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ON A DYNAMIC HEREDITARY SYSTEM SIMULATING ECONOMIC CYCLES

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The paper presents a mathematical model that generalizes the famous Dubovskiy's model used to predict economic crises. This generalization consists in the consideration of the memory effect which occurs frequently in the economic system. With the help of numerical methods, a generalized model solution was received, according to which phase paths were built.

Key words: Kandrat'ev's cycles, economic crisis, Dubovskiy's model, fractional derivative Gerasimov-Caputo operator, memory effect, fractal dimension

Introduction

The importance of mathematical simulation of economic processes is undeniable. The mathematical description of economic systems give quantitative and qualitative representation on economic indicators with the purpose of further forecast for the following time intervals and recommendations to make proper decisions [1]. Economic crises are the most interesting for us as long as they determine the economic welfare of citizents and the degree of social tension in a country. In the 20-is of the latest century, the Soviet economist, N.D. Kondrat'ev, distinguished long-term periodical oscillations (waves) with the duration of 50-55 years in economic time series [2]. Furthermore, in a similar way, other researchers determined the waves of other duration, for example, waves of basic investments [3] etc. From our point of view, the most complete mathematical description of simulation of Kondrat'ev's cycles was carried out in S.V. Dubovskiy's papers [4]-[6]. A mathematical model is suggested in this paper. It generalizes the known Dubovskiy's model in the case of consideration of memory effects in the economic system and is a logical extension of the papers [7] and [8]. Memory effects are described by fractional calculus theory, in particular, by fractional derivatives [9]. Such models are described in the papers by foreign authors [10]-[14] and in the papers of Russian authors [15]-[17].

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Statement of the problem

The generalized model of Kondrat'ev's cycles may be represented as follows:

$$\begin{cases} \partial_{0t}^{\alpha} x(\tau) = -\lambda n x(t)(x(t)-1)(y(t)-y^*), \\ \partial_{0t}^{\beta} y(\tau) = n(1-n) y^2(t)(x(t)-x^*) + f(t), \\ x(0)=a, y(0)=b. \end{cases} \quad (1)$$

where $\partial_{0t}^{\alpha} x(\tau) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\dot{x}(\tau)d\tau}{(t-\tau)^{\alpha}}$ and $\partial_{0t}^{\beta} y(\tau) = \frac{1}{\Gamma(1-\beta)} \int_0^t \frac{\dot{y}(\tau)d\tau}{(t-\tau)^{\beta}}$ are fractional derivatives $0 < \alpha, \beta < 1$ in the sense of Gerasimov-Caputo; $\Gamma(x)$ is Eiler gamma-function; $x(t)$ is new technology efficiency; $y(t)$ is capital productivity efficiency; x^* and y^* is equilibrium stationary solution of system (1); n is the rate of saving; λ is the coefficient determined from time series statistics; $f(t)$ is the external impact of the economic system; $t \in [0,T]$ is the time coordinate; T is the time of process simulation; a and b are the initial conditions, given constants.

We should note that the nonlinear system (1) in the case of parameter values $\alpha = \beta = 1$ and $f(t) = 0$ turns to Dubovskiy's model [4]. Thus, it is obvious that solution of system (1) generalizes the solution of Dubovskiy's model. We solve the nonlinear system ([1]) by numerical methods, finite differential schemes. We divide the time segment $[0,T]$ into N equal parts with the step τ. We approximate fractional derivatives in equation ([1]) based on the paper [18]. Then system (1) is written in the finite-differential statement as follows:

$$\begin{cases} x_0 = a, y_0 = b, \\ x_1 = x_0\left(1 - \frac{\lambda n}{A}(x_0-1)(y_0-y^*)\right), y_1 = y_0\left(1 + \frac{n(1-n)}{B} y_0 (x_0 - x^*)\right), j=0, \\ x_2 = x_1\left(1 - \frac{\lambda n}{A}(x_1-1)(y_1-y^*)\right), y_2 = y_1\left(1 + \frac{n(1-n)}{B} y_1 (x_1 - x^*)\right), j=1, \\ x_{j+1} = x_j\left(1 - \frac{\lambda n}{A}(x_j-1)(y_j-y^*)\right) - \sum_{k=1}^{j-1} p_k (x_{j-k+1} - x_{j-k}), \\ y_{j+1} = y_j\left(1 + \frac{n(1-n)}{B} y_j (x_j - x^*)\right) - \sum_{k=1}^{j-1} q_k (x_{j-k+1} - x_{j-k}) + f_j, j = 2, ..., N-1, \end{cases} \quad (2)$$

