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SOME BOUNDARY VALUE PROBLEMS FOR A THIRD-ORDER PARABOLIC-HYPERBOLIC EQUATION IN A PENTAGONAL DOMAIN

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This article is an example of application of the techniques to develop solutions of integral and differential equations. Here we consider a parabolic-hyperbolic equation $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}\right)(Lu)=0$ in a pentagonal domain. We prove a theorem on the unique solvability of one of two given problems.

Key words: differential and integral equations, a technique for solution development, boundary value problems, parabolic-hyperbolic type, unique solvability

Introduction

The paper is devoted to a method for investigation of some boundary value problems for one class of third-order parabolic-hyperbolic equations in a pentagonal domain which are used to study the problems of mathematical physics. This paper is a logical extension of the papers [1] and [2].

Statement of the problem

In a domain D of a plane xOy we consider the equation

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}\right)(Lu)=0, \tag{1}$$

where

$$Lu=\begin{cases} u_{1xx}-u_{1y}, & (x,y)\in D_1, \\ u_{ixx}-u_{iyy}, & (x,y)\in D_i \ (i=2,3), \end{cases},$$

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$$u(x,y)=u_i(x,y),\ (x,y)\in D_i\ (i=1,2,3,4),$$

$$D=D_1\cup D_2\cup D_3\cup D_4\cup J_1\cup J_2\cup J_3\cup J_4, D_1=\left\{(x,y)\in R^2:\ 0<x<1,\ 0<y<1\right\},$$

$$D_2=\left\{(x,y)\in R^2:\ -1<y<0,\ 0<x<y+1\right\}, D_3=\left\{(x,y)\in R^2:\ -1<x<0,\ -x-1<y<0\right\},$$

$$D_4=\left\{(x,y)\in R^2:\ -1<x<0,\ 0<y<x+1\right\}, J_1=\left\{(x,y)\in R^2:\ y=0,\ 0<x<1\right\}$$

$$J_2=\left\{(x,y)\in R^2:\ y=0,\ -1<x<0\right\}, J_3=\left\{(x,y)\in R^2:\ x=0,\ -1<y<0\right\},$$

$J_4=\left\{(x,y)\in R^2:\ x=0,\ 0<y<1\right\}$, that is D_1 is a rectangle with vertexes at the points $A(0;0)$, $B(1;0)$, $B_0(1,1)$, $A_0(0,1)$, D_2 is a triangle with vertexes at the points $A(0;0)$, $B(1;0)$, $C(0,-1)$, D_3 is a triangle with vertexes at the points $A(0;0)$, $D(-1,0)$, $C(0,-1)$, D_4 is a triangle with vertexes at the points $A(0;0)$, $D(-1,0)$, $A_0(0,1)$, J_1 is an open segment with vertexes at the points $A(0;0)$, $B(1;0)$, J_2 is an open segment with vertexes at the points $A(0;0)$, $D(-1,0)$, J_3 is an open segment with vertexes at the points $A(0;0)$, $C(0,-1)$, J_4 is an open segment with vertexes at the points $A(0;0)$, $A_0(0,1)$.

