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MSC 35M13

STATEMENT AND STUDY OF SOME BOUNDARY VALUE PROBLEMS FOR THIRD ORDER PARABOLIC-HYPERBOLIC EQUATION OF TYPE $\frac{\partial}{\partial x}(Lu) = 0$ IN A PENTAGONAL DOMAIN

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The paper paper considers two boundary value problems, and examines one of these problems for a third order parabolic-hyperbolic equation of type $\frac{\partial}{\partial x}(Lu) = 0$ in a pentagonal domain. The unique solvability of the problem is proved

Key words: differential and integral equations, boundary value problems, parabolic-hyperbolic type

Introduction

Methods of differential and integral equations are applied for the solution of boundary value problems in mathematical physics. This paper is an example of application of these methods to solve one boundary value problem for third order parabolic-hyperbolic equation in a pentagonal domain.

Statement of the problem

We consider domain D on domain xOy, where $D = D_1 \cup D_2 \cup D_3 \cup D_4 \cup \cup AB \cup AE_2 \cup AE_1 \cup AA_0 \cup \{(0,0)\}$, and D_1 is a rectangle with vertexes at the points $A(0,0)$, $B(1,0)$, $B_0(1,1)$, $A_0(0,1)$, D_2 is a triangle with vertexes at the points $A(0,0)$, $B(1,0)$, $E_1(0,-1)$, D_3 is a triangle with vertexes at the points $A(0,0)$, $E_1(0,-1)$, $E_2(-1,0)$, D_4 is a triangle with vertexes at the points $A(0,0)$, $A_0(0,1)$, $E_2(-1,0)$, AB is an open segment with vertexes at the points $A(0,0)$ and $B(1,0)$, AE_2 is an open segment with vertexes at the points $A(0,0)$ and $E_2(-1,0)$, AE_1 is an open segment with vertexes at the points $A(0,0)$ and $E_1(0,-1)$, AA_0 is an open segment with vertexes at the points $A(0,0)$ and $A_0(0,1)$.

In domain D we consider the equation

$$\frac{\partial}{\partial x}(Lu) = 0, \tag{1}$$

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where

$$Lu \equiv \begin{cases} u_{xx} - u_y, & (x, y) \in D_1, \\ u_{xx} - u_{yy}, & (x, y) \in D_i, \ i = 2, 3, 4. \end{cases}$$

Introduce the following notations: $u(x, y) = u_i(x, y)$, $(x, y) \in D_i (i = 1, 2.3, 4)$.

Now we consider the following problem for equation (1):

Task 1. *Find a function $u(x, y)$, which*

1) is continuous in a closed domain $\overline{D}$;

2) satisfies equation (1) in domain D or $x \neq 0$ and $y \neq 0$;

3) satisfies the following boundary conditions:

$$u_1(1, y) = \varphi_1(y), \ 0 \le y \le 1, \tag{2}$$

$$u_2|_{BE_1} = \psi_1(x), \ 0 \le x \le 1, \tag{3}$$

$$u_4|_{A_0C_3} = \psi_2(x), \ -\frac{1}{2} \le x \le 0 \tag{4}$$

$$\left.\frac{\partial u_2}{\partial n}\right|_{BE_1} = \psi_4(x), \ 0 \le x \le 1, \tag{5}$$

$$\left.\frac{\partial u_4}{\partial n}\right|_{A_0E_2} = \psi_6(x), \ -1 \le x \le 0, \tag{6}$$