Here $A = \frac{\tau^{-\alpha}}{\Gamma(2-\alpha)}$, $B = \frac{\tau^{-\beta}}{\Gamma(2-\beta)}$, $p_k = (1+k)^{1-\alpha} - k^{1-\alpha}, q_k = (1+k)^{1-\beta} - k^{1-\beta}$.

Solution of (2) in the case when $\alpha = \beta = 1$ and $f_j = 0$ turns to the solution for Dubovskiy's model [4]. We investigate the solution of (2) depending on different values of fractional parameters α and β and draw the phase paths. In this paper we do not discuss the issues of stability or convergence of an explicit finite differential scheme (2).

Simulation results

We take the simulation parameters from the paper [4] $x^* = 1.3, y^* = 0.5, n = 0.2, \lambda = 2.25, x(0) = 1.35, y(0) = 0.5, T = 250, \alpha = \beta = 1$.

Fig. 1 illustrates a case which corresponds to the case considered in the paper [5] in the simulation of Kondrat'ev's cycle with the period of 50.1 years. We can also see that the phase path (Fig. 1*b*) has an elliptical closed form, the equilibrium state of a system is called a center. The oscillation amplitude in this case is constant (Fig. 1*a*).

Fig. 2 shows a case when the system (1) suffers from external periodic impact $f(t) = \delta\cos(\omega t)$, investment cycles. The values of the parameters are $\delta = 0.01$ and $\omega = 1$. We may conclude that the external periodic impact results in the cycle with the period of 7 years that corresponds to the basic investment Zhuglyar's cycles [6]. The main cycle is 60 years that

