

## ABOUT A PROBLEM FOR THE DEGENERATING MIXED TYPE EQUATION FRACTIONAL DERIVATIVE

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The existence and the uniqueness of solution of local problem for degenerating mixed type equation is investigated. Considering parabolic-hyperbolic equation involve the Caputo fractional derivative. The uniqueness of solution is proved using the method of the extremum principle and integral energy, the existence is proved by the method of integral equations.

*Keywords: boundary value problem, degenerating equation, parabolic-hyperbolic type, Gauss hypergeometric function, Cauchy problem, existence and uniqueness of solution, a principle an extremum, method of integral equations, Caputo fractional derivative.*

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### Introduction

In the works [1],[2],[3],[4] we can see significant development in the fractional differential equations in recent years. The fractional calculus is widely applied to investigation of partial differential equations of mixed type and hyperbolic type with degenerations (see [4],[5],[6]). In a series of papers (see [7], [8],[9]) the authors considered some classes of boundary value problems for mixed type non degenerating and degenerating differential equations involving Caputo and Riemann-Liouville fractional derivatives of order  $0 < \alpha \leq 1$ .

### Preliminaries

DEFINITION. Caputo fractional derivatives  ${}_C D_{ax}^\alpha f$  and  ${}_C D_{xb}^\alpha f$  of order  $\alpha > 0$ , ( $\alpha \notin N \cup \{0\}$ ) are defined by [1.p.92]:

$$({}_C D_{ax}^\alpha f)_x = \frac{1}{\Gamma(n-\alpha)} \int_a^x \frac{f^{(n)}(t)}{(x-t)^{\alpha-n+1}} dt, n = [\alpha] + 1, x > a; \quad (1)$$

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$$({}_c D_{xb}^\alpha f)x = \frac{(-1)^n}{\Gamma(n-\alpha)} \int_x^b \frac{f^{(n)}(t)}{(t-x)^{\alpha-n+1}} dt, n = [\alpha] + 1, x < b; \tag{2}$$

respectively.

From (1), (2), as a conclusion we will have:  $k - 1 < \alpha \leq k, k \in N$ ; consequently, while for  $\alpha \in N \cup \{0\}$  we have

$$({}_c D_{ax}^0 f)x = f(x), ({}_c D_{xb}^0 f)x = f(x), ({}_c D_{ax}^n f)x = f^{(n)}(x);$$

$$({}_c D_{xb}^n f)x = (-1)^n f^{(n)}(x), n \in N.$$

Gauss hypergeometric function  $F(a, b, c, z)$  is defined in the unit disk as the sum of the hypergeometric series (see [1. p.27]):

$$F(a, b, c, z) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{(c)_k} \frac{z^k}{k!}, \tag{3}$$

where

$$|z| < 1,$$

$$a, b \in \mathbb{C}, c \in \mathbb{C} \setminus Z_0^- \text{ and } (a)_0 = 1, (a)_n = a(a+1)\dots(a+n-1) = \frac{\Gamma(a+n)}{\Gamma(a)}, (n = 1, 2, \dots).$$

One such analytic continuation is given by Euler integral representation:

$$F(a, b, c; z) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_0^1 x^{b-1} (1-x)^{c-b-1} (1-zx)^{-a} dx, \tag{4}$$

$$0 < \operatorname{Re} b < \operatorname{Re} c, \quad |\arg(1-z)| < \pi.$$

The Gauss hypergeometric function  $F(a, b, c, z)$  allows the following estimation:

$$F(a, b, c; z) \leq \begin{cases} c_1, & \text{if } c-a-b > 0, \quad 0 \leq z \leq 1 \\ c_2(1-z)^{c-a-b}, & \text{if } c-a-b < 0, \quad 0 < z < 1 \\ c_3(1+|\ln(1-z)|), & \text{if } c-a-b = 0 \end{cases} . \tag{5}$$

$$F(a, 1-a, c, z) = (1-z)^{c-1} F\left(\frac{c-a}{2}, \frac{c+a-1}{2}, c, 4z(1-z)\right) \tag{6}$$

