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MATHEMATICS

MSC 35M10

ON A PROBLEM FOR THE LOADED MIXED TYPE EQUATION WITH FRACTIONAL DERIVATIVE

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An existence and an uniqueness of solution of local boundary value problem with discontinuous matching condition for the loaded parabolic-hyperbolic equation involving the Caputo fractional derivative and Riemann-Liouville integrals have been investigated in this research work. The uniqueness of solution is proved by the method of integral energy and the existence is proved by the method of integral equations.

Key words: Caputo derivative, boundary value problem, Riemann-Liouville integral, energy integral

Introduction and formulation of a problem

BVPs for the mixed type equations involving the Caputo and the Riemann-Liouville fractional differential operators were investigated in works [1],[2].

It was given the most general definition of a “loaded equations” and various loaded equations are classified in detail by A.M.Nakhushev [3]. After this work very interesting results on the theory of boundary value problems for the loaded equations parabolic, parabolic-hyperbolic and elliptic-hyperbolic types were published, for example, see [4]-[6].

In this direction was investigated, some local and non-local problems for the loaded elliptic-hyperbolic type equations of the second and third order in double-connected domains (see [7]-[10]).

BVPs for the loaded mixed type equations with fractional derivative have not been investigated yet.

In the given work, we consider the equation:

$$\begin{cases} u_{xx} - D_{oy}^{\alpha} u + p(x,y) \sum\limits_{k=1}^{n} D_{x1}^{-\beta_k} u(x,0) = 0, \quad \text{at } y>0, \\ u_{xx} - u_{yy} + q(x+y) \sum\limits_{k=1}^{n} \int\limits_{x+y}^{1} (t-x-y)^{\gamma_k - 1} u(t,0)dt = 0, \ \text{ at } y<0, \end{cases} \tag{1}$$

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with operation [11]

$$ {}_CD_{0y}^{\alpha} f = \frac{1}{\Gamma(1-\alpha)} \int\limits_0^y (y-t)^{-\alpha} f'(t)dt,\ D_{xa}^{-\beta} f(x) = \frac{1}{\Gamma(\beta)} \int\limits_x^a (t-x)^{\beta-1} f(t)dt, \tag{2} $$

where

$$ 0 < \alpha,\ \beta_k, \gamma_k < 1. \tag{3} $$

Let's, Ω is domain, bounded with segments: $A_1A_2 = \{(x,y):\ x = 1,\ 0 < y < h\}$, $B_1B_2 = \{(x,y):\ x = 0,\ 0 < y < h\}$,$B_2A_2 = \{(x,y):\ y = h,\ 0 < x < 1\}$ at the $y > 0$, and characteristics: $A_1C: x - y = 1$; $B_1C: x + y = 0$ of the equation (1) at $y < 0$, where $A_1(1;0)$,$A_2(1;h)$,$B_1(0;0)$, $B_2(0;h)$, $C\left(\frac{1}{2};-\frac{1}{2}\right)$.

Let's enter designations: $\Omega^+ = \Omega \cap (y > 0)$, $\Omega^- = \Omega \cap (y < 0)$, $I_1 = \left\{x : \frac{1}{2} < x < 1\right\}$, $I_2 = \{y : 0 < y < h\}$. In the domain of Ω the following problem is investigated.

Task I. *To find a solution $u(x,y)$ of the equation (1) from the following class of functions:*

$$ W = \left\{u(x,y):\ u(x,y) \in C(\bar{\Omega}) \cap C^2(\Omega^-),\ u_{xx} \in C\left(\Omega^+\right), {}_CD_{oy}^{\alpha} u \in C\left(\Omega^+\right)\right\}, $$

satisfies boundary conditions:

$$ u(x,y)\Big|_{A_1A_2} = \varphi(y), 0 \le y \le h, \tag{4} $$

$$ u(x,y)\Big|_{B_1B_2} = \psi(y), 0 \le y \le h, \tag{5} $$

$$ u(x,y)\Big|_{A_1C} = \omega(x),\ x \in \overline{I_1} \tag{6} $$

and gluing condition:

$$ \lim_{y\to+0} y^{1-\alpha} u_y(x,y) = \lambda u_y(x,-0),\ (x,0) \in A_1B_1, \tag{7} $$

where $\varphi(y)$, $\psi(y)$, $\omega(x)$ are given functions, $\lambda = const$ $(\lambda \neq 0)$, besides: $\omega(1) = \varphi(0)$.

