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ESTIMATION OF PREDICTIVE EFFICIENCY OF SEISMIC REGIME PROBABILISTIC MODEL PARAMETERS

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On the basis of theoretical-probabilistic approach to the Kamchatka Earthquakes Catalog, a subset of random events was specified and their probability values were calculated. The received probabilities of random events are considered as predictors of strong earthquakes with the energy class of $K_S \geq 14$. Their efficiency V, reliability R and forecast efficiency (informative value) J were calculated for the periods of seismic activity and seismic quiescence.

Key words: probability, seismic regime, predictive efficiency

Introduction

In the papers [1, 2, 3] a probabilistic approach was applied to Kamchatka Earthquake Catalogue. Considering each earthquake as an elementary event and the whole catalogue or its part as elementary event space, this approach allows us to specify a set of random event subsets. Using static processing of the Earthquake Catalogue, we can determine probability distributions for defined random events. Variations in random event probability distributions for different time periods allow us to monitor the changes in seismic regime of the region and, consequently, give us the opportunity to detect the regions and the periods of seismic activity increases.

In the algorithm of strong earthquake forecast "Anticipated Earthquake Map"(AEM) [4], we analyzed a complex of predictors (seismogenic fracture density, angle of inclination, earthquake repetition graph, released seismic energy, event number per time unit), the time series of which were calculated from the Earthquake Catalogue for the defined seismically active cells. In order to select the predictors for the calculation of the conditional probability of a strong earthquake occurrence, informative value J was estimated. The predictive efficiency of each parameter was calculated in retrospect as the relation of strong earthquake flux average density during alarms to their average density within the observation time.

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In the papers [6, 7], to estimate the seismic predictive informative value of seismic quiescence anomalies which are precursors of strong earthquakes, we applied the notions of precursor efficiency V and precursor reliability R.

In the paper we calculated the defined random event probabilities P for six seismically active areas of Kamchatka region. They were determined on the basis of Kamchatka Earthquake Catalogue. Considering the anomalous values of the probabilities P during seismically active and calm periods as predictors preceding strong earthquakes with the energy class of $K_S \geq 14$, their efficiency V, reliability R and forecast efficiency (informative value) J.

Seismic regime probabilistic model

In probability-theoretical approach, the Earthquake Catalogue can be represented in the form of probabilistic space of three mathematical objects, they are: Ω is the elementary event space, $\tilde{F}$ is the set of random event subsets, P is the probabilities of these events [1, 2]. In this case, each earthquake is considered as a point in space Ω. Each outcome ω_i is determined by a system of random continuous quantities including the latitude φ_i, longitude λ_i, depth h_i, energy class $K_i = \lg E$ and time t_i. A single event time will be removed from the further analysis in the suggested model as a random value. As long as the random values φ_i, λ_i, h_i and K_i are within the corresponding intervals between the minimum and the maximum values, than the following is fair for elementary event space:

$$\Omega = \{\omega : \varphi_{min} \leq \varphi \leq \varphi_{max}; \lambda_{min} \leq \lambda \leq \lambda_{max}; h_{min} \leq h \leq h_{max}; K_{min} \leq K \leq K_{max}\} \tag{1}$$

The maximum and the minimum values of random quantities occurring in (1) are defined by real seismic region geometry as well as by its inner properties determining the event energy. To turn from the ideal probabilistic space to real experiment, it is necessary to define the elementary event space boundaries Ω and the probabilities P from the Earthquake Catalogue based on its statistical analysis. For continuous values determining a seismic event, the distribution density $f(\varphi,\lambda,h,K)$ can be presented by the following relation:

$$f(\varphi,\lambda,h,K) = f(\varphi) \cdot f(\lambda|\varphi) \cdot f(h|\varphi,\lambda) \cdot f(K|\varphi,\lambda,h) \tag{2}$$

The following notions are introduced in (2): $f(\varphi)$ is the unconditional distribution density of seismic events depending on φ; $f(\lambda|\varphi)$ is the seismic event distribution dansity on λ under the condition that their latitude equals φ; $f(h|\varphi,\lambda)$ is the seismic event distribution density on h under the condition that their latitude and longitude equal φ and λ, respectively; $f(K|\varphi,\lambda,h)$ is the seismic event distribution dansity on K under the condition that their latitude, longitude and depth equal φ, λ and h, respectively. When the analytic form of distribution density (2) is known, we can calculate the probability of a seismic event falling within the defined intervals in regards to latitude $\Delta\varphi = \varphi_i - \varphi_{i-1}$, longitude $\Delta\lambda = \lambda_j - \lambda_{j-1}$, depth $\Delta h = h_m - h_{m-1}$ and energy class $\Delta K = K_n - K_{n-1}$

