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MATHEMATICAL MODELING

MSC 74S60

ON THE LOG-NORMAL LAW OF FREQUENCY DISTRIBUTION IN HIGH-FREQUENCY GEOACOUSTIC EMISSION PULSES

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The paper is devoted to the detection of the law of frequency distribution in high-frequency geoacoustic emission signals generated in the result of dislocation changes in sedimentary rocks. Sparse approximation is used for emission pulse processing. As a consequence, a signal is decomposed into atoms with a definite frequency corresponding to the size of a shear source. On the basis of Kolmogorov criteria, it was ascertained that distribution of atoms in a geoacoutstic signal corresponds to the log-normal law.

Key words: high-frequency acoustic emission, sparse approximation, log-normal law

Introduction

Acoustic emissions in solid bodies are elastic oscillations occurring in the result of dislocation changes in a medium. The characteristics of excited pulse radiation are directly associated with the features of plastic processes which determines the interest to the investigation of the emission to develop methods of acoustic diagnostics of a medium. In geophysics the acoustic emission is considered in three main ranges, they are: infrasonic range to register earthquakes and to estimate their characteristics; sound range to investigate deformation processes, to search and to map minerals and to study strong earthquake precursors; ultrasonic range in laboratory deformations of rock samples to investigate crack formation mechanisms.

Investigations of high-frequency geoacoustic emission (GAE) have been carried out in Kamchatka since 1999 in a wide range of sound frequencies from hundreds of hertz to the first tens of kilohertz. The relation of emission signals with plastic process dynamics in one of the most seismically active regions of the globe has been detected [1, 2].

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A GAE signal is composed of a sequence of relaxation pulses of different amplitude and duration with impulse excitation and the basic frequency from units to tens of kilohertz. The repetition frequency during calm periods is units per a second and during anomalies, preceding seismic events, it reaches tens and even hundreds per a second [3].

Registration and preliminary analysis of GAE signals was carried out both within the whole frequency range (0,1 – 10000 Hz) and at the output of band-pass filters dividing the frequency range into several subranges. Such a system allows us to detect acoustic signals of different nature in real time and to analyze them in a wide range [4].

Classic methods of time-and-frequency analysis are usually used to analyze the signals of pulse nature, they are: Fourier transform, short time Fourier transform, wavelet analysis. However, in the result of short duration, temporal localization of anomalies and high noisiness, the classical time-and-frequency analysis of GAE signals does not give desirable results. In 2011 sparse approximation method was suggested to analyze the inner structure of GAE signals [5].

Signal approximation is a problem of its presentation in the form of superposition of some set of function from a defined family (dictionary) [3]

$$f(t) = \sum_{m=0}^{N-1} a_m g_m(t) + R_N, \|R_N\| \to min, \tag{1}$$

where $f(t)$ is a signal under investigation, $g_m(t)$ is an element (atom) of a dictionary $D = \{g_m(t), \ \|g_m\| = 1\}$, a_m are expansion coefficients, N is the number of expansion elements, R_N is an approximation error.

Sparse approximation suggests construction of a signal model containing the least number of elements. Methods of sparse approximation are used for signal decomposition in redundant dictionaries that means in the dictionaries the number of atoms of which is much more than dimensionality of a signal. We should note that the problem of search for an optimal expansion basis, containing the least number of elements and minimizing the approximation error at the same time, has high computational complexity. It was ascertained in the course of computational experiments that among the algorithms of spare approximation the least demanding for the analysis of GAE signals is the algorithm of Matching Pursuit suggested by Mallat S. and Zhang S. The essence of this algorithm is the interactive search expansion elements minimizing the approximation error at each step [3].

Description of the approach of GAE signal processing and the obtained results

In the case of strong attenuation, which takes place in sedimentary rocks, the spatial scale of an emission signal is comparable with the length of its attenuation which, in its turn, is comparable with the distance to a source. Thus, we can determine the distance to the source by the duration of a GAE signal [1]. As long as shear sources of acoustic emission, located at the distance of the first tens of meters from a receiver, prevail in the area of investigation of GAE, we can use J.Brown formula to estimate their size. In spite of the maximal error, it is the most simple one for calculation [1]

$$l = \frac{2.34 V_p}{2\pi f}, \tag{2}$$

where l is the size of a source, V_p is the velocity of longitudinal oscillations in sedimentary rocks, f is the signal frequency from a source.