Moreover, we write the domains D_i $(i=2,3,4)$ as follows: $D_i=D_{i1}\cup D_{i2}\cup AC_{i-1}$, where D_{21} is a triangle with vertexes at the points $A(0;0)$, $B(1;0)$, $C_1\left(\frac{1}{2},-\frac{1}{2}\right)$, D_{22} is a triangle with vertexes at the points $A(0;0)$, $E_1(0;-1)$, $C_1\left(\frac{1}{2},-\frac{1}{2}\right)$, D_{31} is a triangle with vertexes at the points $A(0;0)$, $E_1(0;-1)$, $C_2\left(-\frac{1}{2},-\frac{1}{2}\right)$, D_{32} is a triangle with vertexes at the points $A(0;0)$, $E_2(-1;0)$, $C_2\left(-\frac{1}{2},-\frac{1}{2}\right)$, D_{41} is a triangle with vertexes at the points $A(0;0)$, $A_0(0;1)$, $C_3\left(-\frac{1}{2},\frac{1}{2}\right)$, D_{42} is a triangle with vertexes at the points $A(0;0)$, $E_2(-1;0)$, $C_3\left(-\frac{1}{2},\frac{1}{2}\right)$, AC_1 is an open segment with vertexes at the points $A(0;0)$, $C_1\left(\frac{1}{2},-\frac{1}{2}\right)$, AC_2 is an open segment with vertexes at the points $A(0;0)$, $C_2\left(-\frac{1}{2},-\frac{1}{2}\right)$, AC_3 is an open segment with vertexes at the points $A(0;0)$, $C_3\left(-\frac{1}{2},\frac{1}{2}\right)$, that is $AC_1=\left\{(x,y)\in R^2:\ 0<x<\frac{1}{2},\ y=-x\right\}$, $D_{21}=\left\{(x,y)\in R^2:\ -\frac{1}{2}<y<0,\ -y<x<y+1\right\}$, $D_{22}=\left\{(x,y)\in R^2:\ 0<x<\frac{1}{2},\ x-1<y<-x\right\}$, $AC_2=\left\{(x,y)\in R^2:\ -\frac{1}{2}<x<0,\ y=x\right\}$, $D_{31}=\left\{(x,y)\in R^2:-\frac{1}{2}<x<0,\ -x-1<y<x\right\}$, $D_{32}=\left\{(x,y)\in R^2:-\frac{1}{2}<y<0,\ -y-1<x<y\right\}$, $AC_3=\left\{(x,y)\in R^2:\ -\frac{1}{2}<x<0,\ y=-x\right\}$, $D_{41}=\left\{(x,y)\in R^2:\ -\frac{1}{2}<x<0,\ -x<y<x+1\right\}$, $D_{42}=\left\{(x,y)\in R^2:\ 0<y<\frac{1}{2},\ y-1<x<-y\right\}$.

For equation (1) we formulate the following problem:

Task 1. *Find a function $u(x,y)$, which*

1) is continuous in a closed domain $\overline{D}$;

2) satisfies equation (1) in the domain D for $x\neq 0$, $y\neq 0$;

3) satisfies the following boundary conditions:

$$u_1(1,y)=\varphi_1(y),\ \ 0\le y\le 1, \tag{2}$$

$$u_2|_{E_1}=\psi_1(x),\ 0\le x\le \frac{1}{2}, \tag{3}$$

$$u_3|_{E_2}=\psi_2(x),\ -\frac{1}{2}\le x\le 0, \tag{4}$$

$$\left.\frac{\partial u_3}{\partial n}\right|_D=\psi_3(x),\ -1\le x\le 0, \tag{5}$$

$$u_4|_{A_0E_3}=\psi_4(x),\ -\frac{1}{2}\le x\le 0, \tag{6}$$

4) satisfied the following continuous condition of gluing on the segments J_1 and J_2:

$$u_1(x,0)=u_2(x,0)=\tau_1(x),\ 0\le x\le 1, \tag{7}$$

$$u_{1y}(x,0)=u_{2y}(x,0)=\nu_1(x),\ 0\le x\le 1, \tag{8}$$

$$u_{1yy}(0,y)=u_{2yy}(0,y)=\mu_1(x),\ 0\le x\le 1, \tag{9}$$

$$u_3(x,0)=u_4(x,0)=\tau_2(x),\ -1\le x\le 0, \tag{10}$$

$$u_{3y}(x,0)=u_{4y}(x,0)=\nu_2(x),\ -1\le x\le 0, \tag{11}$$

$$u_{3yy}(x,0)=u_{4yy}(x,0)=\mu_2(x),\ -1\le x\le 0, \tag{12}$$

$$u_2(0,y)=u_3(0,y)=\tau_3(y),\ -1\le y\le 0, \tag{13}$$

$$u_{2x}(0,y)=u_{3x}(0,y)=\nu_3(y),\ -1\le y\le 0 \tag{14}$$

$$u_{2xx}(0,y)=u_{3xx}(0,y)=\mu_3(y),\ -1\le y\le 0, \tag{15}$$

$$u_3(0,y)=u_4(0,y)=\tau_4(y),\ 0\le y\le 1, \tag{16}$$

$$u_{3x}(0,y)=u_{4x}(0,y)=\nu_4(y),\ 0\le y\le 1, \tag{17}$$

$$u_{3xx}(0,y)=u_{4xx}(0,y)=\mu_4(y),\ 0\le y\le 1. \tag{18}$$

Here $\psi_i\ (1,2,3,4)$, ϕ_1 (defined sufficiently smooth functions and $\tau_1,\nu_1,\mu_1,\tau_2,\nu_2,\mu_2,$ $\tau_3,\nu_3,\mu_3,\tau_4,\nu_4,\mu_4$ (unknown so far sufficiently smooth functions, and the matching conditions $\tau_1(1)=\phi_1(0)=\psi_1(1)$ are fulfilled.

Task 2. *This problem differs from problem 1 only by the fact that instead of condition (4) and (6) we take the condition*

$$u_3|_D=\psi_2(x),\ -1\le x\le .0$$

Other conditions are unchanged.

We will be limited here only by investigation of problem 1.

Theorem. *If $\phi_1\in C^3[0,1]$, $\psi_1\in C^3\left[0,\frac{1}{2}\right]$, $\psi_3\in{}^2[-1,0]$, $\psi_2,\psi_4\in{}^3\left[-\frac{1}{2},0\right]$, and the matching condition $\psi_1(0)=\psi_2(0)$ is fulfilled, the problem 1 admits a unique solution.*

Proof. The theorem is proved by a technique for development of a solution . To do that, we write equation (1) as follows:

$$u_{1xx}-u_{1y}=\omega_1(x-y),\ (x,y)\in D_1, \tag{19}$$

$$u_{ixx}-u_{iyy}=\omega_i(x-y),\ (x,y)\in D_i\ \ (i=2,3,4), \tag{20}$$

were we introduce the notation $u(x,y)=u_i(x,y),\ (x,y)\in D_i\ \left(i=\overline{1,4}\right)$, and functions $\omega_i(x-y)$, $i=\overline{1,4}$ are unknown so far sufficiently smooth functions.

If we take into account the types of domains D_i, $(i=2,3,4)$ which are written on the top, equation (20) $(i=2,3,4)$ may be written as

$$u_{ikxx}-u_{ikyy}=\omega_{ik}(x-y),\ (x,y)\in D_{ik}\ \ (i=2,3,4;\ k=1,2), \tag{21}$$

where the following notations are introduced: $u_i(x,y)=u_{ik}(x,y)$, $\omega_i(x-y)=\omega_{ik}(x-y)$.

At first, we consider the problem in the domain D_{31}. We write the solution of equation (21) $(i=3;\ k=1)$, satisfying the conditions (13),(14) as follows:

$$u_{31}(x,y)=\frac{\tau_3(y+x)+\tau_3(y-x)}{2}+\frac{1}{2}\int\limits_{y-x}^{y+x}\nu_3(t)\,dt+\frac{1}{2}\int\limits_0^x d\eta\int\limits_{y-x+\eta}^{y+x-\eta}\omega_{31}(\eta-\xi)\,d\xi \tag{22}$$

Condition (5) may be written in the form

$$\left.\left(\frac{\partial u_{31}}{\partial x}+\frac{\partial u_{31}}{\partial y}\right)\right|_{y=-x-1}=\sqrt{2}\psi_3(x). \tag{23}$$

Differentiating (22) with respect to x and y and substituting them into (23), then differentiating the obtained equation and changing $2x-1$ by $x-y$, we find

$$\omega_{31}(x-y)=\sqrt{2}\psi_3'\left(\frac{x-y-1}{2}\right),\ 0\le x-y\le 1. \tag{24}$$

Now proceeding to the limint for $x \to 0$ in equations (21) $(i=2;\ k=2)$ and (21) $(i=3;\ k=1)$ and taking into account (13), (15), we obtain the equations $\mu_3(y)-\tau_3''(y)=\omega_{22}(-y)$ and $\mu_3(y)-\tau_3''(y)=\omega_{31}(-y)$. It is clear from these equations that $\omega_{22}(-y)=\omega_{31}(-y)$. In this equality, changing $-y$ by $x-y$ and owing to (24), we obtain

$$\omega_{22}(x-y)=\omega_{31}(x-y)=\sqrt{2}\psi_3'\left(\frac{x-y-1}{2}\right),\ 0\le x-y\le 1 \tag{25}$$

Substituting (22) into (4), then differentiating the obtained equation and changing $-2x-1$ by y, we obtain the relation

$$\tau_3'(y)-\nu_3(y)=\delta_1(y),\ -1\le y\le 0, \tag{26}$$

where $\delta_1(y)$ is the known function.