4) satisfies the following continuous conditions of gluing:

$$u_1(x, 0) = u_2(x, 0) = \tau_1(x), \ 0 \le x \le 1, \tag{7}$$

$$u_{1y}(x, -0) = u_{2y}(x, +0) = \nu_1(x), \ 0 \le x \le 1, \tag{8}$$

$$u_3(x, 0) = u_4(x, 0) = \tau_2(x), \ -1 \le x \le 0, \tag{9}$$

$$u_{3y}(x, 0) = u_{4y}(x, 0) = \nu_2(x), \ -1 \le x \le 0 \tag{10}$$

$$u_1(0, y) = u_4(0, y) = \tau_4(y), \ 0 \le y \le 1, \tag{11}$$

$$u_{1x}(0, y) = u_{4x}(0, y) = \nu_4(y), \ 0 \le y \le 1, \tag{12}$$

$$u_{1xx}(0, y) = u_{4xx}(0, y) = \mu_4(y), \ 0 \le y \le 1, \tag{13}$$

$$u_2(0, y) = u_3(0, y) = \tau_3(y), \ -1 \le y \le 0, \tag{14}$$

$$u_{2x}(0, y) = u_{3x}(0, y) = \nu_3(y), \ -1 \le y \le 0, \tag{15}$$

$$u_{2xx}(0, y) = u_{3xx}(0, y) = \mu_3(y), \ -1 \le y \le 0. \tag{16}$$

Here n is an inner normal to the straight lines BE_1 or E_1E_2, $\varphi_1, \psi_1, \psi_2, \psi_4, \psi_6$ are given sufficiently smooth functions, and τ_j, ν_j $(j = 1, 2, 3, 4)$, μ_1, μ_2 are for now unknown sufficiently smooth functions to be determined, and the following matching conditions are fulfilled: $\psi_4'(0) = -\psi_5'(0)$, $\tau_1(1) = \psi_1(1) = \varphi(0)$, $\tau_1'(1) = \varphi'(0) - \sqrt{2}\psi_4(1)$, $\tau_1''(1) = \varphi''(0) - \sqrt{2}\psi_4'(1)$.

Task 2. *This problem differs from* ***Problem 1*** *by the fact that instead of condition (4) we take the condition*

$$u_3|_{E_2C_2} = \psi_3(x), -1 \le x \le -\frac{1}{2}.$$

Other conditions remain the same.

In this paper we consider only **Problem 1**. To solve it, equation (1) is written as follows:

$$u_{1xx} - u_{1y} = \omega_1(y), \ (x, y) \in D_1, \tag{17}$$

$$u_{ixx} - u_{iyy} = \omega_i(y), \ (x, y) \in D_i \ (i = 2, 3, 4), \tag{18}$$

where we introduce the notation: $u(x,y)=u_i(x,y)$, $(x,y)\in D_i$ $\left(i=\overline{1,4}\right)$, and $\omega_i(y)$ $\left(i=\overline{1,4}\right)$ are for now unknown continuous functions.

