = \alpha - \beta + \left(\frac{2n}{r} + \gamma\right)\frac{1}{p} < 0, \ 0 < p < \infty,$$

with some restrictions on β and α and more generally

$$\|f_{\tilde{z}}\|_{A^{\vec{p}}_{\vec{\gamma}}(T^m_{\Omega})} = \left(\left(\int_{T_{\Omega}} \cdots \int_{T_{\Omega}} |f_{\vec{z}}(\vec{w})|^{p_1} d\nu_{\alpha_1}(w_1)\right)^{\frac{p_2}{p_1}} \cdots d\nu_{\alpha_m}(w_m)\right)^{\frac{1}{p_m}}$$

$$= \prod_{j=1}^{m} \|\widetilde{f}_{z_j}(w_j)\|_{A^{p_j}_{\gamma_j}(T^m_{\Omega})} \le c\left(\prod_{j=1}^{m} \delta^{\tilde{\tau}_j}(z_j)\right),$$

for some $\tilde{\tau}_j, \ j = 1, \ldots, m$.

Also, then we have

$$\|f_{\tilde{z}}\|_{S^{p,0}_{\alpha,\beta}(d\vec{\mu})} \ge c\left(\prod_{j=1}^{m} \delta^{V}(z_j)\right)\left(\prod_{j=1}^{m} \mu_j(B_{\delta}(z_j))\right), \ \delta > 0,$$

and also

$$\|f_{\tilde{z}}\|_{\tilde{S}^{p,0}_{\alpha,\beta}(d\vec{\mu})} \ge c_1\left(\prod_{j=1}^{m} \delta^{\tilde{V}}(z_j)\right)\prod_{j=1}^{m} \mu_j\left(B_{\delta}(z_j)\right),$$

for some parameters V, $\tilde{V}$ and $z_j \in T_{\Omega}, \ j = 1, \ldots, m$.

First two estimates are based directly on Forelly-Rudin estimate in tube and the last two estimates are based on standard arguments related with estimates from below of Bergman kernel on Bergman ball, (see [10], [12], for example for similar type argument).

Remark 3. Similar type results (maximal and embedding theorems) by similar methods can be shown in the polydisk, in the unit ball and in pseudconvex domains with smooth boundary in $\mathbb{C}^m$. This will be done in out next papers.

A maximal theorem in the unit ball related with the Poisson type integral

In this section we extend the notion of classical Poisson integral in a simple natural way in the unit ball and extend some well known estimates related with it, in particular an extension of a known maximal theorem will be provided in the unit ball. Such type results probably can be proved in context of more general pseudoconvex domains with smooth boundary.

Poisson type integrals on product domains considered recently also in papers [4] and [14].

In this short section we also in particular provide an extension of a known result concerning estimate from below of Poisson kernel (see, for example, [18]).

Let $B_n = \{z \in \mathbb{C}^n : |z| < 1\}$ be the unit ball and let $S_n = \{z \in \mathbb{C}^n : |z| = 1\}$ be the unit sphere.

Let $\xi \in S_n,\ r > 0,\ Q_r(\xi) = \{z \in B_n,\ d(z,\xi) < r\},\ d(z,w) = |1 - <z,w>|^{\frac{1}{2}},\ z,w \in \overline{B}_n$.

We recall $Q_r(\xi)$ is Carleson tube at ξ. Let $M_\alpha(f)(\xi) = \sup_{z \in D_\alpha(\xi)} |f(z)|$, where

$$D_\alpha(\xi) = \{z \in B_n : |1 - <z,\xi>| < \frac{\alpha}{2}(1-|z|^2),\ \alpha > 1\}$$

where $\xi \in S_n$.

Let

$$P[\mu](z) = \int_{S_n} \frac{(1-|z|)^n d\mu(\xi)}{|1-\overline{\xi}z|^{2n}},\ z \in B_n,$$

$$(M\mu)(\xi) = \sup_{\delta > 0} \frac{1}{\sigma(Q(\xi,\delta))} \int_{Q(\xi,\delta)} d\mu(\tilde{\xi}),$$

where $Q(\xi,\delta) = \{\eta \in S_n : d(\xi,\eta) < \delta\},\ \delta > 0,\ \xi \in S_n,\ z \in B_n$.

Theorem A. *(a maximal theorem in the ball, see [11], [18]) Let $\alpha > 1$, then*

$$I_\mu(\xi) = (M_\alpha P[\mu])(\xi) \le \tilde{c} M\mu(\xi),\ \xi \in S_n,$$

and $\|I_f(\xi)\|_{L^p(S_n)} \le \tilde{c}\|f\|_{L^p}$, $p > 1$ for a μ positive complex finite Borel measure on B_n.

This maximal theorem result has many applications (see, for example, [11], [18]). We now show an extension of this, that the following result is valid also.