Generalized fractional integro-differential operators with Gauss hypergeometric function  $F(a, b, c; z)$  defined for real  $a, b, c$  and  $x > 0$  will be given by formulate:

$$F_{ox} \left[ \begin{matrix} a, b \\ c, x^k \end{matrix} \right] f(x) = \frac{1}{\Gamma(c)} \int_0^x f(t) (x^k - t^k)^{c-1} F\left(a, b, c; \frac{x^k - t^k}{x^k}\right) k t^{t-1} dt,$$

$$c > 0, \quad k > 0$$

(7)

The elementary definition of the Wright type function at  $\alpha > \beta, \alpha > 0$  and for all  $z \in \mathbb{C}$ , is [10]

$$e_{\alpha, \beta}^{\mu, \delta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \mu) \Gamma(\delta - \beta k)}. \tag{8}$$

If  $\alpha = \mu = 1$ , then owing to (3) from (8) we have:

$$e_{1, \beta}^{1, \delta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{n! \Gamma(\delta - \beta n)}. \tag{9}$$

### Problem formulation and main functional relation

This work deals the existence and uniqueness of solution of the problem for the mixed type equation with two lines and different order of degenerating which involve the Caputo fractional derivative.

We consider equation:

$$0 = \begin{cases} u_{xx} - cD_{oy}^\alpha u, & \text{at } y > 0 \\ (-y)^m u_{xx} - x^n u_{yy}, & \text{at } y < 0 \end{cases} \tag{10}$$

with operators (see (1)):

$$cD_{oy}^\alpha u = \frac{1}{\Gamma(1-\alpha)} \int_0^y (y-t)^{-\alpha} u_t(x,t) dt, \tag{11}$$

where  $0 < \alpha < 1$ ,  $m, n = const$ .

Let's  $\Omega$  domain, bounded with segments:

$$A_1A_2 = \{(x,y) : x=0, 0 < y < h_2\},$$

$$B_1B_2 = \{(x,y) : x=h_1, 0 < y < h_2\}, A_2B_2 = \{(x,y) : y=h_2, 0 < x < h_1\} \text{ at the } y > 0,$$

and by the characteristics:  $A_1C : \frac{1}{q}x^q - \frac{1}{p}(-y)^p = 0, B_1C : \frac{1}{q}x^q + \frac{1}{p}(-y)^p = 1$ ; of equation (10) at  $y < 0$ , where  $A_1(0;0), A_2(0;h_2), B_1(h_1;0), B_2(h_1;h_2)$  and  $C\left(\left(\frac{q}{2}\right)^{1/q}, -\left(\frac{p}{2}\right)^{1/p}\right)$ . Here  $2q = n + 2$ ,  $2p = m + 2$ ,  $h_1 = q^{1/q}$ ,  $h_2 > 0$ , and that  $m > n$ . Introduce designations:  $2\alpha_1 = n/(n + 2)$ ,  $2\beta_1 = m/(m + 2)$ ,

$$0 < \alpha_1 < \beta_1 < \frac{1}{2}, \tag{12}$$

$$\Omega^+ = \Omega \cap (y > 0), \Omega^- = \Omega \cap (y < 0), I_1 = \{x : 0 < x < h_1\}, I_2 = \{y : 0 < y < h_2\}.$$

For the equation (10), we consider the following problem: Find a solution  $u(x,y)$  of equation (10) from the following class of functions:

$$\Delta = \{u(x,y) : u(x,y) \in C(\bar{\Omega}) \cap C^2(\Omega^-), u_{xx} \in C(\Omega^+), cD_{oy}^\alpha u \in C(\Omega^+)\}$$

satisfies boundary conditions:

$$u(x,y) \Big|_{A_1A_2} = \varphi_1(y), 0 \leq y \leq h_2, \tag{13}$$

$$u(x,y) \Big|_{B_1B_2} = \varphi_2(y), h_1 \leq y \leq h_2, \tag{14}$$

$$u(x,y) \Big|_{A_1C} = h(x), x \in I_1. \tag{15}$$

and gluing condition:

$$\lim_{y \rightarrow +0} y^{1-\alpha} u_y(x,y) = u_y(x,-0), (x,0) \in A_1B_1 \tag{16}$$

where  $\varphi_1(y), \varphi_2(y), h(x)$  are given functions.