We write the domains D_i $(i=2,3,4)$ in the following form: $D_i=D_{i1}\cup D_{i2}\cup AC_{i-1}$, where D_{21} are triangles with vertexes at the points $A(0;0)$, $B(1;0)$, $C_1\left(\frac{1}{2},-\frac{1}{2}\right)$, D_{22} is a triangle with vertexes at the points $A(0;0)$, $E_1(0;-1)$, $C_1\left(\frac{1}{2},-\frac{1}{2}\right)$, D_{31} is a triangle with vertexes at the points $A(0;0)$, $E_1(0;-1)$, $C_2\left(-\frac{1}{2},-\frac{1}{2}\right)$, D_{32} is a triangle with vertexes at the points $A(0;0)$, $E_2(-1;0)$, $C_2\left(-\frac{1}{2},-\frac{1}{2}\right)$, D_{41} is a triangle with vertexes at the points $A(0;0)$, $A_0(0;1)$, $C_3\left(-\frac{1}{2},\frac{1}{2}\right)$, D_{42} is a triangle with vertexes at the points $A(0;0)$, $E_2(-1;0)$, $C_3\left(-\frac{1}{2},\frac{1}{2}\right)$, AC_1 is an open segment with vertexes at the points $A(0;0)$, $C_1\left(\frac{1}{2},-\frac{1}{2}\right)$, AC_2 is an open segment with vertexes at the points is an open segment with vertexes at the points $A(0;0)$, $C_2\left(-\frac{1}{2},-\frac{1}{2}\right)$, AC_3 is an open segment with vertexes at the points $A(0;0)$, $C_3\left(-\frac{1}{2},\frac{1}{2}\right)$, that is $AC_1=\left\{(x,y)\in R^2:\ 0<x<\frac{1}{2},\ y=-x\right\}$, $D_{21}=\left\{(x,y)\in R^2:\ -\frac{1}{2}<y<0,\ -y<x<y+1\right\}$, $D_{22}=\left\{(x,y)\in R^2:\ 0<x<\frac{1}{2},\ x-1<y<-x\right\}$, $AC_2=\left\{(x,y)\in R^2:\ -\frac{1}{2}<x<0,\ y=x\right\}$, $D_{31}=\left\{(x,y)\in R^2:\ -\frac{1}{2}<x<0,\ -x-1<y<x\right\}$, $D_{32}=\left\{(x,y)\in R^2:\ -\frac{1}{2}<y<0,\ -y-1<x<y\right\}$, $AC_3=\left\{(x,y)\in R^2:\ -\frac{1}{2}<x<0,\ y=-x\right\}$,$D_{41}=\left\{(x,y)\in R^2:\ -\frac{1}{2}<x<\right.$ $D_{42}=\left\{(x,y)\in R^2:\ 0<y<\frac{1}{2},\ y-1<x<-y\right\}$.

Then equation (18) $(i=2,3,4)$ takes the following form:

$$u_{ikxx}-u_{ikyy}=\omega_{ik}(y),\ (x,y)\in D_{ik}\ (i=2,3,4;\ k=1,2), \tag{19}$$

where the following notations are introduced: $u_i(x,y)=u_{ik}(x,y)$, $\omega_i(y)=\omega_{ik}(y)$, $(x,y)\in D_{ik}$ $(i=2,3,4;\ k=1,2)$.

First of all, we study **Problem 1** in domain D_2. We write the solution of equation (19) $(i=2;\ k=1)$, satisfying conditions (7), (8) as follows:

$$u_{21}(x,y)=\frac{\tau_1(x+y)+\tau_1(x-y)}{2}+\frac{1}{2}\int\limits_{x-y}^{x+y}\nu_1(t)\,dt-\int\limits_0^y(y-\eta)\,\omega_{21}(\eta)\,d\eta. \tag{20}$$

Condition (5) may be written in the following form:

$$\left.\left(\frac{\partial u_{21}}{\partial x}-\frac{\partial u_{21}}{\partial y}\right)\right|_{y=x-1}=-\sqrt{2}\psi_4(x). \tag{21}$$

Differentiating equation (20) with respect to x and y and substituting them into (21), we find after some transformations

$$\omega_{21}(y)=-\sqrt{2}\psi_4'(y+1),\ -\frac{1}{2}\le y\le 0. \tag{22}$$

Now substituting (20) into (3) and differentiating the obtained relation and then changing $2x-1$ by x, we come to the equation

$$\nu_1(x)=-\tau_1'(x)+\alpha_1(x),\ 0\le x\le 1, \tag{23}$$

where $\alpha_1(x)$ is a known function.

Then proceeding to the limit of $y\to 0$ in equation (17), we obtain

$$\tau_1''(x)-\nu_1(x)=\omega_1(0),\ 0\le x\le 1,$$

where $\omega_1(0)$ is for now unknown constant number. Substituting (23) into the latest equality, we obtain the equation

$$\tau_1''(x)+\tau_1'(x)=\alpha_1(x)+\omega_1(0),\ 0\le x\le 1.$$

Integrating this equation from 1 to x, we have

$$\tau_1'(x)+\tau_1(x)=\int\limits_1^x\alpha_1(t)\,dt+\omega_1(0)(x-1)+b,\ 0\le x\le 1,$$

where b is for now unknown constant number. Solving this equation under the conditions