The Uniqueness of solution of the Problem I

It's known that, the equation (1) at $y \le 0$ on the characteristics coordinate $\xi = x + y$ and $\eta = x - y$ totally looks like:

$$ u_{\xi\eta} = \frac{q(\xi)}{4} \int\limits_{\xi}^{1} (t-\xi)^{\gamma-1} u(t,0)dt. \tag{8} $$

Let's enter designations:

$$ u(x,0) = \tau(x),\ 0 \le x \le 1,\ u_y(x,-0) = \nu^-(x),\ 0 < x < 1, $$

$$ \lim_{y\to+0} y^{1-\alpha} u_y(x,y) = \nu^+(x),\ 0 < x < 1. $$

Known, that solution of the Cauchy problem for the equation (1) in the domain of Ω^- can be represented as follows:

$$ u(x,y) = \frac{\tau(x+y)+\tau(x-y)}{2} - \frac{1}{2}\int\limits_{x+y}^{x-y} \nu^-(t)dt + \frac{1}{4}\int\limits_{x+y}^{x-y} q(\xi)d\xi \int\limits_{\xi}^{x-y} d\eta \int\limits_{\xi}^{1} (t-\xi)^{\gamma-1}\tau(t)dt. \tag{9} $$

After using condition (6) and taking (3) into account from (9) we will get:

$$\nu^{-}(x) = \frac{1-x}{2}\Gamma(\gamma)q(x)\sum_{k=1}^{n} D_{x\,1}^{-\gamma_k}\tau(x) - \tau'(x) + \omega'\left(\frac{x+1}{2}\right). \tag{10}$$

Considering designations and gluing condition (7) we have

$$\nu^{+}(x) = \lambda\nu^{-}(x). \tag{11}$$

Further from the Eq. (1) at $y \to +0$ owing to account (2), (11) and

$$\lim_{y\to 0} D_{0y}^{\alpha-1} f(y) = \Gamma(\alpha)\lim_{y\to 0} y^{1-\alpha} f(y),$$

we get:

$$\tau''(x) - \lambda\Gamma(\alpha)\nu^{-}(x) + p(x,0)\sum_{k=1}^{n} D_{x1}^{-\beta_k}\tau(x) = 0. \tag{12}$$

Theorem 1. *If satisfies conditions*

$$p(0,0) \le 0,\; p'(x,0) \le 0,\; \lambda q(0) \ge 0,\; \lambda((1-x)q(x))' \ge 0, \tag{13}$$

*then, the solution $u(x,y)$ of the **Problem I** is unique.*

Proof. Known, that, if homogeneous problem has only trivial solution, then we can state that original problem has unique solution. For this aim we assume that the **Problem I** has two solutions, then denoting difference of these as $u(x,y)$ we will get appropriate homogenous problem.

Equation (12) we multiply to $\tau(x)$ and integrated from 0 to 1:

$$\int_0^1 \tau''(x)\tau(x)dx - \lambda\Gamma(\alpha)\int_0^1 \tau(x)\nu^{-}(x)dx + \int_0^1 \tau(x)p(x,0)\sum_{k=1}^{n} D_{x1}^{-\beta_k}\tau(x)dx = 0. \tag{14}$$

We will investigate the integral

$$I = \lambda\Gamma(\alpha)\int_0^1 \tau(x)\nu^{-}(x)dx - \int_0^1 \tau(x)p(x,0)\sum_{k=1}^{n} D_{x1}^{-\beta_k}\tau(x)dx.$$

Taking (10) into account $\omega(x) = 0$ we get:

$$\begin{gathered} I = \frac{\lambda\Gamma(\alpha)}{2}\sum_{r=1}^{n}\Gamma(\gamma_k)\int_0^1 \tau(x)\,(1-x)\,q(x)D_{x\,1}^{-\gamma_k}\tau(x)dx - \lambda\Gamma(\alpha)\int_0^1 \tau(x)\tau'(x)dx - \\ - \int_0^1 \tau(x)p(x,0)\sum_{k=1}^{n} D_{x1}^{-\beta_k}\tau(x)dx = \\ = \frac{\lambda\Gamma(\alpha)}{2}\int_0^1 q(x)\tau(x)\,(1-x)\,dx\int_x^1 \sum_{k=1}^{n}(t-x)^{\gamma_k-1}\tau(t)dt - \frac{\lambda\Gamma(\alpha)}{2}\int_0^1 d\left(\tau^2(x)\right) - \\ - \int_0^1 \tau(x)p(x,0)dx\int_x^1 \sum_{k=1}^{n}\frac{(t-x)^{\beta_k-1}}{\Gamma(\beta_k)}\tau(t)dt. \end{gathered} \tag{15}$$