In fact, that functional relation between  $\tau(x)$  and  $v(x)$  transferred from the parabolic part  $\Omega^+$  (hyperbolic part  $\Omega^-$  to the line  $y=0$ ) is played important role on the proved unique and existence of solution.

It is well know, that the solution of the Cauchy problem for the equation (10) in domain  $\Omega^-$  satisfies conditions

$$u(x,-0) = \tau^-(x), 0 \leq x \leq 1 \text{ and } u_y(x,-0) = v^-(x), 0 < x < 1, \tag{17}$$

presented on the form [11]:

$$u(x,y) = \frac{\Gamma(2\alpha_1)}{\Gamma^2(\alpha_1)} \left(\frac{1}{q}x^q\right)^{-\alpha_1} \int_0^1 \left[\frac{1}{p}(-y)^p(2z-1) + \frac{1}{q}x^q\right]^{\alpha_1} [z(1-z)]^{\beta_1-1} \times$$

$$\begin{aligned} & \times \tau^- \left\{ \left[ \frac{q}{p} (-y)^p \cdot 2z - 1 + x^q \right]^{\frac{1}{q}} \right\} F(\alpha_1, 1 - \alpha_1, \beta_1, \rho) dz - \\ & - \frac{\Gamma(1 - 2\alpha_1)}{\Gamma^2(1 - \alpha_1)} p^{-2\beta_1} \left( \frac{1}{p} (-y)^p \right)^{1-2\beta_1} \int_0^1 \left[ \frac{1}{p} (-y)^p (2z - 1) + \frac{1}{q} x^q \right]^{\alpha_1} [z(1 - z)]^{-\beta_1} \times \\ & \times \nu^- \left\{ \left[ \frac{q}{p} (-y)^p \cdot (2z - 1) + x^q \right]^{\frac{1}{q}} \right\} F(\alpha_1, 1 - \alpha_1, \beta_1, \rho) dz \end{aligned} \tag{18}$$

where  $\rho = \frac{q(-y)^{\frac{1}{p}} z(1-z)}{p^2 x^q \left[ \frac{1}{p} (-y)^p (2z-1) + \frac{1}{q} x^q \right]}$ .

Due to condition (15) from (18), by using formulate (6) and () we will get

$$\begin{aligned} h^*(x) &= \gamma_1 (x^{2q})^{\frac{2-\alpha_1-3\beta_1}{2}} F_{0x} \left[ \begin{matrix} \frac{\beta_1-\alpha_1}{2}, \frac{\alpha_1+\beta_1-1}{2} \\ \beta_1, x^{2q} \end{matrix} \right] (x^{2q})^{\frac{\alpha_1+\beta_1-2}{2}} \tau^-(x) - \\ &- \gamma_2 (x^{2q})^{\frac{\beta_1-\alpha_1}{2}} F_{0x} \left[ \begin{matrix} \frac{1-\beta_1-\alpha_1}{2}, \frac{\alpha_1-\beta_1}{2} \\ 1-\beta_1, x^{2q} \end{matrix} \right] (x^{2q})^{\frac{\alpha_1-\beta_1-1}{2}} \nu^-(x), (x, 0) \in I_1, \end{aligned} \tag{19}$$

where  $h^*(x) = h \left[ \left( \frac{x^q}{2} \right)^{1/q} \right]$ ,  $\gamma_1 = \frac{\Gamma(2\alpha_1)}{\Gamma^2(\alpha_1)} 2^{\alpha_1-\beta_1}$ ,  $\gamma_2 = \frac{2^{\alpha_1+3\beta_1-2} \Gamma(1-2\alpha_1)}{\Gamma(1-\alpha_1)} \left( \frac{p}{q} \right)^{1-2\alpha_1}$ .