$$\tau_1(1) = \psi_1(1) = \varphi(0), \tau_1'(1) = \varphi'(0) - \sqrt{2}\psi_4(1), \tau_1''(1) = \varphi''(0) - \sqrt{2}\psi_4'(1),$$

we obtain

$$\tau_1(x) = \int_1^x (1 - \exp(t-x))\,\alpha_1(t)\,dt + \omega_1(0)(x - 2 + \exp(1-x)) + \tag{24}$$

$$+b(1 - \exp(1-x)) + c\exp(1-x)$$

where

$$c = \varphi(0), b = \varphi'(0) + \varphi(0) - \sqrt{2}\psi_4(1),$$

$$\omega_1(0) = \varphi_1''(0) - \sqrt{2}\psi_4'(1) + \varphi'(0) - \sqrt{2}\psi_4(1) - \psi_1'(1).$$

Thus, we have found the function $\tau_1(x)$, and, consequently, the functions $\nu_1(x)$, $u_{21}(x, y)$.

Now we pass on to domain D_{22}. We write the solution of equation (19) ($i = 2$; $k = 2$), satisfying conditions (14), (15) as follows:

$$u_{22}(x, y) = \frac{\tau_3(y+x) + \tau_3(y-x)}{2} + \frac{1}{2}\int_{y-x}^{y+x} \nu_3(t)\,dt + \frac{1}{2}\int_0^x d\eta \int_{y-x+\eta}^{y+x-\eta} \omega_{22}(\xi)\,d\xi. \tag{25}$$

Differentiating (25) with respect to x and y and substituting them into (21), we find after some transformations

$$\omega_{22}(y) = -\sqrt{2}\psi_4'(y+1),\ -1 \le y \le -\frac{1}{2} \tag{26}$$

Now we apply from the condition $\left(\frac{\partial u_{22}}{\partial x} + \frac{\partial u_{22}}{\partial y}\right)\Big|_{y=-x} = \left(\frac{\partial u_{21}}{\partial x} + \frac{\partial u_{21}}{\partial y}\right)\Big|_{y=-x}$.

$$\tau_3'(0) + \nu_3(0) + \int_0^x \omega_{22}(-\eta)\,d\eta = \tau_1'(0) + \nu_1(0) - \int_0^{-x} \omega_{21}(\eta)\,d\eta.$$

Differentiating this equality and changing $-x$ by y in the obtained equality, and taking into account (22), we obtain $\omega_{22}(y) = -\sqrt{2}\psi_4'(y+1),\ -\frac{1}{2} \le y \le 0$. It is clear from this equality and (26) that

$$\omega_{22}(y) = -\sqrt{2}\psi_4'(y+1),\ -1 \le y \le 0.$$

Then, substituting (25) into (3) and differentiating the obtained equality, we have

$$\tau_3'(y) + \nu_3(y) = \delta_1(y),\ -1 \le y \le 0, \tag{27}$$

where $\delta_1(y)$ is a known function.

Now we apply the condition $u_{22}(x, -x) = u_{21}(x, -x)$, where $u_{21}(x, -x)$ is a known function.

Substituting (25) into this condition and differentiating the obtained equality and then changing $-2x$ by y, we obtain the following relation:

$$\tau_3'(y) - \nu_3(y) = \delta_2(y),\ -1 \le y \le 0, \tag{28}$$

here $\delta_2(y)$ is a known function.

From (27) and (28) we find

$$\nu_3(y) = \frac{1}{2}[\delta_1(y) - \delta_2(y)],\ -1 \le y \le 0. \tag{29}$$

Integrating

$$\tau_3'(y) = \frac{1}{2}\left[\delta_1(y) + \delta_2(y)\right], \ -1 \le y \le 0.$$

the latest equality from -1 to y, we obtain

$$\tau_3(y) = \frac{1}{2}\int_{-1}^{y} \left[\delta_1(\eta) + \delta_2(\eta)\right] d\eta + \tau_1(0), \ -1 \le y \le 0 \tag{30}$$

where it is assumed that $\tau_3(0) = \tau_1(0)$.

