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TEACHING MATERIALS

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SOLUTIONS OF THE PROBLEMS OF MATHEMATICAL COMPETITION «VITUS BERING – 2015»

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The paper considers solutions of the problems of Mathematical Competition «Vitus Bering – 2015» for senior pupils. It took place at Kamchatka State University in November 2015.

Key words: problems for mathematical competition for senior pupils.

Introduction

The proposed paper does not keep within the limits of the main theme of the Journal "Bulletin of KRAESC. Phys.-Math. Sciences"which publishes the results of original investigations of physical-mathematical profile. It is devoted to the Mathematical Competition «Vitus Bering– 2015» for senior pupils which was held at Vitus Bering Kamchatka State University in November 2015. We hope that such competitions at the Faculty of Physics and Mathematics will traditionally take place every year.

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Thematic, in particular mathematical, competitions are known to be a good method to make science popular, from one side, and to choose talented pupils for higher schools, from the other. Mathematical competitions for pupils were initiated in the 30's of the last century and became very popular. From 60's they were traditional for many towns of the Soviet Union. Such competitions take place at the levels of towns, regions and the country. Many educational institutions organize such activities. The system of preparation of pupils to solve very specific problems has become very extensive. Involvement of the best young mathematicians and physicists of the county in the competition movement is so high that it is possible to enter some leading universities of the country, for example the famous «Moscow Institute of Physics and Technology», only by the results of competitions.

The changeover of the latest years to the Common State Examination (CSE) when entering a university has promoted higher schools to organize such competitions. It is quite reasonable, since higher schools want not only to admit students on the basis of formal CSE grades but to make a meaningful selection within admission regulations. Winners of competitions of higher educational institutions get additional grades to CSE grades when entering a higher school.

The Mathematical Competition «Vitus Bering– 2015» had one round and included 6 problems of diverse complexity for the senior pupils of 9-11 forms. The participants of the competition had 3 hours to complete the tasks. The organizers used some standard problems from the workbook [1] for the competition tasks.

Further are some tasks of the Competition and the solutions.

Competition tasks

1) (5 grades) Find b^b, when the relations $a^b = 81$, $b^c = 2$, $a^c = 3$ hold.

2) (5 grades) A freight train catches up with a passenger train on a by-pass route. The freight train speed is 105 km/h, the passenger train speed is 85 km/h. The length of the passenger train is 800 meters. Find the length of the freight train if it overtook the passenger train in 252 seconds. Please give the answer in meters.

3) (5 grades) Solve the equation

$$(1-(2-(3-(...(2013-(2014-(2015-x)))...)))) = 1000.$$

4) (10 grades) There is an isosceles triangle MNK with the base MK. Point A belong to the base MK. Points B and C are marked on the lateral sides MN and NK so that $MN||AC$ and $NK||AB$. Find the relation of the triangle ABC and MNK areas if $AC : NC = 4 : 7$.

5) (15 grades) Find the area of the figure limited on the coordinate plane by the following system

$$\begin{cases} \sqrt{2-x}+4\cdot x \geq 0, \\ -2+\sqrt{x} \leq y \leq 1+\sqrt{x} \end{cases}$$

6) (10 grades) During a term, a teacher gave his pupils $<< 1 >>$, $<< 2 >>$, $<< 3 >>$, $<< 4 >>$, $<< 5 >>$. Petya's mean arithmetic mark is $<< 3{,}5 >>$. Petya asked the

teacher to change one mark $<< 4 >>$ by two marks $<< 3 >>$ and $<< 5 >>$. Prove that Petya's average mark increased.

Solutions

1) From the problem situation we have that the values $a > 0,\ b > 0,\ c > 0$.

   We consider $a^b = 81$, then $\log_3 a^b = 4$, from which $b \cdot \log_3 a = 4$. Analogously, it follows from $a^c = 3$ that $c \cdot \log_3 a = 1$. Find the relation $\dfrac{b \cdot \log_3 a}{c \cdot \log_3 a} = \dfrac{4}{1}$, from which $b = 4c$.

   Then, applying the condition $b^c = 2$, we obtain the sought expression $b^b = b^{4c} = (b^c)^4 = (2)^4 = 16$.

   Answer: 16.

2) The approach speed of the freight train with the passenger one is $(105 - 85) = 20$ km/h. Then with the velocity of 20 km/h the train will travel a distance equal to the sum of the lengths of the passenger and the freight trains in 252 seconds.

   We transfer all the data into similar units (meters and seconds): 20 km/h = $\frac{50}{9}$ m/s.

   Assume l is the length of the freight train, then

$$\begin{aligned} l + 800 &= \tfrac{50}{9} \cdot 252, \\ l + 800 &= 1400. \end{aligned}$$

   From which $l = 600$ m.

   Answer: 600 m.

3) We expand the brackets and in the left side of the equation obtain an alternating sequence of summands

$$1 - 2 + 3 - \ldots + 2013 - 2014 + 2015 - x = 1000.$$

   We group the summands in the following manner

$$\underbrace{(1 + 2015)}_{2016} + \underbrace{(-2 - 2014)}_{-2016} + \underbrace{(3 + 2013)}_{2016} + \ldots + \underbrace{(-1006 - 1010)}_{-2016} + \underbrace{(1007 + 1009)}_{2016} - 1008 - x = 1000$$

   and see that there will be 504 expressions equal to 2016 and 503 expressions equal to (–2016).

   Thus, our equation can be modified into

$$2016 - 1008 - x = 1000.$$

   From which $x = 8$.

   Answer: $x = 8$.