Applying operator

$$\frac{d}{d(x^{2q})} (x^{2q})^{\frac{1-\alpha_1-\beta_1}{2}} F_{0x} \left[ \begin{matrix} \frac{\alpha_1+\beta_1-1}{2}, \frac{\alpha_1+\beta_1}{2} \\ \beta_1, x^{2q} \end{matrix} \right] (x^{2q})^{\frac{2\alpha_1-1}{2}}$$

to both parts of the equality (19), we obtain functional relation between  $\tau^-(x)$  and  $\nu^-(x)$  transferred from hyperbolic domain  $\Omega^-$  on the line  $y = 0$ :

$$\tilde{\nu}^-(x) = \frac{\gamma_1}{\gamma_2} x^{\frac{1-2\alpha_1}{2}} \frac{d}{dx} x^{\frac{1-2\beta_1}{2}} F_{0x} \left[ \begin{matrix} \alpha_1 + \beta_1, \frac{2\beta_1-1}{2} \\ 2\beta_1, x \end{matrix} \right] x^{\frac{2\alpha_1-1}{2}} \tilde{\tau}^-(x) - \frac{x^{\frac{1-\alpha_1+\beta_1}{2}}}{\gamma_2} h^*(x), 0 < x < h_1, \tag{20}$$

where  $\tilde{\tau}^-(x) = u \left[ ((qk)^2 x)^{1/2q}, 0 \right]$ ,  $\tilde{\nu}^-(x) = u_y \left[ ((qk)^2 x)^{1/2q}, -0 \right]$ .

On the other hand, considering designations (17) and  $\lim_{y \rightarrow +0} y^{1-\alpha} u_y(x, y) = \nu^+(x)$ ,  $0 < x < h_1$  from gluing condition (16) we have

$$\nu^+(x) = \nu^-(x) \tag{21}$$

For further supposes, from Eq. (10) at  $y \rightarrow +0$  considering (11), (21) and

$$\lim_{y \rightarrow 0} D_{0y}^{\alpha-1} f(y) = \Gamma(\alpha) \lim_{y \rightarrow 0} y^{1-\alpha} f(y)$$

we get:

$$\tau''(x) - \Gamma(\alpha) \nu^+(x) = 0 \tag{22}$$

### Uniqueness of the solution

**Theorem 1.** *If satisfy conditions  $0 < \alpha < 1$  and (12) then, the solution is unique.*

**Proof.**

As usual we consider corresponding homogeneous problem  $[\varphi_1(y) \equiv \varphi_2(y) \equiv 0]$  and prove that  $u(x, y) \equiv 0$ . With this aim we multiply to  $\tau(x)$  equation (22) and integrate from 0 to  $h_1$ :

$$\Gamma(\alpha) \int_0^{h_1} \tau(x) \nu^+(x) dx = \int_0^{h_1} \tau''(x) \tau(x) dx. \tag{23}$$

Integrating by part and using the relations  $\tau(0) = \tau(h_1) = 0$ , owing to (21) we obtain

$$\int_0^{h_1} \tau(x) v^-(x) dx = \int_0^{h_1} \tau(x) v^+(x) dx = - \int_0^{h_1} (\tau'(x))^2 dx \leq 0. \tag{24}$$

Now, we prove that  $\int_0^{h_1} \tau(x) v^-(x) dx \geq 0$ .

At first, using by formulate ( ) we make some simplifications in (20):

$$v^-(x) = \frac{\gamma_1 (x^{2q})^{\frac{1-2\alpha_1}{2}}}{\gamma_2 \Gamma(2\beta_1)} \frac{d}{dx^{2q}} (x^{2q})^{\frac{1-2\beta_1}{2}} \int_0^x (x^{2q} - t^{2q})^{2\beta_1-1} (t^{2q})^{\frac{2\alpha_1-1}{2}} \times \\ \times \tau^-(t) F \left( \alpha_1 + \beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{x^{2q} - t^{2q}}{x^{2q}} \right) dt^{2q} - \frac{(x^{2q})^{\frac{1-\alpha_1+\beta_1}{2}}}{\gamma_2} h^*(x), \tag{25}$$