Let further

$$P_{\overrightarrow{\alpha}}(\overrightarrow{z},\xi) = \frac{(1-|z|)^n}{\prod_{j=1}^m |1-z_j\xi|^{\alpha_j}},$$

where $\xi \in S_n,\ z_j \in B_n,\ |z_j| = |z|,\ \alpha_j > 0,\ j = 1,\ldots,m$.

Theorem 4. *Let*

$$P_{\vec{\alpha}}[\mu](\vec{z}) = \int_{S_n} \frac{(1-|z|)^n d\mu(\xi)}{\prod_{j=1}^m |1-z_j\xi|^{\alpha_j}},\ z_j \in B,\ j = 1,\ldots,m,\ \sum_{j=1}^m \alpha_j = 2n.$$

Then we have that

$$K_\xi(\mu) = \sup_{z_j \in D_\tau(\xi), |z_j| = r} P_{\vec{\alpha}}[\mu](\vec{z}) \le c_1 M\mu(\xi),\ r \in (0,1),$$

where μ is a positive Borel measure, and

$$\|K_\xi(f)\|_{L^p(S_n, d\sigma(\xi))} \le c_2 \|f\|_{L^p(S_n, d\sigma(\xi))},\ p > 1.$$

Proof.

Let $\vec{z} \in D_\alpha(\xi)$. Let $z = (|z_1|\varphi_1,\ldots,|z_m|\varphi_m),\ |z_j| = |z_i|,\ 1 \le i \le j \le m,\ r = |z_j|,\ t = 8\alpha(1-r)$. Let $V_0 = \{\eta \in S_n : |1 - <\eta,\xi>| < t\}$, $1 \le k \le N$, $2^N t > 2$, $V_k = \{\eta \in S_n : 2^{k-1}t \le |1 - <\eta,\xi>| < 2^k t\}$.

Note that we have obviously

$$\int_{S_n} P_{\vec{\alpha}}(\vec{z},\eta)d\mu(\eta) = \int_{V_0} P_{\vec{\alpha}}(\vec{z},\eta)d\mu(\eta) + \sum_{k=1}^N \int_{V_k} P_{\vec{\alpha}}(\vec{z},\eta)d\mu(\eta).$$

It is enough to show that

$$\int_{S_n} P_{\vec{\alpha}}(\vec{z},\eta)d\mu(\eta) \le c(\alpha,n)(M\mu(\xi)),\ \xi \in S_n,\ z_j \in D_\alpha(\xi),\ j=1,\ldots,m,$$

for some constant $c(\alpha,n)$. We have by definition that $V_k \subset Q(\xi,\sqrt{2^k t})$, $1 \le k \le N$ and hence

$$\mu(V_k) \le \mu(Q(\xi,\sqrt{2^k t})) \le ((M\mu)(\xi))\sigma(Q(\xi,\sqrt{2^k t})) \le c(2^k t)^n M\mu(\xi),\ 0 \le k \le N.$$

$$P_{\vec{\alpha}}(\vec{z},\eta) \le 2^n(1-r)^{-n}.$$

We have hence, since $\mu(V_0) \le ct^n M\mu(\xi)$

$$\int_{V_0} P_{\vec{\alpha}}(\vec{z},\eta)d\mu(\eta) \le c\left(\frac{t}{1-r}\right)^n (M\mu(\xi)) \le C_{\alpha,n}(M\mu)(\xi).$$

Then if $\eta \in S_n$ we have

$$|1-<\tilde{z},\xi>| \le \tilde{c}_\alpha(1-|\tilde{z}|^2) \le \tilde{c}'_\alpha|1-<\tilde{z},\eta>|$$

$$d(\tilde{z},\xi) < \sqrt{\alpha'}d(\tilde{z},\eta).$$

Hence we have from definition of d

$$d(\xi,\eta) \le d(\xi,\tilde{z}) + d(\tilde{z},\eta) \le \tilde{c}''_\alpha d(\tilde{z},\eta),\ \tilde{z}=z_j,\ j=1,\ldots,m.$$

Hence

$$\int_{V_k}(P_{\vec{\alpha}}(\overrightarrow{z},\eta))d(\mu(\eta)) \le (\frac{c(\alpha,n)}{2^{kn}})(M\mu)(\xi),$$

where $1 \le k \le N$, the rest is clear.

The last estimate follows from the fact that

$|1-<\xi,\eta>| \le \tilde{c}(\alpha,n)|1-<z_j,\eta>|$ for every z_j, $j=1,\ldots,m$, $1 \le k \le N$, $\eta \in V_k$. And hence we have that

$$P_{\vec{\alpha}}(\vec{z},\eta) \le \frac{\tilde{c}'(\alpha,n)t^n}{|1-<\xi,\eta>|^{2n}} \le \frac{\tilde{c}''(\alpha,n)}{4^{kn}t^n}.$$

The second assertion of theorem follows from the first part of our theorem and the well known maximal theorem, (see [11], [18]).