$$\begin{aligned} P(\Delta\varphi_i,\Delta\lambda_j,\Delta h_m,\Delta K_n) &= \int\limits_{\varphi_{i-1}}^{\varphi_i} d\varphi \int\limits_{\lambda_{j-1}}^{\lambda_j} d\lambda \int\limits_{h_{m-1}}^{h_m} dh \int\limits_{K_{n-1}}^{K_n} f(\varphi,\lambda,h,K)dK = \\ &= F(\varphi_i,\lambda_j,h_m,K_n) - F(\varphi_{i-1},\lambda_{j-1},h_{m-1},K_{n-1}) = \\ &= P(\Delta\varphi_i)P(\Delta\lambda_j|\Delta\varphi_i)P(h_m|\Delta\varphi_i,\Delta\lambda_j)P(K_n|\Delta\varphi_i,\Delta\lambda_j,h_m) \end{aligned} \tag{3}$$

where i, j, m and n are the indices corresponding to random quantity intervals.

Statistical analysis of the Catalogue according to formula (3) gives the opportunity not only to answer the question on the average probability of seismic event occurrence in a defined interval of geographic coordinates, depth and energy class but to obtain numerical values of step distribution function $F(\Delta\varphi,\Delta\lambda,\Delta h,\Delta K)$. As the event number n increases and the interval Δ decreases, random event relative frequency tends to its mathematical analogue P, and $F(\Delta\varphi,\Delta\lambda,\Delta h,\Delta K)$ tends to the stable continuous distribution $F(\varphi,\lambda,h,K)$. When calculating function F for some time interval T in a chosen volume V, the condition set is averaged causing one or another random event realization and, consequently, F describes a seismic event on average. Thus, function F determines on average the potentials of a seismic regime of a chosen volume within the considered period in probability theory terms. Under such an approach, the Earthquake Catalogue over the whole period of instrumental observations T_{inst}, represented in the form of a mathematical object of three elements $\{\Omega,\tilde{F},P\}$, can be regarded as a reference model. At its background, seismic activity changes can be monitored, i.e. we can register variations in probability distributions in seismic region local areas which are determined by activity change of some volume within different time intervals T $(T < T_{\text{inst}})$.

Based on the Kamchatka Earthquake Catalogue compound by KF FITs EGS RAS [9], we can determine the following random events:

Event A : Falling of seismic events, occurred in some seismically active volume V, within the defined regions S_i;

Event B: Falling of seismic events, occurred in some seismically active volume V, within the defined depth intervals Δh_m;

Event C: Falling of seismic events, occurred in some seismically active volume V, within the energy class intervals ΔK_n.

Probabilities of random events A, B and C were calculated for each of six regions S_i with the dimensions of 150×250 km, located along the eastern coast of Kamchatka (Fig.1).

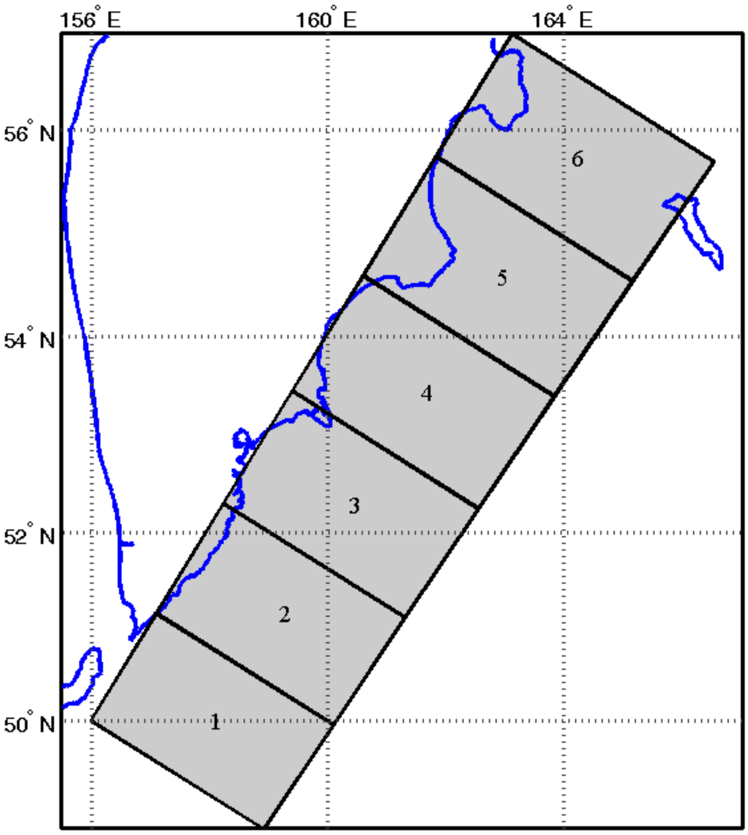

Fig. 1. Investigated regions S_i

Table 11

Seismic event number with the energy class $K_S \geq 14$, which occurred in the regions S_i at the depths up to 100 km for the period 01.01.1962 – 01.10.2016