Considering $\tau(1)=0, \tau(0)=0$ (which deduced from the conditions (4),(5) in homogeneous case) and on a base of the formula [12]:

$$|x-t|^{-\delta}=\frac{1}{\Gamma(\gamma)\cos\left(\frac{\pi\delta}{2}\right)}\int\limits_0^\infty z^{\delta-1}\cos[z(x-t)]\,dz,\ 0<\delta<1.$$

After some simplifications from (15) we will get:

$$I=\frac{\lambda\Gamma(\alpha)q(0)}{2\pi}\int\limits_0^\infty\sum_{k=1}^n\Gamma(\gamma_k)z^{-\gamma_k}\cos\frac{\pi\gamma_k}{2}\left[\left(\int\limits_0^1\tau(t)\cos zt\,dt\right)^2+\left(\int\limits_0^1\tau(t)\sin zt\,dt\right)^2\right]dz+$$

$$+\frac{\lambda\Gamma(\alpha)}{2\pi}\int\limits_0^\infty\sum_{k=1}^n\Gamma(\gamma_k)z^{-\gamma_k}\cos\frac{\pi\gamma_k}{2}dz\int\limits_0^1\frac{\partial}{\partial x}[(1-x)q(x)]\left[\left(\int\limits_x^1\tau(t)\cos zt\,dt\right)^2+\left(\int\limits_x^1\tau(t)sinzt\,dt\right)^2\right]dx-$$

$$-\frac{p(0,0)}{\pi}\int\limits_0^\infty\sum_{k=1}^n z^{-\beta_k}\cos\frac{\pi\beta_k}{2}\left[\left(\int\limits_0^1\tau(t)\cos zt\,dt\right)^2+\left(\int\limits_0^1\tau(t)\sin zt\,dt\right)^2\right]dz- \tag{16}$$

$$-\frac{2}{\pi}\int\limits_0^\infty\sum_{k=1}^n z^{-\beta_k}\cos\frac{\pi\beta_k}{2}dz\int\limits_0^1\frac{\partial}{\partial x}[p(x,0)]\left[\left(\int\limits_x^1\tau(t)\cos zt\,dt\right)^2+\left(\int\limits_x^1\tau(t)sinzt\,dt\right)^2\right]dx.$$

Thus, owing to (13) from (16) it is concluded, that $\tau(x)\equiv 0$. Hence, based on the solution of the first boundary problem for the Eq. (1) [2],[11] owing to account (4) and (5) we will get $u(x,y)\equiv 0$ in $\overline{\Omega}^+$. Further, from functional relations (10), taking into account $\tau(x)\equiv 0$we get that $\nu^-(x)\equiv 0$. Consequently, based on the solution (9) we obtain $u(x,y)\equiv 0$ in closed domain $\overline{\Omega}^-$. □

The existence of solution of the Problem I

Theorem 2. *If satisfies conditions (13) and*

$$\varphi(y),\ \psi(y)\in C\left(\overline{I_2}\right)\cap C^1(I_2),\omega(x)\in C^1\left(\overline{I_1}\right)\cap C^2(I_1), \tag{17}$$

$$p(x,0),q(x)\in C\left(\overline{A_1B_1}\right)\cap C^2(A_1B_1). \tag{18}$$

then the solution of the investigating problem is exist.

Proof. Taking (10) into account from Eq.(12) we will obtain:

$$\tau''(x)-\frac{\lambda}{2}\Gamma(\alpha)(1-x)q(x)\sum_{k=1}^n\Gamma(\gamma_k)D_{x1}^{-\gamma_k}\tau(x)+\lambda\Gamma(\alpha)\tau'(x)=$$

$$=\lambda\Gamma(\alpha)\omega'\left(\frac{x+1}{2}\right)-p(x,0)\sum_{k=1}^n D_{x1}^{-\beta_k}\tau(x).$$