Theorem is proved. □

Let further, as usual, $P(z,\xi) = \frac{(1-|z|^2)^n}{|1-<z,\xi>|^{2n}}$, $z \in B_n$, $\xi \in S_n$ be the Poisson kernel on B_n unit ball (see [11], [17], [18]). Let $P[f](z) = \int_{S_n} P(z,\xi)f(\xi)d\sigma(\xi)$, $f \in L^1(S_n,d\sigma)$, where σ is a normalized measure on S_n.

It is natural to study the following extension of $P[f]$, $\tilde{P}[f](\vec{z})$, $z_j \in B_n$, $j=1,\ldots,m$, where

$$\tilde{P}[f](\vec{z}) = \int_{S_n} \tilde{P}(\vec{z},\xi)f(\xi)d\sigma(\xi),$$

where

$$\tilde{P}(\vec{z},\xi) = \tilde{P}_{\vec{\alpha},\vec{\beta}}(\vec{z},\xi) = \frac{\prod_{j=1}^m (1-|z_j|^2)^{\alpha_j}}{\prod_{j=1}^m |1-<z_j,\xi>|^{\beta_j}},$$

$\alpha_j, \beta_j > 0,\ j = 1, \dots, m,\ \sum_{j=1}^m \alpha_j = n,\ \sum_{j=1}^m \beta_j = 2n$ and even more generally.

$$G(z_1^1, \dots, z_m^1, \dots, z_1^k, \dots, z_m^k, \xi_1, \dots, \xi_k) = \prod_{i=1}^k \widetilde{P}^i(\overrightarrow{z}, \xi),\ \widetilde{P}^i = \widetilde{P}_{\overrightarrow{\alpha_j}^i, \overrightarrow{\beta_j}^i},$$

$z_i^j, \xi_j \in B_n,\ i, j = 1, \dots, k$ and try to expand classical known assertions.

Very similar procedure of extension of Bergman projection was provided and used intensively in connection with trace problem (see [15], [16]).

We found the following results for Poisson $\tilde{P}$ kernel as modification of known proofs for P kernel (see [18] for $m = 1$ case).

Proposition 1. Let $\mu,\ \mu_j$ be positive Borel measures on B_n, and let $p_i \in (0, \infty)$, $i = 1, \dots, m$. Then we have that

1)

$$\sup_{z_j \in B_n, j=1\dots m} \int_{B_n} \tilde{P}(w, \vec{z}) d\mu(w) \geq c\ \frac{\mu(Q_r(\xi))}{r^{2n}},\ \xi \in S_n,\ r \in (0,1),$$

and even generally we have for $r_j \in (0,1),\ j = 1, \dots, m.$

2)

$$\sup_{w_j \in B_n, j=1\dots m} \left(\int_{B_n} \cdots \left(\int_{B_n} |G(\vec{z}, \vec{w})|^{p_1} d\mu_1(z_1) \right)^{\frac{p_2}{p_1}} d\mu_2(z_2) \cdots d\mu_m(z_m) \right)^{\frac{1}{p_m}} \geq$$

$$\geq \tilde{C_1}\ \frac{\mu_1(Q_{r_1}(\xi_1)) \dots \mu_m(Q_{r_m}(\xi_m))}{r_1^{\alpha_0^1} \dots r_m^{\alpha_0^m}},\ 0 < p_j < \infty,\ j = 1 \dots m,$$

for some $\alpha_0^j,\ \alpha_0^j > 0,\ j = 1, \dots, m$, where $\mu,\ \mu_j,\ j = 1, \dots, m$ are positive Borel measures on B_n for some constants $\tilde{c}_1,\ c$ where $\xi_j \in S_n,\ j = 1, \dots, m.$

Proofs are heavily based on arguments of proofs of $m = 1$ case and they will be given elsewhere.

Remark 4. These results of Proposition extend some known assertions from [11] and [18].

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О НЕКОТОРЫХ НОВЫХ ОЦЕНКАХ, СВЯЗАННЫХ С ТЕОРЕМАМИ ОБ ОГРАНИЧЕННОСТИ ПРОЕКТОРОВ ТИПА БЕРГМАНА И ИНТЕГРАЛОМ ПУАССОНА В ТРУБЧАТОЙ ОБЛАСТИ И ЕДИНИЧНОМ ШАРЕ

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Введены новые аналитические пространства типа Герца, основанные на шарах Бергмана в трубчатых областях над симметричными конусами. Мы предлагаем для этих пространств типа Герца новые максимальные теоремы и теоремы вложения, расширяющие известные результаты в единичном круге. Кроме того, мы определяем новый интеграл типа Пуассона в единичном шаре и распространяем известную классическую максимальную теорему, связанную с ним. Соответствующие результаты для интегралов такого типа также будут приведены

Ключевые слова: трубчатые области над симметричными конусами, пространства типа Герца, интегральные операторы типа Бергмана, максимальные теоремы, теоремы вложения, интеграл типа Пуассона, единичный шар