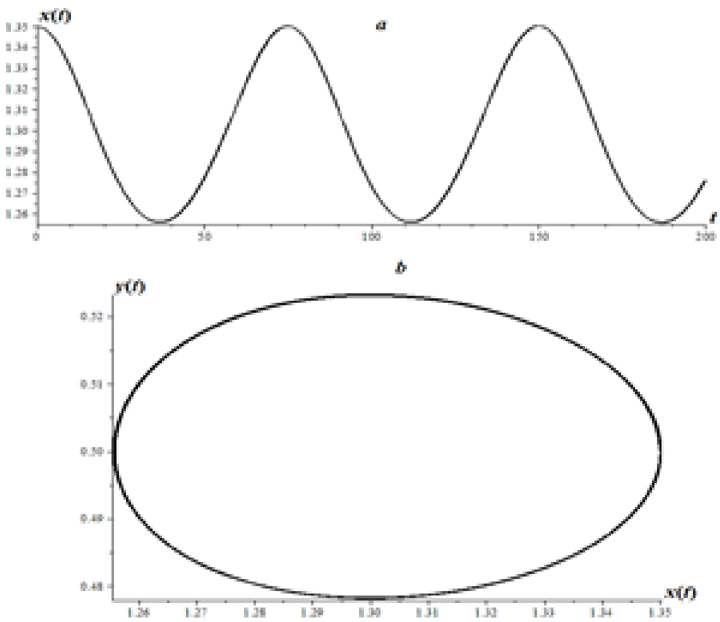

Fig. 1. Calculated curve a and phase path b in the case when $f(t) = 0$

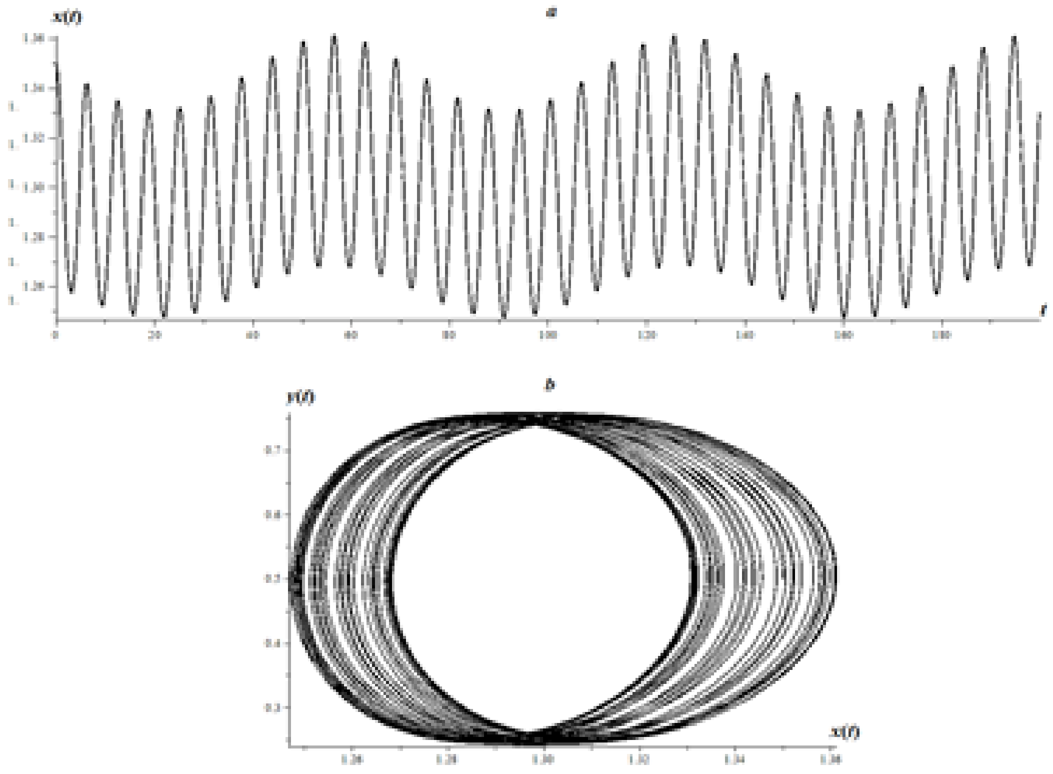

Fig. 2. Calculated curve a and phase path b in the case when $f(t) = \delta \cos(\omega t)$

corresponds to the upper limit of Kondrat'ev's cycle [2]. Such a combined model describes the economic crises in the most flexible way.

Fig. 3 shows a case when $f(t) = 0$, $\alpha = 0.8$ and $\beta = 1$. The rest of the parameters are unchanged.

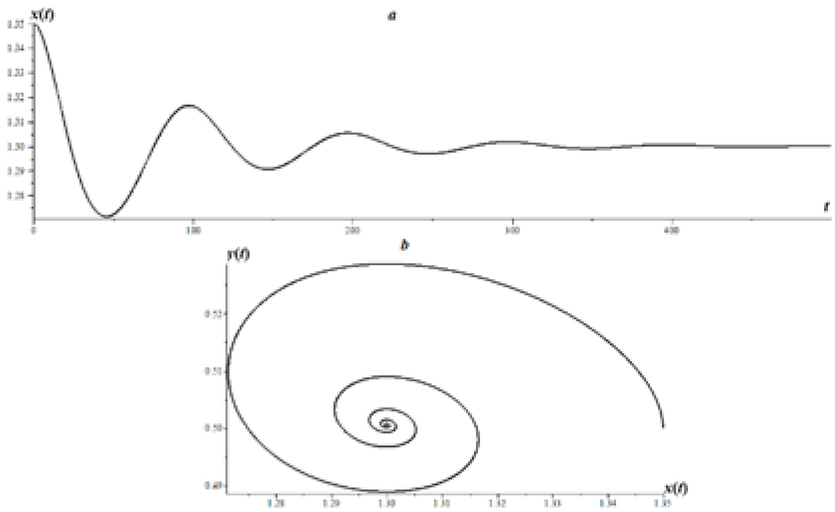

Fig. 3. Calculated curve *a* and phase path *b* in the case when $\alpha = 0.8, \beta = 1$ and $f(t) = 0$

It is clear from Fig. 3*a* that the oscillation process is attenuating and the phase path in Fig. 3 *b* is open. The equilibrium position of the system is called a stable focus. In this case, there are no cycles, however, if we introduce an external impact function $f(t) = \delta \cos(\omega t)$, which can be interpreted as investment cycles, we come to the following result (Fig. 4).