Region	S_1	S_2	S_3	S_4	S_5	S_6
Number of earthquakes	11	3	7	1	5	6

Table 11 illustrates the number of events with the energy class $K_S \geq 14$, which occurred in the regions S_i within the instrumental observation interval T_{inst}=01.01.1962 – 01.10.2016. The areas under investigation are in the southern and northern segments of Kamchatka seismic focal zone and partially in the Komandorskiy segment of Aleutian arc and Pacific Ocean zone. About 93% of earthquakes with the energy class $K_S \geq 8.6$ and about 79% of the released seismic energy $\sum E$ fall within the indicated zones [8]. In the calculation of the probabilities of a random event B, we considered three intervals of a random quantity Δh which had the values of $0 \leq \Delta h_1 < 25$ km, $25 \leq \Delta h_2 < 50$ km and $50 \leq \Delta h_3 < 100$ km. The probabilities of a random event C were calculated for the energy class intervals $9 \leq \Delta K_1 < 10$, $9 \leq \Delta K_2 < 11$, $10 \leq \Delta K_3 < 11$, $10 \leq \Delta K_4 < 14$, $11 \leq \Delta K_5 < 14$ and $12 \leq \Delta K_6 < 14$. Probabilities of random events were estimated for the earthquakes with the energy class $K_S \geq 9$ and hypocenter depths $h \leq 100$ km. The time window, in which the probabilities were estimated, was chosen to be $\Delta T = 3$ years. The step, with which the time window was moved along the interval under investigation T_{inst}, equals $\Delta t = 1$ month.

As an example, Fig. 2-4 shows the graphs of probabilities P, calculated for random events A, B, C for the region S_1.

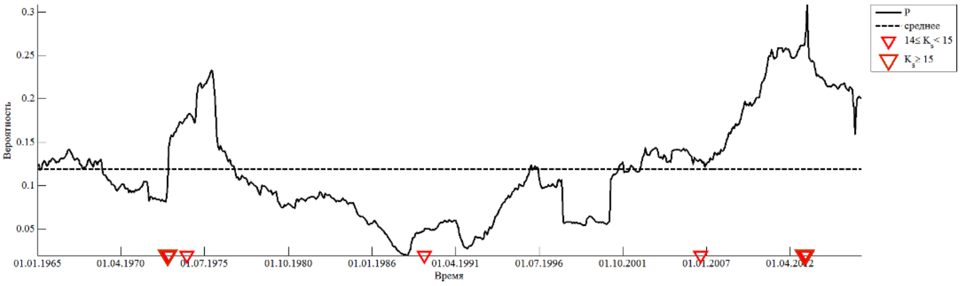

Fig. 2. Time series of probability $P(A)$ for the region S_1. Seismic events with the energy class $K_S \geq 14$, occurred in this region, are marked by triangles

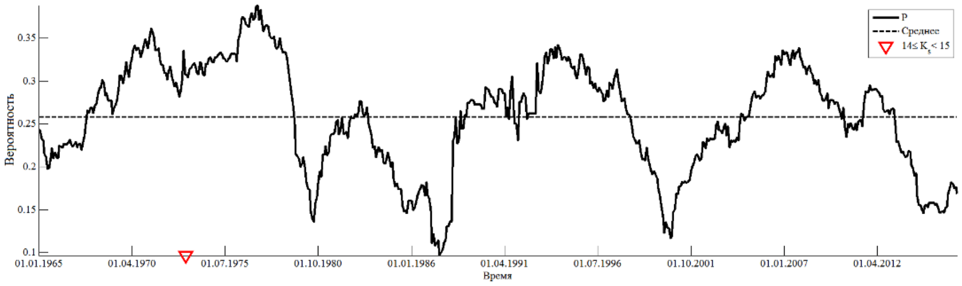

Fig. 3. Time series of probability $P(B)$ for $0 \leq \Delta h_1 < 25$ km for the region S_1.