We pass on to the domain D_{22}. We write the solution of equation (21) $(i=2;\ k=2)$, satisfying conditions (13), (14) as follows:

$$u_{22}(x,y)=\frac{\tau_3(y+x)+\tau_3(y-x)}{2}+\frac{1}{2}\int\limits_{y-x}^{y+x}\nu_3(t)\,dt+\frac{1}{2}\int\limits_0^x d\eta\int\limits_{y-x+\eta}^{y+x-\eta}\omega_{22}(\eta-\xi)\,d\xi \tag{27}$$

Substituting (27) into (3) and differentiating the obtained equation and changing $2x-1$ by y, we obtain the relation

$$\tau_3'(y)+\nu_3(y)=\delta_2(y),\ -1\le y\le 0, \tag{28}$$

where $\delta_2(y)$ is the known function.

From (26) and (28) we find

$$\nu_3(y)=\frac{1}{2}\left[\delta_2(y)-\delta_1(y)\right], \tag{29}$$

$$\tau_3'(y)=\frac{1}{2}\left[\delta_2(y)+\delta_1(y)\right].$$

Integrating the latest equality from -1 to y, we have

$$\tau_3(y)=\frac{1}{2}\int\limits_{-1}^{y}\left[\delta_2(t)+\delta_1(t)\right]dt+\psi_1(0).$$

Thus, we have found the functions $u_{31}(x,y)$ and $u_{22}(x,y)$.

Now we pass on to the domain D_{21}. We write the solution of equation (21) $(i=2;\ k=2)$, satisfying conditions (7), (8) as follows:

$$u_{21}(x,y)=\frac{\tau_1(x+y)+\tau_1(x-y)}{2}+\frac{1}{2}\int\limits_{x-y}^{x+y}\nu_1(t)\,dt-\frac{1}{2}\int\limits_0^y d\eta\int\limits_{x-y+\eta}^{x+y-\eta}\omega_{21}(\xi-\eta)\,d\xi \tag{30}$$

Differentiating (27) and (30) with respect to x and y and substituting them into the condition $\left.\left(\frac{\partial u_{21}}{\partial x}+\frac{\partial u_{21}}{\partial y}\right)\right|_{y=-x}=\left.\left(\frac{\partial u_{22}}{\partial x}+\frac{\partial u_{22}}{\partial y}\right)\right|_{y=-x}$ and differentiating the obtained equality then changing $2x$ by $x-y$, we find

$$\omega_{21}(x-y)=\omega_{22}(x-y)=\sqrt{2}\psi_3'\left(\frac{x-y-1}{2}\right),\ 0\le x-y\le 1. \tag{31}$$

Now we use the condition $u_{21}(x,-x)=u_{22}(x,-x)$. Substituting (31) into this condition and differentiating the obtained equality then changing $2x$ by x, we have the following relation:

$$\nu_1(x)=\tau_1'(x)-\alpha_1(x),\ 0\le x\le 1, \tag{32}$$

where $\alpha_1(x)$ is the known function.

Applying the operator $\dfrac{\partial}{\partial x}+\dfrac{\partial}{\partial y}$ to equation (18) and proceeding to the limit for $y\to 0$ in the obtained equality, we obtain the following relations:

$$\tau_1'''(x)+\nu_1''(x)-\nu_1'(x)-\mu_1(x)=0. \tag{33}$$

Analogously, proceeding to the limit for $y\to 0$ in equation (20) ($i=2$; $k=1$), we obtain the relation

$$\tau_1''(x)-\mu_1(x)=\omega_{21}(x) \tag{34}$$

Eliminating the functions $\nu_1(x)$ and $\mu_1(x)$ from (32), (33) and (34), we obtain the equation

$$\tau_1'''(x)-\tau_1''(x)=\frac{1}{2}\left[\alpha''{}_1(x)-\alpha'{}_1(x)-\omega_{21}(x)\right].$$

Integrating the latest equation twice from 1 to x, we obtain

$$\tau_1'(x)-\tau_1(x)=\alpha_2(x)+k_1(x-1)+k_2, \tag{35}$$

where $\alpha_2(x)$ is the known function.