Entering replacement  $t = xz$  and after some simplifications we have:

$$v^-(x) = \frac{2q\gamma_1 (\alpha_1 + \beta_1) (x^{2q})^{\frac{2\beta_1-1}{2}}}{\gamma_2 \Gamma(2\beta_1)} \int_0^1 (1 - z^{2q})^{2\beta_1-1} (z^{2q})^{\frac{2\alpha_1+1}{2}} \times \\ \times \tau^-(xz) F \left( \beta_1 - \alpha_1, \frac{2\beta_1+1}{2}, 2\beta_1, 1 - z^{2q} \right) dz + \\ + \frac{2q\gamma_1 (x^{2q})^{\frac{2\beta_1+1}{2}}}{\gamma_2 \Gamma(2\beta_1)} \int_0^1 (1 - z^{2q})^{2\beta_1-1} z^{2q} \tau'^-(xz) \times \\ \times F \left( \beta_1 - \alpha_1, \frac{2\beta_1+1}{2}, 2\beta_1, 1 - z^{2q} \right) dz - \frac{(x^{2q})^{\frac{1-\alpha_1+\beta_1}{2}}}{\gamma_2} h^*(x),$$

Consequently, using invers replacements  $s = xz$  we can receive

$$v^-(x) = \frac{2q\gamma_1 (\alpha_1 + \beta_1)}{\gamma_2 \Gamma(2\beta_1)} (x^{2q})^{-\alpha_1-\beta_1} \int_0^x (x^{2q} - s^{2q})^{2\beta_1-1} (s^{2q})^{2\alpha_1} \times \\ \times \tau^-(s) F \left( \alpha_1 + \beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{x^{2q} - s^{2q}}{x^{2q}} \right) ds + \\ + \frac{2q\gamma_1}{\gamma_2 \Gamma(2\beta_1)} (x^{2q})^{\frac{1}{2}-\beta_1} \int_0^x (s^{2q})^{\alpha_1+\frac{1}{2}} (x^{2q} - s^{2q})^{2\beta_1-1} \tau'^-(s) \times \\ \times F \left( \alpha_1 + \beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{x^{2q} - s^{2q}}{x^{2q}} \right) ds - \frac{(x^{2q})^{\frac{1-\alpha_1+\beta_1}{2}}}{\gamma_2} h^*(x), \tag{26}$$

□

There holds the following preliminary assertion.

**Lemma.** *If a function  $\tau(x)$  has a positive maximum (respectively a negative minimum) at the point  $x = x_0 \in (0, h_1)$ , then  $v^-(x_0) > 0$  (respectively  $v^-(x_0) < 0$ ).*

**Proof.** Let's a function  $\tau(x)$  has a positive maximum at the point  $x = x_0 \in (0, h_1)$  and  $h^*(x) \equiv 0$ , then from (26) we have:

$$v^-(x_0) = \frac{2q\gamma_1 (\alpha_1 + \beta_1)}{\gamma_2 \Gamma(2\beta_1)} (x_0^{2q})^{-\alpha_1-\beta_1} \int_0^{x_0} \tau^-(t) dt \int_t^{x_0} (x_0^{2q} - s^{2q})^{2\beta_1-1} \times \\ \times (s^{2q})^{2\alpha_1} F \left( \alpha_1 + \beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{x_0^{2q} - s^{2q}}{x_0^{2q}} \right) ds +$$

$$\begin{aligned}
 & + \frac{2q\gamma_1}{\gamma_2\Gamma(2\beta_1)} \left(x_0^{2q}\right)^{\frac{1}{2}-\beta_1} \int_0^x \tau'(s) (s^{2q})^{\alpha_1+\frac{1}{2}} \left(x_0^{2q}-s^{2q}\right)^{2\beta_1-1} \times \\
 & \times F\left(\alpha_1+\beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{x_0^{2q}-s^{2q}}{x_0^{2q}}\right) ds
 \end{aligned}$$

Due to  $\gamma_1 > 0, \gamma_2 > 0, \Gamma(2\beta_1) > 0, 0 \leq \frac{x_0^{2q_1}-s^{2q_1}}{x_0^{2q_1}} \leq 1,$

$$\int_0^{x_0} \tilde{\tau}'(s) ds = \int_0^{x_0} \lim_{x_0 \rightarrow s} \frac{\tilde{\tau}(x_0) - \tilde{\tau}(s)}{x_0 - s} ds > 0$$

and taking (12), (5) into account, from here we deduce that  $v^-(x_0) > 0.$

Similarly, we can prove that on the point of negative minimum  $\tilde{v}^-(x_0) < 0.$  **Lemma is proved.**