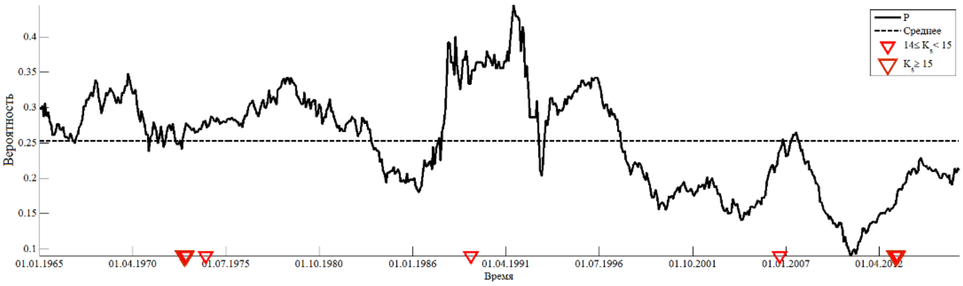

Fig. 4. Time series of probability $P(C)$ for $10 \leq \Delta K_3 < 11$ for the region S_1.

Informative value of predictors

Considering the estimated probability time series as predictors, the anomalous values of which can be the precursors of strong seismic events of Kamchatka region, their predictive efficiency was estimated for each region S_i. It was determined by the following formula [5]:

$$J = \frac{N_{\text{pr}}}{N_{\text{tot}} \cdot (T_{\text{w}}/T_{\text{o}})} \tag{4}$$

where N_{pr} is the number of earthquakes corresponding to successful prediction; N_{tot} is the total number of occurred earthquakes which should have been predicted; T_{w} is the total waiting time (of alarms); T_{o} is the observation total time. The efficiency J shows how many time the number of predicted earthquakes exceeds the number of earthquakes falling within the alarm time in random manner. At random guessing, the efficiency is $J = 1$. In the paper [7] a modified formula (4) was presented. It allows one to calculate J for forecasts (by the same method), which spatial-energy parameters do not coincide:

$$J = \frac{N_{\text{pr}}}{\sum_i^I N_{\text{tot}}(i) \cdot (T_{\text{w}}(i)/T_{\text{o}})} \tag{5}$$

where i is one of I of different variants of forecast "spatial-energy"formulation. Thus, all quantities with index i refer just to this variant. Then the expression under the index of summation gives the average number of "randomly"successful realizations of the forecast with these parameters and their summation over i gives the total number of random guessing. In this situation, the sense of J and N_{pr} is without any changes.

Table 12 shows the informative value characteristics of the predictors [4].

Table 12

Informative value characteristic of a predictor

Informative value characteristic of a predictor	Efficiency, J
Useless (application of the parameter will not substantially improve the forecast)	<1.2
Not very useful (the parameter is not very informative but it should not be ignored since it will turn to be effective in combination with other parameters)	$\cong 1.2$
Useful	1.5 - 2.0
Very useful	>2.7

In order to estimate the seismic predictive informative value of a strong earthquake precursor, the notions of its efficiency and reliability were used in the papers [6, 7]. Precursor efficiency V is the relation

$$V = \frac{n(sA)}{n(A)} \tag{6}$$

where $n(sA)$ is the number of anomalies preceding an earthquake, $n(A)$ is the number of detected anomalies. Precursor reliability R is the relation

$$R = \frac{n(sE)}{n(E)} \tag{7}$$

where $n(sE)$ is the number of earthquakes which were predicted by the considered precursor, $n(E)$ is the total number of earthquakes which should have been predicted.

In this investigation, the predictor efficiency was estimated both for the anomalies during seismic activity intensification and for the anomalies during seismic quiescence periods which could precede strong earthquakes with the energy class $K_S \geq 14$. In the analysis of probability P time series, seismic activity intensification periods were considered to be those within which the probability values exceeded the alarm ratio $P_{\text{al.1}} = M(P) + \sigma(P)$. Seismic quiescence periods were considered to be the time intervals within which probability time series were lower than the alarm ratio $P_{\text{al.2}} = M(P) - \sigma(P)$. At that, the values $M(P)$ and $\sigma(P)$ are the mathematical expectation and probability root-mean-square deviation of random events under consideration. According to the paper [10], the manifestation time of a precursor with the energy class $K = 12.5 \div 16.5$ is $T = 0.6 \div 12$ years. As the waiting time T_{w} of a strong earthquake with $K_S \geq 14$ we considered the time intervals from 1 to 10 years with the step of 1 year. The beginning of an alarm period was chosen so that it coincided with the moment when the investigated predictor exceeded the alarm ratio $P_{\text{al.1}}$ in case of seismic activity intensification or coincided with the moment when the predictor value became lower than the alarm ratio $P_{\text{al.2}}$ in case of seismic quiescence. From all the obtained values of the parameter J, calculated for all the waiting periods $T_{\text{ож}}$, we chose only maximum values for further research.

Results of investigation

The tables 13-7 show the estimation results based on relation (4) of forecast efficiency J for probabilities P of random events A, B and C.