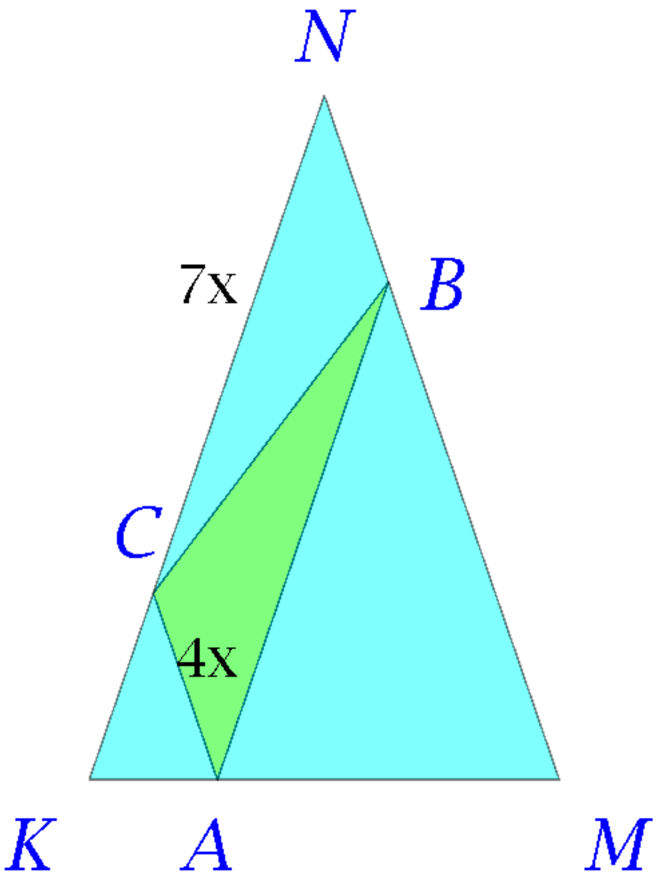

Fig. 1. Figure for task 4

4) We draw the given figure. It is shown in Fig. 1.

   $ACNB$ is a parallelogram since its sides are parallel according to the situation.

   Then $AC : AB = 4 : 7$. Denote $\alpha = \angle CNB = \angle CAB$.

   We find the area of the triangle ABC: $S_{ABC} = 1/2 \cdot AC \cdot AB \cdot \sin\alpha = 1/2 \cdot 4x \cdot 7x \cdot \sin\alpha = 14\ x^2 \sin\alpha$.

   The triangles KCA and KNM are similar with respect to three angles, consequently, $KC = CA = 4x$ and $KN = KC + CN = 4x + 7x = 11x$.

   As long as the triangle is isosceles then $S_{MNK} = 1/2 \cdot KN^2 \cdot \sin\alpha = 1/2 \cdot (11x)^2 \sin\alpha = 60,5\ x^2 \sin\alpha$.

   Then the desired relation is equal to $\dfrac{S_{ABC}}{S_{MNK}} = \dfrac{14\ x^2 \sin\alpha}{60,5\ x^2 \sin\alpha} = \dfrac{28}{121}$

   Answer: $\dfrac{28}{121}$.

5) We draw the given region denoted by D on the coordinate plane.

   The first inequality results in the restriction on the variable x: $2 - x \geq 0$, from which $x \in (-\infty, 2]$, and from the second inequality $x \in [0, \infty)$. Therefore, $x \in [0, 2]$.

   The first inequality $\sqrt{2-x} + 4x \geq 0$ fulfills at all $x \in [0, 2]$, that will be potted by a band parallel to Oy on the coordinate plane. The second inequality determins the domain between the graphs of the functions $y_1 = \sqrt{x}+1$ and $y_2 = \sqrt{x}-2$, where the graph of the function y_2 can be obtained for the graph of the function y_1 by a parallel shift along the axis Oy by three units down (Fig. 2).

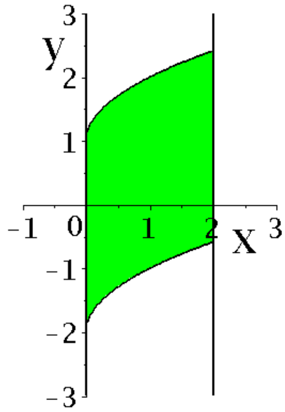

Fig. 2. Figure for the task 5

We note that the domain D is equivalent to a rectangle with the sides 2 and 3. In this regard, it is enough to slit the region D, for example, along the line $y = 1$, and to move the upper part along the axis Oy by three units down.

Thus, the area of the domain D will be equal to $S = 2 \cdot 3 = 6$.

Answer: 6.

6) We assume that the total number of Petya's marks is n, and the sum of all the marks without one «4» is S.

Then the average mark will be $\frac{S+4}{n} = 3,5$, from which $S = 3,5n-4$.

Substitution of one «4» by two marks, «3» and «5», increases the number of marks by one and results in a new average mark $\frac{S+3+5}{n+1} = \frac{S+8}{n+1}$.

We consider the difference of the new and the previous marks

$$\frac{S+8}{n+1} - \frac{S+4}{n} = \frac{4n-4-S}{n(n+1)} = \frac{4n-4-3,5n+4}{n(n+1)} \frac{0,5n}{n(n+1)}.$$

For all natural n, the difference $\frac{0,5n}{n(n+1)} > 0$. Consequently, Petya's average mark increased after the change.

Conclusion

The Mathematical Competition which took place in November 2015 at the Faculty of Physics and Mathematics of Vitus Bering Kamchatka State University had mainly career guidance aims and was oriented to potential students. Winners and participants had additional grades to CSE in Mathematics if they applied to this University.

The authors hope that the problems and their solutions will allow the senior pupils to understand the level and subjects of competition tasks and the teachers of Mathematics to prepare the pupils for Mathematical competitions.

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