Solving equation (35) under the conditions $\tau_1(1)=\varphi_1(0)$, $\tau_1'(1)=\varphi_1'(0)+\alpha_1(1)$, $\tau_1''(1)=\varphi_1''(0)+\sqrt{2}\psi_3'(0)$, we obtain

$$\tau_1(x)=\int_1^x\exp(x-t)\,\alpha_2(t)\,dt+k_1(\exp(x-1)-x)+ \tag{36}$$

$$+k_2(\exp(x-1)-1)+k_3\exp(x-1),$$

where $k_3=\varphi_1(0)$, $k_2=\varphi_1'(0)-\varphi_1(0)+\frac{1}{2}\alpha_1(1)$, $k_1=\sqrt{2}\psi_3'(0)+\varphi_1''(0)-\varphi_1'(0)-\frac{1}{2}[\alpha'{}_1(1)+\alpha_1(1)]$.

Thus, we have found the function $u_{21}(x,y)$. It is determined by formula (30), and the functions $\omega_{21}(x-y)$, $\nu_1(x)$ and $\tau_1(x)$ are determined by formulae (31), (32), (36), respectively.

We pass on to the domain D_{32}. We write the solution of equation (21) ($i=3$; $k=2$), satisfying conditions (10), (11) as follows:

$$u_{32}(x,y)=\frac{\tau_2(x+y)+\tau_2(x-y)}{2}+\frac{1}{2}\int_{x-y}^{x+y}\nu_2(t)\,dt-\frac{1}{2}\int_0^y d\eta\int_{x-y+\eta}^{x+y-\eta}\omega_{32}(\xi-\eta)\,d\xi \tag{37}$$

Differentiating (37) with respect to x and y and substituting them into the condition

$$\left.\left(\frac{\partial u_{32}}{\partial x}+\frac{\partial u_{32}}{\partial y}\right)\right|_{y=-x-1}=\sqrt{2}\psi_3(x),$$

then differentiating the obtained equation and changing $2x+1$ by $x-y$, we obtain

$$\omega_{32}(x-y)=\sqrt{2}\psi_3'\left(\frac{x-y-1}{2}\right),\ -1\le x-y\le 0 \tag{38}$$

Now applying from the condition $u_{32}(x,x)=u_{31}(x,x)$ and differentiating the obtained equations and changing $2x$ by x, we obtain

$$\nu_2(x)=-\tau_2'(x)+\beta_1(x),\ -1\le x\le 0, \tag{39}$$

where $\beta_1(x)$ is the known function.

We pass on to the domain D_{42}. Proceeding to the limit for $y \to 0$ in the equations (21) $(i=4;\ k=2)$ and (21) $(i=3;\ k=2)$, we obtain the equations $\tau_2''(x) - \mu_2(x) = \omega_{42}(x)$ and $\tau_2''(x) - \mu_2(x) = \omega_{32}(x)$. It is clear from these equations that $\omega_{42}(x) = \omega_{32}(x)$, $-1 \le x \le 0$. Then owing to (38), we have

$$\omega_{42}(x-y) = \sqrt{2}\psi_3'\left(\frac{x-y-1}{2}\right), \ -1 \le x-y \le 0 \tag{40}$$

We write the solution of equation (21) $(i=4;\ k=2)$, satisfying the conditions (10), (11) as follows:

$$u_{42}(x,y) = \frac{\tau_2(x+y)+\tau_2(x-y)}{2} + \frac{1}{2}\int\limits_{x-y}^{x+y} \nu_2(t)\,dt - \frac{1}{2}\int\limits_0^y d\eta \int\limits_{x-y+\eta}^{x+y-\eta} \omega_{42}(\xi-\eta)\,d\xi.$$