Table 13 illustrates the maximum values of predictive efficiency for parameter $P(A)$, calculated both for the periods of seismic activity intensification and for the periods of seismic quiescence. The predictor $P(A)$ has the highest efficiency J during seismic activity intensification for the regions S_1, S_2, S_5 and S_6 during waiting periods T_{w} equal to 5, 1, 1 and 5 years, respectively. For the regions S_3 and S_4, the value J of the parameter $P(A)$ is useless for all the considered waiting periods T_{w}. In the six regions we detected 18 anomalies of seismic activity intensification, 10 of which preceded 16 earthquakes with $K_S \geq 14$. Thus, the precursor efficiency was $V = 0.55$, and precursor reliability was $R = 0.48$. Forecast efficiency J in terms of the parameter $P(A)$ in all

Table 13

Informative value J of a predictor $P(A)$ during seismic activity intensification and seismic quiescence.

Region	S_1	S_2	S_3	S_4	S_5	S_6
J (seismic activity intensification)	2.7	7.7	1.1	0	5	3.6
J (seismic quiescence)	0.8	0.9	2.2	0	2.9	0.6

the considered regions was 2.2 based on (5). Consequently, this predictor has useful informative value during seismic activity intensification.

During seismic quiescence anomalies, the parameter $P(A)$, as a predictor, is useful for the region S_3 when $T_w = 3$ years and vary useful for the region S_5 when $T_w = 5$ years. For the regions S_1, S_2, S_4 and S_6, the parameter $P(A)$ does not have predictive informative value. In the six regions we detected 19 seismic quiescence anomalies, 11 of which preceded 12 earthquakes with $K_S \geq 14$. Consequently, precursor efficiency was $V = 0.57$, and precursor reliability was $R = 0.36$. Calculated by formula (5), the forecast efficiency J by the parameter $P(A)$ for all the regions was 1.4. Consequently, the parameter is not very useful for the forecast. Thus, the predictive efficiency of the parameter $P(A)$ is higher during seismic activity intensification than during seismic quiescence periods.

Table 14 shows the maximum values of the forecast efficiency J for the predictor $P(B)$ during seismic activity intensification, which was calculated for the depth intervals Δh_1, Δh_2 and Δh_3 in the regions S_1, S_2,. . . , S_6.

Table 14

Informative value J of the predictor $P(B)$ during seismic activity intensification.

Region	S_1	S_2	S_3	S_4	S_5	S_6
J, $0 \leq \Delta h_1 < 25$ km	17.6	–	5.1	4.1	0	2.2
J, $25 \leq \Delta h_2 < 50$ km	1.7	5	3.1	–	5.2	3.6
J, $50 \leq \Delta h_3 < 100$ km	6.3	–	–	–	8.3	0

It is clear from the table 14 that the predictor $P(B)$ for the depth interval Δh_1 has the highest predictive informative value for the regions S_1 and S_3 when the waiting periods are T_w=1 year and for the regions S_4 and S_6 when T_w equals 7 and 9 years, respectively. For the regions S_2 and S_5, this predictor is not informative. In all the regions we detected 19 anomalies, 6 of which can be attributed to 5 earthquakes out of 9 which occurred within the depth interval Δh_1. Thus, the precursor efficiency was $V = 0.26$, and the precursor reliability is $R = 0.55$. Forecast efficiency J in terms of the parameter $P(B)$ within the interval Δh_1 for all the regions under consideration was 2.15 according to formula (5).

The predictor $P(B)$ for the depth interval Δh_2 is the most informative in the regions S_1, S_2, S_3, S_5 and S_6 for the waiting periods of 9, 2, 5, 1 and 10 years, respectively. For S_5 this feature is useless. We detected 26 anomalies in all the regions, 9 of which preceded 12 earthquakes with $K_S \geq 14$. At that, 15 earthquakes with $K_S \geq 14$ were registered within the depth interval Δh_2. Consequently, the precursor efficiency was $V = 0.35$, and its reliability is $R = 0.8$. Calculated by formula (5), the forecast efficiency J in terms of the parameter $P(B)$ within the depth interval Δh_2 was 2.9 for all the considered regions.

The predictor $P(B)$ for the depth interval Δh_3 is the most informative in the regions S_1 and S_5 for the waiting period of 4 years. For the regions S_2, S_3, S_4 and S_6, this feature turned to be useless. We detected 8 anomalies, 3 of which preceded 7 earthquakes with $K_S \geq 14$. On the whole, we registered 9 earthquakes with $K_S \geq 14$ within the interval Δh_3. Thus, the precursor efficiency was V=0.375, and the precursor reliability was R=0.78. Calculated by formula (5), the forecast efficiency J in terms of the parameter $P(B)$ within the interval Δh_3 was 6.2 for all the regions.

Analysis of $P(B)$ predictive efficiency showed that this predictor is useful during seismic activity intensification anomalies for all the considered depth intervals Δh_1, Δh_2 and Δh_3.