Thus, we have found the function $u_{22}(x, y)$, and, consequently, the function $u_2(x, y)$ completely.

Now we pass on to domain D_3. If in the equations (19) ($i = 2$; $k = 2$) and (19) ($i = 3$; $k = 1$) we proceed to the limit of $x \to 0$, then we obtain the equations $\mu_3(y) - \tau_3''(y) = \omega_{22}(y)$, $\mu_3(y) - \tau_3''(y) = \omega_{31}(y)$. It is clear from these relations that

$$\omega_{31}(y) = \omega_{22}(y) = -\sqrt{2}\psi_4'(y+1), \ -1 \le y \le 0.$$

Consequently, the function $u_{31}(x, y)$ becomes known. It is determined by the formula

$$u_{31}(x, y) = \frac{\tau_3(y+x) + \tau_3(y-x)}{2} + \frac{1}{2}\int_{y-x}^{y+x} \nu_3(t)\, dt + \frac{1}{2}\int_0^x d\eta \int_{y-x+\eta}^{y+x-\eta} \omega_{31}(\xi)\, d\xi. \tag{31}$$

Then we apply the following conditions: $\left.\left(\frac{\partial u_{32}}{\partial x} - \frac{\partial u_{32}}{\partial y}\right)\right|_{y=x} = \left.\left(\frac{\partial u_{31}}{\partial x} - \frac{\partial u_{31}}{\partial y}\right)\right|_{y=x}$.

We write the solution of equation (19) ($i = 3$; $k = 2$) satisfying conditions (9) (10) as follows:

$$u_{32}(x, y) = \frac{\tau_2(x+y) + \tau_2(x-y)}{2} + \frac{1}{2}\int_{x-y}^{x+y} \nu_2(t)\, dt - \int_0^y (y-\eta)\, \omega_{32}(\eta)\, d\eta. \tag{32}$$

Differentiating (31) and (32) with respect to x and y and substituting them into the condition $\left.\left(\frac{\partial u_{32}}{\partial x} - \frac{\partial u_{32}}{\partial y}\right)\right|_{y=x} = \left.\left(\frac{\partial u_{31}}{\partial x} - \frac{\partial u_{31}}{\partial y}\right)\right|_{y=x}$, we obtain

$$\tau_2'(0) - \nu_2(0) + \int_0^x \omega_{32}(\eta)\, d\eta = -\tau_3'(0) + \nu_3(0) + \int_0^x \omega_{31}(\eta)\, d\eta.$$

Differentiating this equation and changing x by y in the obtained equality, we find

$$\omega_{32}(y) = \omega_{31}(y) = -\sqrt{2}\psi_4'(y+1), \ -\frac{1}{2} \le y \le 0.$$

Now, taking into account the condition $u_{32}(x, x) = u_{31}(x, x)$, we have

$$\frac{\tau_2(2x) + \tau_2(0)}{2} + \frac{1}{2}\int_0^{2x} \nu_2(t)\, dt - \int_0^x (x-\eta)\, \omega_{32}(\eta)\, d\eta = \frac{\tau_3(2x) + \tau_3(0)}{2} +$$

$$+ \frac{1}{2}\int_0^{2x} \nu_3(t)\, dt + \frac{1}{2}\int_0^x d\eta \int_{\eta}^{2x-\eta} \omega_{31}(\xi)\, d\xi.$$

Differentiating this equality and changing $2x$ by x in the obtained equation, we obtain

$$\nu_2(x) = -\tau_2'(x) + \alpha_2(x), \ -1 \le x \le 0, \tag{33}$$

where $\alpha_2(x)$ is a known function.