Substituting (39) into the latest equality, we have

$$u_{42}(x,y) = \tau_2(x-y) + \frac{1}{2}\int\limits_{x-y}^{x+y} \beta_1(t)\,dt - \frac{1}{2}\int\limits_0^y d\eta \int\limits_{x-y+\eta}^{x+y-\eta} \omega_{42}(\xi-\eta)\,d\xi \tag{41}$$

Then we pass on to the domain D_{41}. We write the solution of equation (21) $(i=4;\ k=1)$, satisfying conditions (16), (17) as follows:

$$u_{41}(x,y) = \frac{\tau_4(y+x)+\tau_4(y-x)}{2} + \frac{1}{2}\int\limits_{y-x}^{y+x} \nu_4(t)\,dt + \frac{1}{2}\int\limits_0^x d\eta \int\limits_{y-x+\eta}^{y+x-\eta} \omega_{41}(\eta-\xi)\,d\xi \tag{42}$$

Differentiating (41) and (42) with respect to x and y and substituting the obtained equalities into the condition $\left.\left(\frac{\partial u_{41}}{\partial x} + \frac{\partial u_{41}}{\partial y}\right)\right|_{y=-x} = \left.\left(\frac{\partial u_{42}}{\partial x} + \frac{\partial u_{42}}{\partial y}\right)\right|_{y=-x}$, then differentiating the obtained equation and taking into account (40) and changing $2x$ by $x-y$, we find

$$\omega_{41}(x-y) = \omega_{42}(x-y) = \sqrt{2}\psi_3'\left(\frac{x-y-1}{2}\right), \ -1 \le x-y \le 0. \tag{43}$$

Then, taking into account the conditions $u_{41}(x,y)|_{y=-x} = u_{42}(x,y)|_{y=-x}$ and differentiating the obtained equation and then changing $-2x$ by y, we obtain

$$\tau_4'(y) - \nu_4(y) = -2\tau_2'(-y) + \delta_3(y), \ 0 \le y \le 1 \tag{44}$$

where $\delta_3(y)$ is the known function.

Now substituting (42) into (6) and differentiating the obtained equation, then changing $2x+1$ by y, we have

$$\tau_4'(y) + \nu_4(y) = \delta_4(y), \ 0 \le y \le 1 \tag{45}$$

where $\delta_4(y)$ is the known function.

From (44) and (45) we obtain

$$\nu_4(y) = \tau_2'(-y) + \frac{1}{2}[\delta_4(y) - \delta_3(y)], \ 0 \le y \le 1, \tag{46}$$

$$\tau_4'(y) = -\tau_2'(-y) + \frac{1}{2}[\delta_4(y) + \delta_3(y)], \ 0 \le y \le 1. \tag{47}$$

Integrating (47) taking into account the conditions $\tau_4(0) = \tau_2(0)$, we find

$$\tau_4(y) = \tau_2(-y) + \frac{1}{2}\int_0^y [\delta_4(t) + \delta_3(t)]\,dt,\ 0 \le y \le 1 \quad (48)$$

Now we pass on to the domain D_1. Proceeding to the limit for $y \to 0$ in equation (19), then changing x by $x-y$ in the obtained equation, we obtain

$$\omega_{12}(x-y) = \tau_1''(x-y) - \nu_1(x-y),\ 0 \le x-y \le 1, \quad (49)$$

where it is assumed that $\omega_1(x-y) = \begin{cases} \omega_{11}(x-y), & -1 \le x \le 0, \\ \omega_{12}(x-y), & 0 \le x \le 1. \end{cases}$

Proceeding to the limit for $x \to 0$ in equations (21) $(i=4;\ k=1)$ and (19), we obtain the equations $\mu_4(y) - \tau_4''(y) = \omega_{41}(-y)$ and $\mu_4(y) - \tau_4'(y) = \omega_{11}(-y)$. Eliminating the function $\mu_4(y)$ from these equations, we find

$$\omega_{11}(-y) = \tau_4''(y) - \tau_4'(y) + \omega_{41}(-y).$$

Changing $-y$ by $x-y$ in this equality, we obtain the following relation:

$$\omega_{11}(x-y) = \tau_4''(y-x) - \tau_4'(y-x) + \omega_{41}(x-y). \quad (50)$$