From here

$$\tau''(x)+\lambda\Gamma(\alpha)\tau'(x)=f(x), \tag{19}$$

where

$$f(x)=\frac{\lambda}{2}\Gamma(\alpha)\left[(1-x)q(x)\sum_{k=1}^n\Gamma(\gamma_k)D_{x1}^{-\gamma_k}\tau(x)+2\omega'\left(\frac{x+1}{2}\right)\right]-p(x,0)\sum_{k=1}^n D_{x1}^{-\beta_k}\tau(x). \tag{20}$$

Solution of the equation (19) together with conditions

$$\tau(0)=\psi(0),\ \tau(1)=\varphi(0), \tag{21}$$

has a form

$$\tau(x)=(1-x)\psi(0)+x\varphi(0)+\int\limits_0^1 G(x,t)f_1(t)dt. \tag{22}$$

where

$$f_1(x)=f(x)+\lambda\left(\psi(0)-\varphi(0)\right)\Gamma(\alpha),$$

$$G(x,t)=\begin{cases}\dfrac{\left(e^{\lambda\Gamma(\alpha)x}-1\right)\left(e^{\lambda\Gamma(\alpha)t}-e^{\lambda\Gamma(\alpha)}\right)}{e^{\lambda\Gamma(\alpha)x}\left(e^{\lambda\Gamma(\alpha)}-1\right)\lambda\Gamma(\alpha)}, & 0\le t\le x,\\ \dfrac{\left(e^{\lambda\Gamma(\alpha)t}-1\right)\left(e^{\lambda\Gamma(\alpha)x}-e^{\lambda\Gamma(\alpha)}\right)}{e^{\lambda\Gamma(\alpha)x}\left(e^{\lambda\Gamma(\alpha)}-1\right)\lambda\Gamma(\alpha)}, & t\le x\le 1.\end{cases} \tag{23}$$

Is the Green's function of the problem (19), (21). Further, considering (20) and using (3) from (22) we will get:

$$\begin{aligned}\tau(x)=&\frac{1}{2}\int\limits_0^x\tau(t)dt\int\limits_0^t(1-s)\sum_{k=1}^n(t-s)^{\gamma_k-1}K_1(x,s)q(s)ds-\\ &-\frac{1}{\lambda\Gamma(\alpha)}\int\limits_0^x\tau(t)dt\int\limits_0^t\sum_{k=1}^n\frac{(t-s)^{\beta_k-1}p(s,0)}{2\Gamma(\beta_k)}K_1(x,s)ds+\\ &+\frac{1}{2}\int\limits_x^1\tau(t)dt\int\limits_0^x(1-s)\sum_{k=1}^n(t-s)^{\gamma_k-1}K_1(x,s)q(s)ds-\\ &-\frac{1}{\lambda\Gamma(\alpha)}\int\limits_x^1\tau(t)dt\int\limits_0^x\sum_{k=1}^2\frac{(t-s)^{\beta_k-1}p(s,0)}{2\Gamma(\beta_k)}K_1(x,s)ds+\\ &+\frac{1}{2}\int\limits_x^1\tau(t)dt\int\limits_t^x(1-s)q(s)\sum_{k=1}^n(t-s)^{\gamma_k-1}K_2(x,s)ds-\\ &-\frac{1}{\lambda\Gamma(\alpha)}\int\limits_x^1\tau(t)dt\int\limits_x^t\sum_{k=1}^n\frac{(t-s)^{\beta_k-1}p(s,0)}{2\Gamma(\beta_k)}K_2(x,s)ds+F(x),\end{aligned} \tag{24}$$

where

$$F(x)=\lambda\Gamma(\alpha)\int\limits_0^1 G(x,t)\left[\omega'\left(\frac{t+x}{2}\right)+(\psi(0)-\varphi(0))\right]dt+(1-x)\psi(0)+x\varphi(0),$$

$$K_1(x,t)=\frac{\left(e^{\lambda\Gamma(\alpha)t}-1\right)\left(e^{\lambda\Gamma(\alpha)x}-e^{\lambda\Gamma(\alpha)}\right)}{e^{\lambda\Gamma(\alpha)x}\left(e^{\lambda\Gamma(\alpha)}-1\right)},K_2(x,t)=\frac{\left(e^{\lambda\Gamma(\alpha)x}-1\right)\left(e^{\lambda\Gamma(\alpha)t}-e^{\lambda\Gamma(\alpha)}\right)}{e^{\lambda\Gamma(\alpha)x}\left(e^{\lambda\Gamma(\alpha)}-1\right)}. \tag{25}$$