Now we pass on to domain D_{42}. We write the solution of equation (19) ($i=4$; $k=2$) satisfying conditions (9), (10) as follows:

$$u_{42}(x,y)=\frac{\tau_2(x+y)+\tau_2(x-y)}{2}+\frac{1}{2}\int\limits_{x-y}^{x+y}\nu_2(t)\,dt-\int\limits_0^y(y-\eta)\,\omega_{42}(\eta)\,d\eta. \tag{34}$$

Differentiating (34) with respect to x and y and substituting them into (6), we obtain

$$\tau_2'(-1)-\nu_2(-1)+\int\limits_0^{x+1}\omega_{42}(\eta)\,d\eta=\sqrt{2}\psi_6(x),\ -1\le x\le-\frac{1}{2}.$$

Differentiating this equality and changing $x+1$ by y in the obtained equation, we find

$$\omega_{42}(y)=\sqrt{2}\psi_6'(y-1),\ 0\le y\le\frac{1}{2}. \tag{35}$$

Substituting (33) into (34), and after some simplifications, we obtain

$$u_{42}(x,y)=\tau_2(x-y)+\frac{1}{2}\int\limits_{x-y}^{x+y}\alpha_2(t)\,dt-\int\limits_0^y(y-\eta)\,\omega_{42}(\eta)\,d\eta \tag{36}$$

Now we pass on to domain D_{41}. We write the solution of equation (19) ($i=4$; $k=1$) satisfying conditions (11), (12) as follows:

$$u_{41}(x,y)=\frac{\tau_4(y+x)+\tau_4(y-x)}{2}+\frac{1}{2}\int\limits_{y-x}^{y+x}\nu_4(t)\,dt+\frac{1}{2}\int\limits_0^x d\eta\int\limits_{y-x+\eta}^{y+x-\eta}\omega_{41}(\xi)\,d\xi \tag{37}$$

Differentiating (36) and (37) with respect to x and y and substituting them into the condition $\left(\frac{\partial u_{41}}{\partial x}+\frac{\partial u_{41}}{\partial y}\right)\Big|_{y=-x}=\left(\frac{\partial u_{42}}{\partial x}+\frac{\partial u_{42}}{\partial y}\right)\Big|_{y=-x}$, we obtain

$$\tau_4'(0)+\nu_4(0)+\int\limits_0^x\omega_{41}(-\eta)\,d\eta=\tau_2'(0)+\nu_2(0)-\int\limits_0^{-x}\omega_{42}(\eta)\,d\eta,\ -\frac{1}{2}\le x\le 0.$$

Differentiating this equality and changing $-x$ by y in the obtained equation and taking into account (35), we find

$$\omega_{41}(y)=\omega_{42}(y)=\sqrt{2}\psi_6'(y-1),\ 0\le y\le\frac{1}{2}. \tag{38}$$

Substituting (37) into (6), we have

$$-\tau_4'(1)+\nu_4(1)+\int\limits_0^x\omega_{41}(1+\eta)\,d\eta=\sqrt{2}\psi_6(x),\ -\frac{1}{2}\le x\le 0.$$

Differentiating this equality and changing $1+x$ by y in the obtained equation, we find

$$\omega_{41}(y)=\sqrt{2}\psi_6'(y-1),\ \frac{1}{2}\le y\le 1.$$

It is clear from the latest equality and (38) that

$$\omega_{41}(y)=\sqrt{2}\psi_6'(y-1),\ 0\le y\le 1. \tag{39}$$

Substituting (37) into (4) and differentiating the obtained equality, then changing $2x+1$ by y in the obtained equation, we obtain

$$\tau_4'(y)+\nu_4(y)=\alpha_3(y),\ 0\leq y\leq 1, \tag{40}$$

where $\alpha_3(y)$ is a known function.

Taking into account the condition $u_{41}(x,-x)=u_{42}(x,-x)$, differentiating the obtained equation and changing $-2x$ by y, we obtain

$$\tau_4'(y)-\nu_4(y)=-2\tau_2'(-y)+\alpha_4(y),\ 0\leq y\leq 1, \tag{41}$$

where $\alpha_4(y)$ is a known function.