□

Based on the Lemma, we can conclude that,  $\int_0^{h_1} \tau(x)v^-(x)dx \geq 0,$  consequently from (24) we will get  $v^-(x) \equiv \tau(x) \equiv 0.$  Hence, based on the solution of the first boundary problem for the Eq.(10) [7],[13] owing to account (13) and (14) we will get  $u(x,y) \equiv 0$  in  $\bar{\Omega}^+,$  similarly, based on the solution (18) we obtain  $u(x,y) \equiv 0$  in closed domain  $\bar{\Omega}^-.$

### The existence of solution of the Problem I

**Theorem 2.** *If satisfies all conditions of the Theorem 1. and*

$$\varphi_1(y), \varphi_2(y) \in C(\bar{I}_2) \cap C^1(I_2); h(x) \in C^1(\bar{I}_1) \cap C^2(I_1) \tag{27}$$

than the solution of the investigating problem is exist.

**Proof.**

Taking (21) into account from Eq.(22) we will obtain

$$\tau''(x) = f(x) \tag{28}$$

where

$$f(x) = \Gamma(\alpha)v^-(x). \tag{29}$$

Solution of the equation (28) together with conditions  $\tau(0) = \varphi_1(0), \tau(h_1) = \varphi_2(0)$  has a form:

$$\tau(x) = \int_0^x (x-t)f(t)dt - x \int_0^1 (1-t)f(t)dt + \varphi_2(0)(1-x) + x\varphi_1(0),$$

consequently, we can find:

$$\tau'(x) = \int_0^x f(t)dt - \int_0^1 (1-t)f(t)dt + \varphi_1(0) - \varphi_2(0). \tag{30}$$

Further, considering (29) from (30), after some simplifications we will get

$$\tau'(x) = \lambda\Gamma(\alpha) \int_0^x v(t)dt - \lambda\Gamma(\alpha) \int_0^1 (1-t)v(t)dt + \varphi_1(0) - \varphi_2(0) \tag{31}$$

Substituting (26) into (31) we have:

$$\begin{aligned}
 \tau'(x) = & \frac{2q\gamma_1(\alpha_1+\beta_1)\Gamma(\alpha)}{\gamma_2\Gamma(2\beta_1)} \int_0^x (t^{2q})^{-\alpha_1-\beta_1} dt \int_0^t (t^{2q}-s^{2q})^{2\beta_1-1} (s^{2q})^{2\alpha_1} \times \\
 & \times F\left(\alpha_1+\beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{t^{2q}-s^{2q}}{t^{2q}}\right) ds \int_0^s \tau'(z)dz +
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{2q\gamma_1\Gamma(\alpha)}{\gamma_2\Gamma(2\beta_1)} \int_0^x (t^{2q})^{\frac{1-2\beta_1}{2}} dt \int_0^t (t^{2q}-s^{2q})^{2\beta_1-1} (s^{2q})^{\frac{2\alpha_1+1}{2}} \tau'(s) \times \\
 & \quad \times F\left(\alpha_1+\beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{t^{2q}-s^{2q}}{t^{2q}}\right) ds - \\
 & - \frac{2q\gamma_1(\alpha_1+\beta_1)\Gamma(\alpha)}{\gamma_2\Gamma(2\beta_1)} \int_0^1 (1-t)(t^{2q})^{-\alpha_1-\beta_1} dt \int_0^t (t^{2q}-s^{2q})^{2\beta_1-1} (s^{2q})^{2\alpha_1} \times \\
 & \quad \times F\left(\alpha_1+\beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{t^{2q}-s^{2q}}{t^{2q}}\right) ds \int_0^s \tau'(z) dz - \\
 & - \frac{2q\gamma_1\Gamma(\alpha)}{\gamma_2\Gamma(2\beta_1)} \int_0^1 (1-t)(t^{2q})^{\frac{1-2\beta_1}{2}} dt \int_0^t (t^{2q}-s^{2q})^{2\beta_1-1} (s^{2q})^{\frac{2\alpha_1+1}{2}} \tau'(s) \times \\
 & \quad \times F\left(\alpha_1+\beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{t^{2q}-s^{2q}}{t^{2q}}\right) ds + F(x), \tag{32}
 \end{aligned}$$