Table 15 shows the maximal values of the predictive efficiency J for the parameter $P(B)$ during seismic quiescence which was calculated for the depth intervals Δh_1, Δh_2 and Δh_3 in the regions S_1, S_2,. . . , S_6.

Table 15

Informative value J of the predictor $P(B)$ during seismic quiescence periods.

Region	S_1	S_2	S_3	S_4	S_5	S_6
J, $0 \leq \Delta h_1 < 25$ km	0	–	0.5	0	14.9	1
J, $25 \leq \Delta h_2 < 50$ km	8	4.3	2.4	–	5.7	2.4
J, $50 \leq \Delta h_3 < 100$ km	0	–	–	–	0	0

It follows from Table 15 that the parameter $P(B)$ for the depth interval Δh_1 has the highest predictive efficiency only for the region S_5 when the waiting period is $T_{\text{ож}} = 2$ years. For the regions S_1, S_2, S_3, S_4, S_6, this parameter is useless. On the whole, we detected 19 anomalies, 3 of which can be attributed to 3 earthquakes out of 9 occurred within the interval Δh_1. Thus, the precursor efficiency was $V = 0.16$, and the precursor reliability was $R = 0.33$. Based on formula (5), the forecast efficiency J in terms of the parameter $P(B)$ within the depth interval Δh_1 was 0.8 for all the regions.

The predictor $P(B)$ for the depth interval Δh_2 is the most informative in the regions S_1, S_2, S_3, S_5 and S_6 for the waiting periods of 5, 1, 2, 1 and 3 years, respectively. For S_5 this feature turned to be useless. On the whole, 24 earthquakes were detected, 6 of which preceded 7 out of 15 earthquakes with $K_S \geq 14$. Consequently, the precursor efficiency was V=0.25, and the precursor reliability was R=0.47. Based on formula (5), the forecast efficiency J in terms of the parameter $P(B)$ within the depth interval Δh_2 was 4.3 for all the regions.

The predictor $P(B)$ for the depth interval Δh_3 is useless for the forecast in all the considered regions S_i. 7 false anomalies during calm conditions were detected for this parameter. Thus, the precursor efficiency was $V = 0$, the precursor reliability was $R = 0$ and the forecast efficiency was $J = 0$.

Analysis of $P(B)$ predictive efficiency during seismic quiescence anomalies showed that this parameter is quite useful for the depth interval Δh_2 and it is useless for the intervals Δh_1 and Δh_3.

Table 16 presents the maximal values of the efficiency J for the predictor $P(C)$ during seismic activity intensification which was calculated for the energy class intervals ΔK_1, ΔK_2, ΔK_3, ΔK_4, ΔK_5 and ΔK_6 in the regions S_1, $S_2, \ldots$, S_6.

Table 16

The informative value J of the predictor $P(C)$ during seismic activity intensification.

Region	S_1	S_2	S_3	S_4	S_5	S_6
J, $9 \leq \Delta K_1 < 10$	2	5.2	0.9	6.4	1.2	2.3
J, $9 \leq \Delta K_2 < 11$	1.8	6.6	0.6	9.5	1.2	1.5
J, $10 \leq \Delta K_3 < 11$	1.1	0.6	2	0	4.2	1
J, $10 \leq \Delta K_4 < 14$	1.5	0.7	2.8	0	3.1	0.6
J, $11 \leq \Delta K_5 < 14$	2.4	0.9	0.7	1.9	0.9	2.9
J, $12 \leq \Delta K_6 < 14$	5	2.5	1.4	0	1.4	1.1

It is clear from Table 16 that the predictor $P(C)$ for ΔK_1 is the most informative for the regions S_1, S_2, S_4 and S_6 when the waiting periods T_{w} are equal to 5, 2, 1 and 2 years, relatively. For the regions S_3 and S_5 this predictor is not informative. On the whole, 38 anomalies were detected, 14 of which can be attributed to 19 earthquakes. Thus, the precursor efficiency was $V = 0.36$, and the precursor reliability was $R = 0.58$. Based on formula (5), the forecast efficiency J in terms of the parameter $P(C)$ for ΔK_1 was 1.9 for all the regions.

The predictor $P(C)$ for ΔK_2 is informative for the regions S_1, S_2, S_4 and S_6 when the waiting periods T_{w} are 8, 2, 1 and 2 years, respectively. For the regions S_3 and S_5, this predictor is useless. On the whole 32 anomalies were detected, 12 of which can be attributed to 17 earthquakes from 33. Consequently, the precursor efficiency was $V = 0.29$, and the precursor reliabiltiy was $R = 0.33$. Based on formula (5), the forecast efficiency J in terms of the parameter $P(C)$ for ΔK_2 was 1.6 for all the regions.