Taking this into account and applying the algorithms of sparse approximation, we can decompose a GAE signal into atoms where each atom is characterized by its frequency corresponding to the dimensions of shear sources in rocks.

We should also note that noises from different sources may appear in the records of GAE signals at lower (to 700 Hz) and at high (to 12000 Hz) frequencies. Thus, for the signal decomposition, obtained in the course of application of the matching pursuit algorithm, to be more accurate, we need to filter it first. During our work, we constructed and implemented an elliptical band-pass filter with the bandwidth from 700 Hz to 12000 Hz. Data cleaning may be realized by sparse approximation as it was suggested in [6].

As initial data for the analysis we used all the atoms detected in 5 records of GAE signals which were registered in 2014 within different periods of time. The duration of the records was 15 minutes. 18167 geoacoustic pulses were analyzed in total. Having united the obtained decompositions of signals into atoms, the statistics was obtained, i.e. frequency massive of a corresponding shear. Having such s statistics, we can determine if the dimensions of shear sources are distributed randomly relatively each other or their distribution follows a certain law.

To make a hypothesis on the law of distribution, we can use the graphic representation of empirical data that is to plot a histogram based on the obtained statistics (Fig. 1).

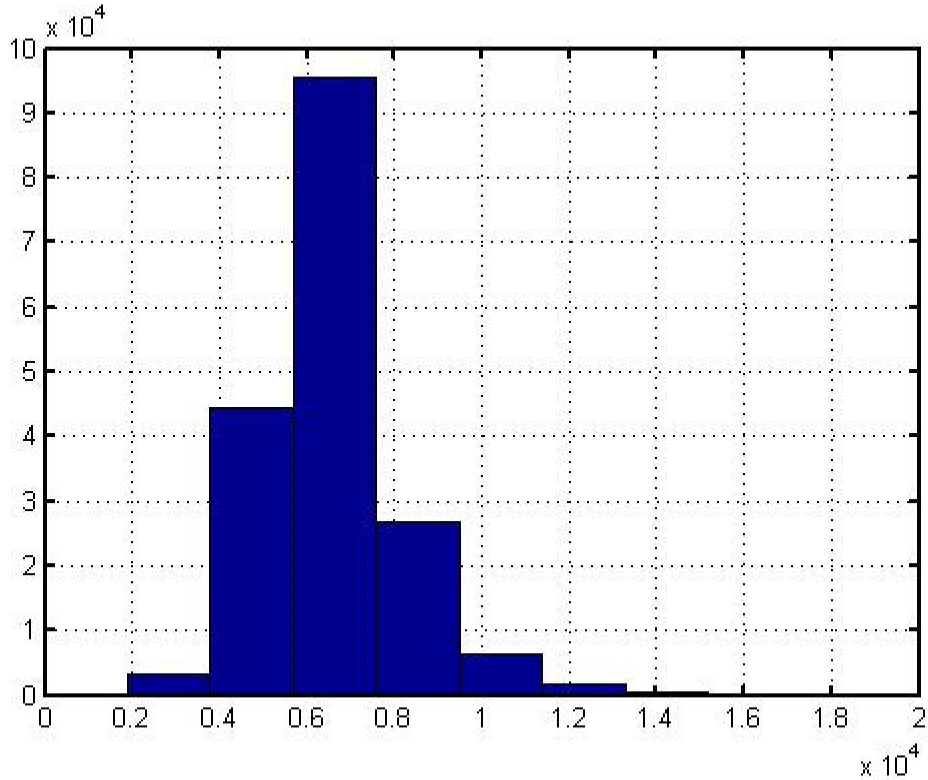

Fig. 1. Presentation of statistical data in the form of a histogram

It follows from the graphic representation of empirical data that probabilistic distribution of frequencies may refer to one of three the most possible distributions the general graphs of which are shown in Fig. 2, in particular exponential (Fig. 2a), normal (Fig. 2b) and log-normal (Fig. 2c) distributions of probabilities.