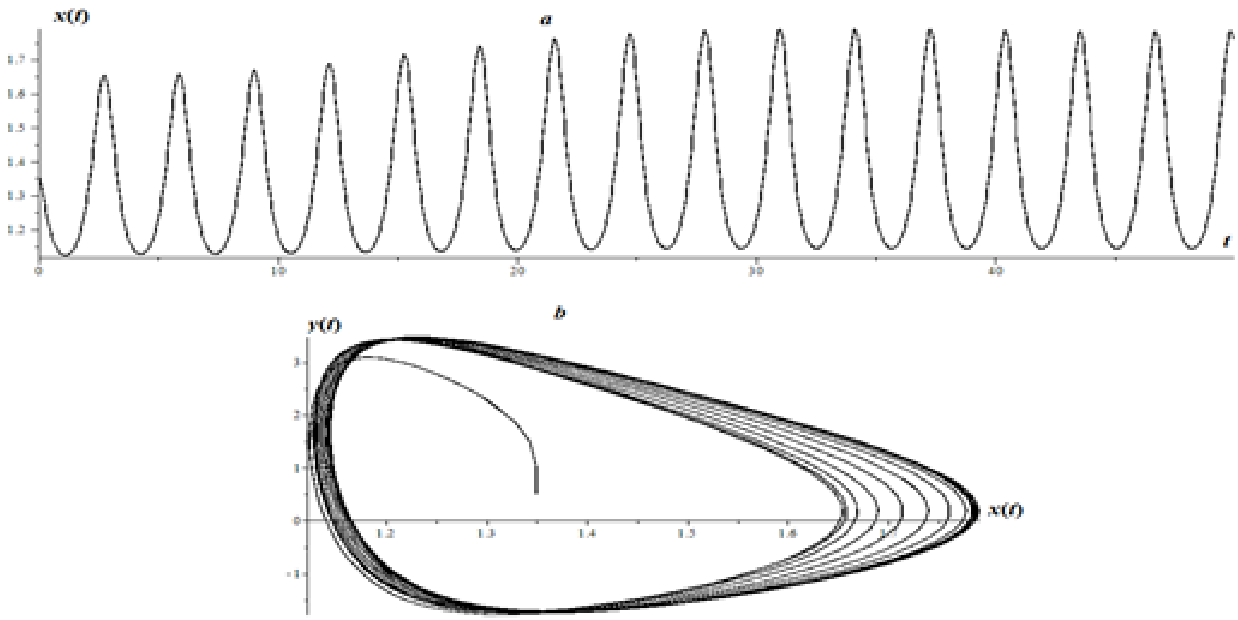

Fig. 4. Calculated curve *a* and phase path *b* in the case when $\alpha = 0.8, \beta = 0.6$ and $f(t) = \delta \cos(\omega t), \delta = 0.5, \omega = 2$

It is clear in Fig. 3*a* that, first, the oscillation amplitude increases and then it comes out to the constant mode. One can see it on the phase path in Fig. 3*b* which, with time, comes out to the constant mode or the boundary cycle that may be applied in the study of Kondrat'ev's cycles. In the following figures (Fig. 5 and Fig. 6) we also see that the phase paths come out to the boundary cycles.