The predictor $P(C)$ for ΔK_3 is informative for the regions S_3 and S_5 when the waiting periods T_{w} are 2 and 1 year, respectively. In the regions S_1, S_2, S_4 and S_6 this predictor is not informative. On the whole, 29 were detected, 9 of which can be attributed to 12 earthquakes out of 33 with $K_S \geq 14$. Thus, the precursor efficiency was $V = 0.3$, and the precursor reliability was $R = 0.36$. Based on formula (5), the forecast efficiency J in terms of the parameter $P(C)$ for ΔK_3 was 1.2 for all the regions.

The predictor $P(C)$ for ΔK_4 is informative for the regions S_1, S_3 and S_5 when the waiting periods T_{w} are 5, 1 and 1 year, respectively. In the regions S_2, S_4 and S_6 this predictor is not informative. On the whole, 27 anomalies were detected, 7 of which can be attributed to 10 earthquakes out of 33 with $K_S \geq 14$. Consequently, the precursor efficiency was $V = 0.26$, and the precursor reliabiltiy was $R = 0.3$. Based on formula (5), the forecast efficiency J in terms of the parameter $P(C)$ for ΔK_4 was 1 for all the regions.

The predictor $P(C)$ for ΔK_5 is informative for the regions S_1, S_4 and S_6 when the waiting periods T_{w} are 1, 9 and 1 year, respectively. In the regions S_2, S_3 and S_5 this predictor is not informative. On the whole, 26 anomalies were detected, 6 of which can be attributed to 7 earthquakes out of 33 with $K_S \geq 14$. Thus, the precursor efficiency was $V = 0.23$, and the precursor reliability was $R = 0.21$. Based on formula (5), the forecast efficiency J in terms of the parameter $P(C)$ for ΔK_5 was 1.3 for all the regions.

The predictor $P(C)$ for ΔK_6 is informative for the regions S_1 and S_2 when the waiting periods T_{w} are 1 and 5 years, respectively. In the regions S_3, S_4, S_5 and S_6 this predictor is not informative. On the whole, 32 anomalies were detected, 10 of which can be attributed to 13 earthquakes out of 33 with $K_S \geq 14$. Consequently, the precursor efficiency was $V = 0.31$, and the precursor reliability was $R = 0.39$. Based on formula (5), the forecast efficiency J in terms of the parameter $P(C)$ for ΔK_6 was 1.7 for all the regions.

Analysis of $P(C)$ predictive efficiency during seismic activity intensification showed that this parameter is informative for the energy class intervals $9 \leq \Delta K_1 < 10$, $9 \leq \Delta K_2 < 11$ and $12 \leq \Delta K_1 < 14$. At that, the parameters V, R and J possess the highest values in the interval $9 \leq \Delta K_1 < 10$.

Table 17 presents the efficiency J maximum values for the predictor $P(C)$ during seismic activity intensification which was calculated for the energy class intervals ΔK_1, ΔK_2, ΔK_3, ΔK_4, ΔK_5 and ΔK_6 in the regions S_1, S_2,..., S_6.

Table 17

Informative value J of the predictor $P(C)$ during seismic quiescence periods.

Region	S_1	S_2	S_3	S_4	S_5	S_6
J, $9 \leq \Delta K_1 < 10$	1.5	0.7	2.8	0	3.1	0.6
J, $9 \leq \Delta K_2 < 11$	2.4	0.9	0.7	2	0.9	2.9
J, $10 \leq \Delta K_3 < 11$	2.2	4.8	1.7	7.3	1.6	2.5
J, $10 \leq \Delta K_4 < 14$	2	5.2	0.9	6.4	1.3	2.3
J, $11 \leq \Delta K_5 < 14$	1.8	6.6	0.6	9.5	1.2	1.5
J, $12 \leq \Delta K_6 < 14$	1.9	1	1.2	0.8	2.3	2.5

As can be seen from Table 17, during seismic quiescence periods, the predictor $P(C)$ for ΔK_1 is the most useful for the regions S_1, S_3, and S_5 when the waiting periods T_{w} are 5, 1 and 1 year, respectively. For the regions S_2, S_4 and S_6, this parameter is not informative. On the whole, 27 anomalies were detected, 7 of which can be attributed to 10 earthquakes out of 33 with $K_S \geq 14$. Thus, the precursor efficiency was $V = 0.26$, and the precursor reliability was $R = 0.3$. Calculated by formula (5), the forecast efficiency J in terms of the parameter $P(C)$ for ΔK_1 was 1 for all the regions.