where

$$\begin{aligned}
 F(x) & = \frac{2q\gamma_1(\alpha_1+\beta_1)\Gamma(\alpha)}{\gamma_2\Gamma(2\beta_1)} \int_0^x (t^{2q})^{-\alpha_1-\beta_1} dt \int_0^t (t^{2q}-s^{2q})^{2\beta_1-1} (s^{2q})^{2\alpha_1} \times \\
 & \quad \times \varphi_2(0) F\left(\alpha_1+\beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{t^{2q}-s^{2q}}{t^{2q}}\right) ds - \\
 & - \frac{2q(\alpha_1+\beta_1)\gamma_1\Gamma(\alpha)}{\gamma_2\Gamma(2\beta_1)} \int_0^1 (1-t)(t^{2q})^{-\alpha_1-\beta_1} dt \int_0^t (t^{2q}-s^{2q})^{2\beta_1-1} (s^{2q})^{2\alpha_1} \times \\
 & \quad \times \varphi_2(0) F\left(\alpha_1+\beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{t^{2q}-s^{2q}}{t^{2q}}\right) ds - \\
 & - \frac{\Gamma(\alpha)}{\gamma_2} \int_0^x (t^{2q})^{\frac{1-\alpha_1+\beta_1}{2}} h^*(t) dt + \frac{\Gamma(\alpha)}{\gamma_2} \int_0^1 (1-t)(t^{2q})^{\frac{1-\alpha_1+\beta_1}{2}} h^*(t) dt + \varphi_1(0) - \varphi_2(0). \tag{33}
 \end{aligned}$$

Changing the order of integration in (32), totally we have integral equation

$$\tau'(x) = \int_0^1 K(x,z)\tau'(z) dz + F(x). \tag{34}$$

Here

$$\begin{aligned}
 K(x,z) & = \begin{cases} K_1(x,z); & 0 \leq z \leq x, \\ K_2(x,z); & x \leq z \leq 1. \end{cases} \tag{35} \\
 K_1(x,z) & = k_1\Gamma(\alpha) \int_z^x (t^{2q})^{-\alpha_1-\beta_1} dt \int_z^t (t^{2q}-s^{2q})^{2\beta_1-1} (s^{2q})^{2\alpha_1} \times \\
 & \quad \times F\left(\alpha_1+\beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{t^{2q}-s^{2q}}{t^{2q}}\right) ds - \\
 & - k_1\Gamma(\alpha) \int_z^1 (1-t)(t^{2q})^{-\alpha_1-\beta_1} dt \int_z^t (t^{2q}-s^{2q})^{2\beta_1-1} (s^{2q})^{2\alpha_1} \times \\
 & \quad \times F\left(\alpha_1+\beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{t^{2q}-s^{2q}}{t^{2q}}\right) ds + \\
 & + k_2\Gamma(\alpha) (z^{2q})^{\frac{2\alpha_1+1}{2}} \int_z^x (t^{2q})^{\frac{1-2\beta_1}{2}} (t^{2q}-z^{2q})^{2\beta_1-1} \times \\
 & \quad \times F\left(\alpha_1+\beta_1, \frac{2\beta_1-1}{2}, 2\beta_1, \frac{t^{2q}-z^{2q}}{t^{2q}}\right) dt - \\
 & - k_2\Gamma(\alpha) (z^{2q})^{\frac{2\alpha_1+1}{2}} \int_z^1 (1-t)(t^{2q})^{\frac{1-2\beta_1}{2}} (t^{2q}-z^{2q})^{2\beta_1-1} \times
 \end{aligned}$$