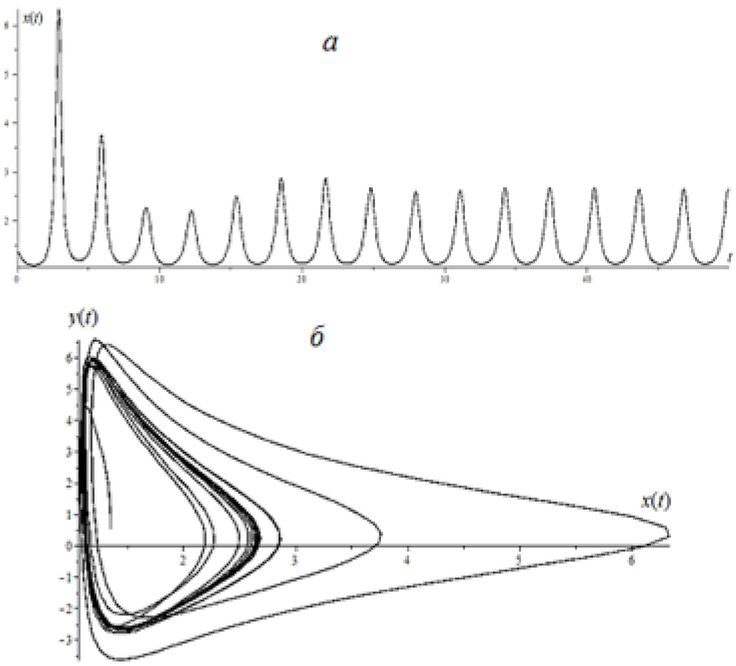

Fig. 5. Calculated curve a and phase path b in the case when $\alpha = 0.8, \beta = 0.8$ and $f(t) = \delta \cos(\omega t), \delta = 0.5, \omega = 2$

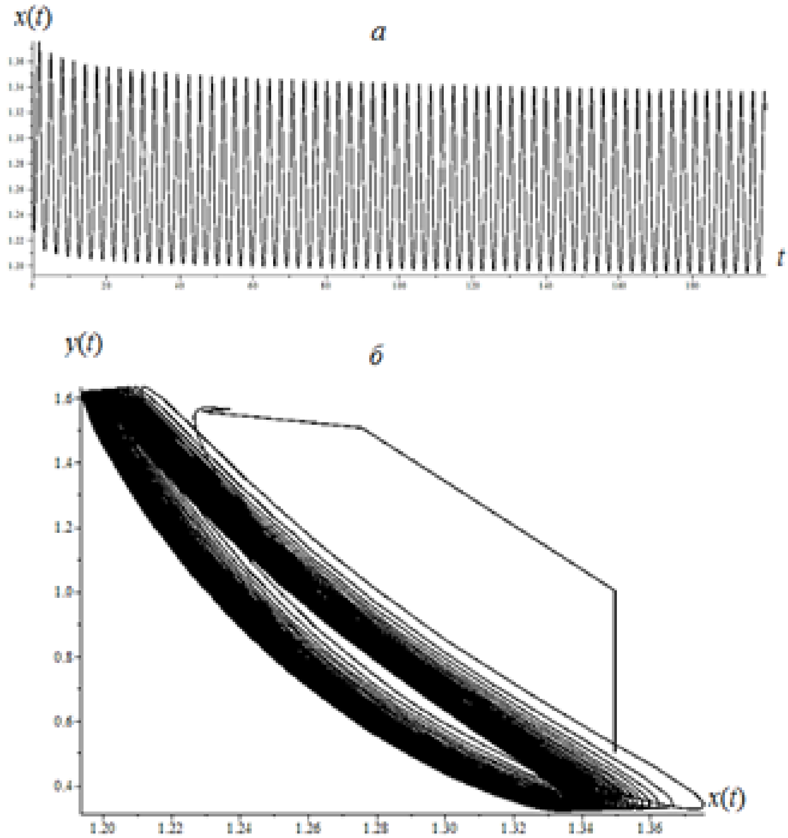

Fig. 6. Calculated curve a and phase path b in the case when $\alpha = 0.1, \beta = 0.1$ and $f(t) = \delta \cos(\omega t), \delta = 0.5, \omega = 2$

Conclusions

The paper suggests a generalized Dubovskiy's model which takes into account the memory effects (economic barriers) in the economic system. Numerical solution for such a model were obtained and phase paths were plotted. It was shown that introductions of fractional derivatives results in attenuating processes. However, if there is an external periodic effect in the system, it comes out to a boundary cycle which may be considered to be one or another economic cycle.

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