After some simplifications (24) we will rewrite on the form:

$$\tau(x)=\int\limits_0^1 K(x,t)\tau(t)dt+F(x). \tag{26}$$

Here

$$K(x,t)=\begin{cases}\int\limits_0^t K_1(x,s)\left[\frac{1}{2}(1-s)\sum\limits_{k=1}^n(t-s)^{\gamma_k-1}q(s)-\sum\limits_{k=1}^n\frac{(t-s)^{\beta_k-1}p(s,0)}{\lambda\Gamma(\alpha)\Gamma(\beta_k)}\right]ds,\ 0\le t\le x,\\ \int\limits_0^x K_1(x,s)\left[\frac{1}{2}(1-s)\sum\limits_{k=1}^n(t-s)^{\gamma_k-1}q(s)-\sum\limits_{k=1}^n\frac{(t-s)^{\beta_k-1}p(s,0)}{\lambda\Gamma(\alpha)\Gamma(\beta_k)}\right]ds+\\ +\int\limits_x^t K_2(x,s)\left[\frac{1}{2}(1-s)\sum\limits_{k=1}^n(t-s)^{\gamma_k-1}q(s)-\sum\limits_{k=1}^n\frac{(t-s)^{\beta_k-1}p(s,0)}{\lambda\Gamma(\alpha)\Gamma(\beta_k)}\right]ds,\ x\le t\le 1.\end{cases}\quad(27)$$

Owing to class (17), (18) of the given functions and after some evaluations from (25) and (27) we will conclude, that, $|K(x,t)|\le const$, $|F(x)|\le const$.

Since kernel $K(x,t)$ is continuous and function in right-side $F(x)$ is continuously differentiable, solution of integral equation (27) we can write via resolvent-kernel:

$$\tau(x)=F(x)-\int\limits_0^1\Re(x,t)F(t)dt,\quad(28)$$

where $\Re(x,t)$ is the resolvent-kernel of $K(x,t)$.

Unknown functions $\nu^-(x)$ and $\nu^+(x)$ we will found accordingly from (10) and (11):

$$\nu^-(x)=\frac{x-1}{2}q(x)\int\limits_x^1(t-x)^{1-\gamma}dt\int\limits_0^1\Re(t,s)F(s)ds+\frac{1-x}{2}q(x)\int\limits_x^1(t-x)^{1-\gamma}F(t)dt-$$

$$-F'(x)+\int\limits_0^1\frac{\partial\Re(x,t)}{\partial x}F(t)dt+\omega'\left(\frac{x+1}{2}\right),$$

and $\nu^+(x)=\lambda\nu^-(x)$.

Solution of the **Problem I** in the domain Ω^+ we write as follows [13]:

$$u(x,y)=\int\limits_0^y G_\xi(x,y,0,\eta)\psi(\eta)d\eta-\int\limits_0^y G_\xi(x,y,1,\eta)\varphi(\eta)d\eta+\int\limits_0^1 G_0(x-\xi,y)\tau(\xi)d\xi-$$

$$-\int\limits_0^y\int\limits_0^1 G(x,y,0,\eta)p(\xi)d\xi d\eta\int\limits_\xi^1(t-\xi)^{\beta-1}\tau(t)dt.$$

Here

$$G_0(x-\xi,y)=\frac{1}{\Gamma(1-\alpha)}\int\limits_0^y\eta^{-\alpha}G(x,y,\xi,\eta)d\eta,$$

$$G(x,y,\xi,\eta)=\frac{(y-\eta)^{\alpha/2-1}}{2}\sum_{n=-\infty}^{\infty}\left[e^{1,\alpha/2}_{1,\alpha/2}\left(-\frac{|x-\xi+2n|}{(y-\eta)^{\alpha/2}}\right)-e^{1,\alpha/2}_{1,\alpha/2}\left(-\frac{|x+\xi+2n|}{(y-\eta)^{\alpha/2}}\right)\right].$$

Is the Green's function of the first boundary problem Eq. (1) in the domain Ω^+ with the Riemanne-Liouville fractional differential operator instead of the Caputo ones [11],

$$e^{1,\delta}_{1,\delta}(z)=\sum_{n=0}^{\infty}\frac{z^n}{n!\Gamma(\delta-\delta n)},$$

is the Wright type function [13].

Solution of the **Problem I** in the domain Ω^- will be found by the formulate (9). Hence, the **Theorem 2** is proved. □

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