There are special algorithms to check the validity of statistical hypotheses. The most known ones are the Pirson criterion and the Kolmogorov criterion. We apply the Kolmogorov criterion for the sake of simple calculation. As a measure of deviation between the theoretical and empirical distributions we consider the maximum value of absolute difference between empirical function of distribution $F_n(x)$ and the corresponding theoretical function of distribution

$$D = |F_n(x) - F(x)|, \tag{3}$$

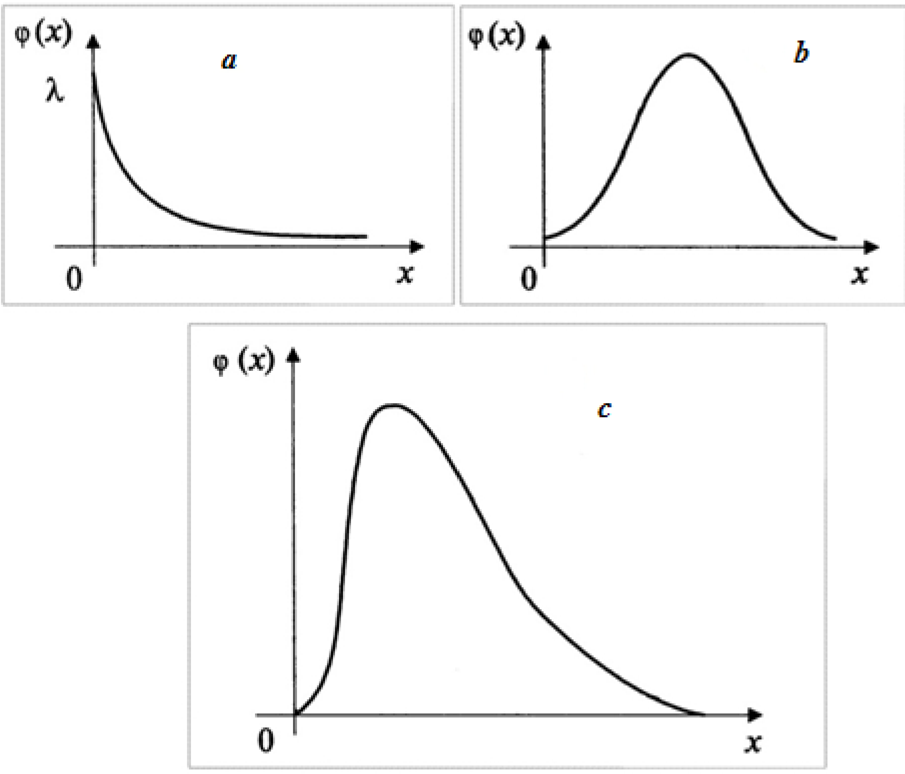

Fig. 2. General graphs of some distribution laws: a) exponential law of distribution; b) normal law of distribution; c) log-normal law of distribution

called the statistics of Kolmogorov criterion [7].

Does not matter what the distribution function $F(x)$ of a continuous random value X is, when the number of observations $(n \to \infty)$ grows, the probability of inequality $P(D\sqrt{n} \geq \lambda)$ tends to the limit

$$P(\lambda) = 1 - \sum_{k=-\infty}^{+\infty} (-1)^k e^{-2k^2\lambda^2}. \quad (4)$$

Then from the equality (4) we determine the probability that due to some random cause, the maximum deviation between $F_n(x)$ and $F(x)$ will turn to be less than that actually observed.

If the probability $P(\lambda)$ is small (less than 0,05), the hypothesis should be rejected as an implausible one. When probability $P(\lambda)$ is large, the hypothesis may be considered to be compatible with experimental data. The values of $P(\lambda)$ are taken from special tables [8].