The predictor $P(C)$ for ΔK_2 is the most informative for the regions S_1, S_4 and S_6 when the waiting periods T_w are 1, 9 and 1 year, respectively. In the regions S_2, S_3 and S_5 this predictor is not informative. On the whole, 26 anomalies were detected, 6 of which can be attributed to 7 earthquakes out of 33 with $K_S \geq 14$. Consequently, the precursor efficiency was $V = 0.23$, and the precursor reliability was $R = 0.21$. Calculated by formula (5), the forecast efficiency J in terms of the parameter $P(C)$ for ΔK_2 was 1.3 for all the regions.

The predictor $P(C)$ for ΔK_3 is informative for all the regions S_1, S_2,..., S_6 when the waiting periods T_w are 5, 2, 6, 1, 6 and 2 years, respectively. On the whole, 35 anomalies were detected, 15 of which can be attributed to 21 earthquakes out of 33 with $K_S \geq 14$. Thus, the precursor efficiency was $V = 0.43$, and the precursor reliability was $R = 0.64$. Calculated by formula (5), the forecast efficiency J in terms of the parameter $P(C)$ for ΔK_3 was 2.17 for all the regions.

The predictor $P(C)$ for ΔK_4 is informative for the regions S_1, S_2, S_4 and S_6 when the waiting periods T_w are 5, 2, 1 and 2 years, respectively. In the regions S_3 and S_5 this predictor is not informative. On the whole, 38 anomalies were detected, 14 of which can be attributed to 19 earthquakes out of 33 with $K_S \geq 14$. Consequently, the precursor efficiency was $V = 0.37$, and the precursor reliability was $R = 0.58$. Calculated by formula (5), the forecast efficiency J in terms of the parameter $P(C)$ for ΔK_4 was 1.9 for all the regions.

The predictor $P(C)$ for ΔK_5 is informative for the regions S_1, S_2, S_4 and S_6 when the waiting periods T_w are 8, 2, 1 and 2 years, respectively. In the regions S_3 and S_5 this predictor is not informative. On the whole, 32 anomalies were detected, 12 of which can be attributed to 17 earthquakes out of 33 with $K_S \geq 14$. Thus, the precursor efficiency was $V = 0.38$, and the precursor reliability was $R = 0.52$. Calculated by formula (5), the forecast efficiency J in terms of the parameter $P(C)$ for ΔK_5 was 1.57 for all the regions.

The predictor $P(C)$ for ΔK_6 is informative for the regions S_1, S_5 and S_6 when the waiting periods T_w are 4, 4 and 2 years, respectively. In the regions S_2, S_3, S_4 this predictor is not informative. On the whole, 41 anomalies were detected, 12 of which can be attributed to 19 earthquakes out of 33 with $K_S \geq 14$. Consequently, the precursor efficiency was $V = 0.29$, and the precursor reliability was $R = 0.58$. Calculated by formula (5), the forecast efficiency J in terms of the parameter $P(C)$ for ΔK_6 was 1.7 for all the regions.

From all the energy class intervals ΔK, the predictor $P(C)$ is informative for the energy class intervals $10 \leq \Delta K_3 < 11$, $10 \leq \Delta K_4 < 14$, $11 \leq \Delta K_5 < 14$ and $12 \leq \Delta K_6 < 14$ during seismic quiescence periods. The parameters V, R and J possess the highest values for the interval $10 \leq \Delta K_3 < 11$.

Conclusions

Application of probabilistic methods to Kamchatka Earthquake Catalogue allowed us to define a subset of random events A, B and C. Within the course of statistics processing of the Catalogue, probability P distributions of these events were calculated for six regions in the time window $\Delta T = 3$ years with a step $\Delta t = 1$ month within the interval from 01.01.1962 to 01.10.2016. Considering the parameters $P(A)$, $P(B)$ and $P(C)$ as predictors, we estimated their efficiency V, reliability R and informative value J for the earthquakes with $K_S \geq 14$. Based on the obtained results, the predictor $P(A)$ was evaluated as a useful one during seismic activity intensification. The predictor $P(B)$ was evaluated as a useful one for all the considered depth intervals $0 \leq \Delta h_1 < 25$ km, $25 \leq \Delta h_2 < 50$ km and $50 \leq \Delta h_3 < 100$ km during seismic activity intensification and for the depth interval $25 \leq \Delta h_2 < 50$ km during seismic quiescence periods. The predictor $P(C)$, calculated during seismic activity intensification, showed the best predictive efficiency within the energy class interval $9 \leq \Delta K_1 < 10$, and that calculated during seismic quiescence periods had the best efficiency within the interval $10 \leq \Delta K_3 < 14$. Thus, application of the probabilistic approach to the Earthquake Catalogue allows us to investigate the dynamics of seismic regime and to determine anomalous changes preceding strong earthquakes.

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