From (40) and (41) we find the functions $\tau_4'(y)$ and $\nu_4(y)$:

$$\tau_4'(y)=-\tau_2'(-y)+\frac{1}{2}\left[\alpha_3(y)+\alpha_4(y)\right],\ 0\leq y\leq 1, \tag{42}$$

$$\nu_4(y)=\tau_2'(-y)+\frac{1}{2}\left[\alpha_3(y)-\alpha_4(y)\right],\ 0\leq y\leq 1. \tag{43}$$

Integrating (42) from 1 to y, we find $\tau_4(y)$:

$$\tau_4(y)=\tau_2(-y)+\frac{1}{2}\int\limits_1^y\left[\alpha_3(t)+\alpha_4(t)\right]dt+\psi_2(0)-\tau_2(-1),\ 0\leq y\leq 1.$$

Now we pass on to domain D_1. Proceeding to the limit of $x\to 0$ in equations (17) and (19) $(i=4;\ k=1)$, we obtain the following relations: $\mu_4(y)-\tau_4'(y)=\omega_1(y),\mu_4(y)-\tau_4''(y)=\omega_{41}(y)$.

Eliminating the function $\mu_4(y)$ from these relations, we obtain $\omega_1(y)=\tau_4''(y)-\tau_4'(y)+\omega_{41}(y)$.

Differentiating (42), we have $\tau_4''(y)=\tau_2''(-y)+\frac{1}{2}\left[\alpha'_3(y)+\alpha'_4(y)\right]$.

Taking into account the latest equality and (42), the function $\omega_1(y)$ can be written as follows:

$$\omega_1(y)=\tau_2''(-y)+\tau_2'(-y)+\gamma_1(y), \tag{44}$$

where $\gamma_1(y)$ is a known function.

Then we write the solution of equation (17) satisfying conditions (2), (7), (11) as follows:

$$u_1(x,y)=\frac{1}{2\sqrt{\pi}}\left[\int\limits_0^y\tau_4(\eta)G_\xi(x,y;0,\eta)d\eta-\int\limits_0^y\varphi_1(\eta)G_\xi(x,y;1,\eta)d\eta+\right. \tag{45}$$
$$\left.+\int\limits_0^1\tau_1(\xi)G(x,y;\xi,0)d\xi-\int\limits_0^y\omega_1(\eta)d\eta\int\limits_0^1 G(x,y;\xi,\eta)d\xi\right].$$

Differentiating (45) with respect to x and proceeding to the limit of $x\to 0$ in the obtained equality, owing to (42) and (43) taking into account the equality $\tau_2'(-y)=\tau_1'(0)-\int\limits_0^y\tau''_2(-\eta)d\eta$, we obtain Abel equation relatively $\tau_2''(-y)$. Using this Abel equation and after some estimations, we obtain the equation

$$\tau_2''(-y)+\int\limits_0^y K(y,\eta)\tau''_2(-\eta)d\eta=g(y), \tag{46}$$

where $K(y,\eta)$, $g(y)$ are known functions.

Equation (46) is an integral Volterra equation of the second kind. The kernel $K(y,\eta)$ has a weak singularity and the right part $g(y)$ is continuous within $0<y<1$. Solving equation (46) in the class of continuous functions within $0<y<1$, we find the function $\tau_2''(-y)$, and, consequently, the functions $\tau_2'(-y)$, $\tau_2(-y)$, $\tau_4(y)$, $\nu_4(y)$, $\omega_1(y)$, $u_{41}(x,y)$, $u_{42}(x,y)$ and $u_1(x,y)$.

Thus, **Problem 1** has been completely solved.

Conclusions

The paper [1] considers a series of boundary value problems for more general third order parabolic-hyperbolic equations in a domain with one characteristic line of type change. The paper [2] discusses boundary value problems for one class of third order parabolic-hyperbolic equations in a concave hexagonal domain.

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