On the assumption of graphic representation of the data, three hypotheses were proposed:

1) distribution follows the demonstrative law;
2) distribution follows the normal law;
3) distribution follows the log-normal law.

To apply the Kolmogorov criteria in order to check the validity of the suggested hypotheses, the following actions should be performed:

1) Derive the distribution function $F_n(x)$ and the suggested theoretical distribution functions $F(x)$ for demonstrative, normal and log-normal distributions, respectively (Fib. 3).

2) Determine the measure of deviation between theoretical and empirical distribution D and estimate the value

$$\lambda = \frac{\sqrt{n}}{D}. \tag{5}$$

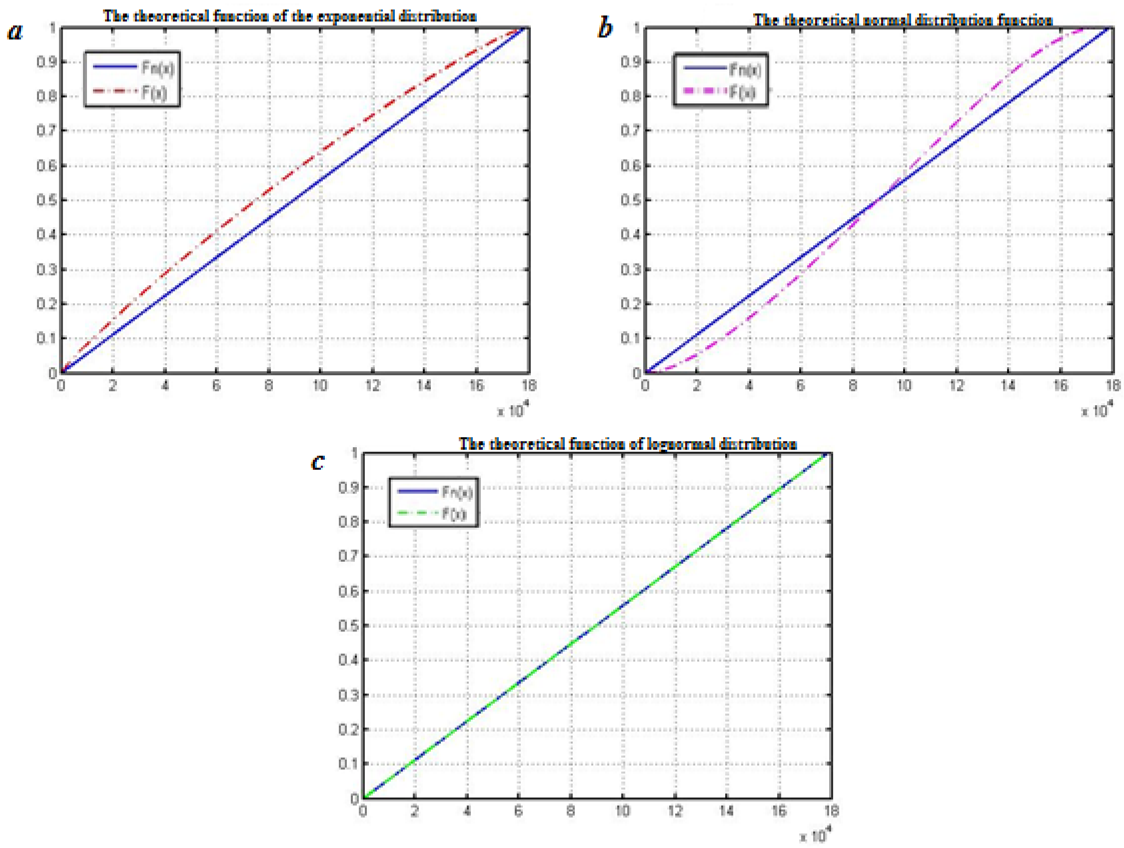

Fig. 3. Graphic comparison of the empiric $F_n(x)$ and theoretical $F(x)$ distribution functions: a) $F(x)$ corresponds to the demonstrative law of distribution; b) $F(x)$ corresponds to the normal law of distribution c) $F(x)$ corresponds to the log-normal law of distribution

3) For the estimated λ, determine the probability $P(\lambda)$ in a special table of values [8].