Differentiating (47), we find

$$\tau_4''(y) = \tau_2''(-y) + \frac{1}{2}\left[\delta'_4(y) + \delta'_3(-y)\right],\ 0 \le y \le 1. \quad (51)$$

Substituting (47) and (51) into (50), we obtain

$$\omega_{11}(x-y) = \tau_2''(x-y) + \tau_2'(x-y) + \gamma_1(x-y) \quad (52)$$

where $\gamma_1(x-y)$ is the known function.

Now we write the solution of equation (19), satisfying conditions (2), (7), (16) as follows:

$$u_1(x,y) = \frac{1}{2\sqrt{\pi}}\left[\int_0^y \tau_4(\eta)G_\xi(x,y;0,\eta)\,d\eta - \int_0^y \varphi_1(\eta)G_\xi(x,y;1,\eta)\,d\eta + \int_0^1 \tau_1(\xi)G(x,y;\xi,0)\,d\xi - \right.$$
$$\left. - \int_0^y d\eta\int_0^\eta \omega_{11}(\xi-\eta)G(x,y;\xi,\eta)\,d\xi - \int_0^y d\eta\int_\eta^1 \omega_{12}(\xi-\eta)G(x,y;\xi,\eta)\,d\xi\right], \quad (53)$$

where the functions

$$\left.\begin{array}{c} G(x,y;\xi,\eta) \\ N(x,y;\xi,\eta) \end{array}\right\} = \frac{1}{\sqrt{y-\eta}}\sum_{n=-\infty}^{+\infty}\left\{\exp\left[-\frac{(x-\xi-2n)^2}{4(y-\eta)}\right] \mp \exp\left[-\frac{(x+\xi-2n)^2}{4(y-\eta)}\right]\right\}$$

are Green functions of the first and the second boundary value problems for equation (17).

Differentiating (53) with respect to x and assuming $x \to 0$ in the obtained equality, we have

$$\nu_4(y) = -\frac{1}{2\sqrt{\pi}}\int_0^y \tau'_4(\eta)N(0,y;0,\eta)\,d\eta +$$
$$+\frac{1}{2\sqrt{\pi}}\int_0^y \varphi'_1(\eta)N(0,y;1,\eta)\,d\eta + \frac{1}{2\sqrt{\pi}}\int_0^1 \tau'_1(\xi)N(0,y;\xi,0)\,d\xi -$$

$$-\frac{1}{2\sqrt{\pi}}\int\limits_0^y \omega_{11}(-\eta)N(0,y;0,\eta)\,d\eta-\frac{1}{2\sqrt{\pi}}\int\limits_0^y d\eta\int\limits_0^\eta \omega'_{11}(\xi-\eta)N(0,y;\xi,\eta)\,d\xi+$$

$$+\frac{1}{2\sqrt{\pi}}\int\limits_0^y \omega_{12}(1-\eta)N(0,y;1,\eta)\,d\eta-\frac{1}{2\sqrt{\pi}}\int\limits_0^y d\eta\int\limits_\eta^1 \omega'_{12}(\xi-\eta)N(0,y;\xi,\eta)\,d\xi.$$

Differentiating this equality and taking into account (51), (52), after some calculations, we have

$$\tau_2'''(-y)+\int\limits_0^y K(y,\eta)\,\tau'''_2(-\eta)\,d\eta=g(y), \tag{54}$$

where $K(y,\eta)$, $g(y)$ are the known functions.

Equation (54) is the Volterra equation of the second kind. When solving it, we find uniquely the function $\tau_2'''(-y)$, and consequently all the unknown functions $\tau_2(-y)$, $\nu_2(-y)$, $\tau_4(y)$, $\nu_4(y)$, $\omega_{11}(y)$, $u_{32}(x,y)$, $u_{41}(x,y)$, $u_{42}(x,y)$, $u_1(x,y)$. □

Conclusions

The papers [3],[4] considered a number of boundary value problems for more generalized third-order parabolic-hyperbolic equations in the domain with one line of type change.

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