$$\begin{aligned}
 & \times F\left(\alpha_1 + \beta_1, \frac{2\beta_1 - 1}{2}, 2\beta_1, \frac{t^{2q} - z^{2q}}{t^{2q}}\right) dt \tag{36} \\
 K_2(x, z) = & k_1 \Gamma(\alpha) \int_z^1 (1-t) (t^{2q})^{-\alpha_1 - \beta_1} dt \int_z^t (t^{2q} - s^{2q})^{2\beta_1 - 1} (s^{2q})^{2\alpha_1} \times \\
 & \times F\left(\alpha_1 + \beta_1, \frac{2\beta_1 - 1}{2}, 2\beta_1, \frac{t^{2q} - s^{2q}}{t^{2q}}\right) ds - \\
 & - k_2 \Gamma(\alpha) (z^{2q})^{\frac{2\alpha_1 + 1}{2}} \int_z^1 (1-t) (t^{2q})^{\frac{1 - 2\beta_1}{2}} (t^{2q} - z^{2q})^{2\beta_1 - 1} \times \\
 & \times F\left(\alpha_1 + \beta_1, \frac{2\beta_1 - 1}{2}, 2\beta_1, \frac{t^{2q} - z^{2q}}{t^{2q}}\right) dt \tag{37}
 \end{aligned}$$

where

$$k_1 = \frac{2q\gamma_1(\alpha_1 + \beta_1)}{\gamma_2\Gamma(2\beta_1)}, k_2 = \frac{2q\gamma_1}{\gamma_2\Gamma(2\beta_1)}$$

Due to properties of hypergeometric function (5) from (37) we will get

$$|K_1(x, z)| \leq \left| \int_z^x (t^{2q})^{-\alpha_1 - \beta_1} dt \left| \int_z^t (t^{2q} - s^{2q})^{2\beta_1 - 1} (s^{2q})^{2\alpha_1} s^{1-2q} ds^{2q} \right| \right|. \tag{38}$$

Hence, due to class of given functions (38) considering (36) and (37) from (33) and (35) respectively we will receive  $|K(x, z)| \leq const$  for all  $0 \leq x, z \leq 1$ ,  $|F(x)| \leq const$ ,  $0 \leq x \leq 1$ .

Since kernel  $K(x, z)$  is continuous and function in right-side  $F(x)$  is continuously differentiable, solution of integral equation (34) we can write via resolvent-kernel:

$$\tau'(x) = F(x) - \int_0^1 \mathfrak{R}(x, z) F(z) dz, \tag{39}$$

where  $\mathfrak{R}(x, z)$  is the resolvent-kernel of  $K(x, z)$ .

Unknown functions  $v^-(x)$  we will in accordingly from (26).

Solution of the Problem I in the domain  $\Omega^+$  we will write as follows [13], [7]:

$$u(x, y) = \int_0^y G_\xi(x, y, 0, \eta) \psi(\eta) d\eta - \int_0^y G_\xi(x, y, 1, \eta) \varphi(\eta) d\eta + \int_0^1 G_0(x - \xi, y) \tau(\xi) d\xi,$$

Here

$$\begin{aligned}
 G_0(x - \xi, y) &= \frac{1}{\Gamma(1 - \alpha)} \int_0^y \eta^{-\alpha} G(x, y, \xi, \eta) d\eta, \\
 G(x, y, \xi, \eta) &= \frac{(y - \eta)^{\alpha/2 - 1}}{2} \sum_{n=-\infty}^{\infty} \left[ e_{1, \alpha/2}^{1, \alpha/2} \left( -\frac{|x - \xi + 2n|}{(y - \eta)^{\alpha/2}} \right) - e_{1, \alpha/2}^{1, \alpha/2} \left( -\frac{|x + \xi + 2n|}{(y - \eta)^{\alpha/2}} \right) \right]
 \end{aligned}$$

Is the Green's function of the first boundary problem Eq. (10) in the domain  $\Omega^+$  with the Riemann-Liouville fractional differential operator instead of the Caputo ones [13],

$$e_{1, \delta}^{1, \delta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{n! \Gamma(\delta - \delta n)}$$

is the Wright type function [10].

Solution of the Problem I in the domain  $\Omega^-$  will be found by the formulate (18). Hence, the **Theorem 2** is proved.  $\square$



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