The results of calculations are shown in the table.

Table

Calculation results for three laws of distribution

Law of distribution	λ	$P(\lambda)$
Demonstrative distribution	34,875737351463920	0
Normal distribution	35,096495617372746	0
Log-normal distribution	0,011829027654287	≈ 1

Based on the data from the Table, the upper limit of an absolute error of an approximated equality

$$F_n(x) \approx F(x) \quad (6)$$

equals $D \approx 0,000027985179$ at any x for log-normal distribution.

It is also clear from the graphs in Fig. 3 that minimum deviation between the empirical and theoretical functions of distributions may be observed in the log-normal law. Thus, we can conclude that the considered statistics which histrogram is shown in Fig. 1 has log-normal law of distribution.

The obtained result agrees well with the data of the paper [9], which describes the process of formation of micro-cracks inside a sample of a stressed solid and illustrates that the observed acoustic emission from cracks should reflect log-normal distribution at each defined frequency.

Conclusions

When analyzing the data obtained by GAE signal processing by sparse approximation method, it was ascertained that distribution of emission frequencies associated with the formation of shear sources in rocks follows the log-normal law of distribution.

References

1. Larionov I. A., Marapulets Ju. V., Mishchhenko M. ,A. Otklik vysokochastotnoy geoakusticheskoy emissii na aktivizatsiyu plasticheskih processov v seysmoaktivnom regione [Response of high-frequency geoacoustic emission on intensification of plastic processes in the seismically active region. O Uchenye Zapiski Fizicheskogo Fakul'teta MGU – Memoirs of the Faculty of Physics. Lomonosov Moscow State University. 2014. no 6. 146311

2. Marapulets Yu. V. High-frequency acoustic emission effect. Bulletin KRASEC. Physical and Mathematical Sciences. 2015. vol.10. no 1. pp. 39-48

3. Afanas'eva A. A., Lukovenkova O. O., Marapulets Yu. V. Primenenie razrezhennoy approksimatsii i metodov klasterizatsii dlya opisaniya struktury vremennykh ryadov akusticheskoy emissii [Application of sparse approximation and clusterization to describe the structure of acoustic emission time series]. Tsifrovaya obrabotka signalov – Digital signal processing. 2013. no. 2. pp. 30-34.

4. Marapulets Yu. V., Mishchenko M. A., Shadrin A. V., Shcherbina A. O. Metody issledovaniya vysokochastotnoy geoakusticheskoy emissii: monografiya [Methods of investigation of high-frequency geoacoustic emission: monograph]. Petropavlovsk-Kamchatskiy. KamchatGTU. 2007. 106 p.

5. Lukovenkova O. O., Tristanov A. B. Adaptivnyy algoritm soglasovannogo presledovaniya s utochneniem na smeshannykh slovaryakh v analize signalov geoakusticheskoy emissii [Adaptive algorithm of matching pursuit with refinement on mixed dictionaries in the analysis of geoacoustic emission signals]. Tsifrovaya obrabotka signalov – Digital signal processing. 2014. no. 2. pp. 54–57

6. Lukovenkova O. O. Denoising of geoacoustic emission signals by sparse approximation method. Bulletin KRASEC. Physical and Mathematical Sciences. 2015. vol. 11. issue 2. pp. 73–78

7. Kremer N. Sh. Teoriya veroyatnostey i matematicheskaya statistika: Uchebnik dlya vuzov [Probability theory and mathematical statistics: textbook for universities]. Moscow. UNITI-DANA. 2004. 573 p.

8. Rebro I. V., Nosenko V. A., Korotkova N. N. Prikladnaya matematicheskaya statistika dlya tekhnicheskikh spetsial'nostey: uchebnoe posobie [Applied mathematical statistics for technicians: textbook]. Volgograd. IUNYa VolgGTU. 2011. 149 p.

9. Gregori G. P., Paparo G. Lognormality and fractality. A unique rationale for different phenomena. Phys. Review. Preprint. 2010

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