

## MATHEMATICAL MODELING

MSC 97A80

### REVERSALS IN A GEODYNAMO MODEL DRIVEN BY 6-CELL CONVECTION

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A large-scale geodynamo model based on indirect data of density inhomogeneities in the Earth's liquid core is described. Convective structure is correlated with a spherical harmonic  $Y_4^2$ , which defines the basic poloidal component of the velocity. The Coriolis drift of this mode determines the toroidal component of the velocity. Thus, a 6-cell convective structure is formed. The model takes into account the inverse effect of the magnetic field on convection. It was ascertained that the model realizes a stable regime of field generation. The convection velocity and the dipole component of the magnetic field are close to the observed ones.

*Key words: geodynamo, geomagnetic field reversals, convection*

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## Introduction

The process of formation of magnetic fields of planets and stars is well explained by the theory of hydromagnetic dynamo. The present models of convection in planet liquid cores and star convective zones admit flow structures capable of generating magnetic fields similar to real ones [1, 2, 3].

Modern computation systems do not allow us to make direct numerical simulation of nonstationary three-dimensional problems of planetary dynamo on the time scales of field lifetime order ( $\sim 10^9$  years) on computational meshes. Thus, the present numerical models either represent MHD-flows with good spatial resolution on relatively short (according to geological scales) time intervals ( $\sim 10^4$  year), or reproduce long-term evolution of the largest-scale spatial structures. In the models of the first type, the spatial structure of flows and field is calculated, and for the models of the second type

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this structure should be defined initially. In addition, a question arises what the real large-scale structure of convection is.

Indirect data on this structure for the Earth's core may be obtained from the data on spatial inhomogeneity of the liquid core density. The paper [4] analyzes the results of investigations of splitting functions of the Earth's self oscillations after the strongest Chile earthquake. Distribution of these functions characterizes the inhomogeneity of the Earth's core at different depths. Having analyzed the functions, the author [4] made a conclusion that there is a 12-zone staggered inhomogeneity in the liquid core. It may be naturally described in the first approximation by the spherical harmonic  $Y_4^2$ . He also proposed a hypothesis that there is a convection with 6 regions of substance ascending and 6 regions of substance descending in the core, i.e. a 6-cell convection. In such a convective structure, in each polar region, 2 cells are located, and there are 2 more cells in the equatorial region.

We also note that a 6-cell structure is the result of direct numerical simulation of convection in the Earth's core at some values of core parameters [5]. In these calculations, 3 cells are located in each hemisphere. Such a structure is described by the spherical harmonic  $Y_4^3$ .

The known property of real space dynamo systems is the presence of reversals, field dipole component sign change. At that, reversals not associated with convection rearrangement are of great interest.

This paper investigates the questions on generation of a field by a 6-cell structure described by the harmonic  $Y_4^2$  taking into account the turbulent  $\alpha$ -effect and on reversals in such a system.

## Basic equations

We consider a spherical shell of viscous incompressible liquid (liquid core) rotating around the  $z$ -axis with constant angular velocity  $\mathbf{\Omega}$ . In the spherical coordinate system  $(r, \theta, \varphi)$ , the inner boundary is  $r = r_i$  and the outer boundary is  $r = r_o$ . Temperatures at the inner and the outer boundaries are constant and are  $T_i$  and  $T_o$ , respectively.

The geodynamo equations, taking into account the  $\alpha$ -effect in Boussinesq approximation are:

$$\begin{aligned} \frac{E}{Pm} \left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \mathbf{v} &= E \nabla^2 \mathbf{v} - \nabla p - 2\mathbf{e}_z \times \mathbf{v} + Ra Pm T \mathbf{r} + \mathbf{rot} \mathbf{B} \times \mathbf{B}, \\ \frac{\partial T}{\partial t} + (\mathbf{v} \cdot \nabla) (T + T_s) &= \frac{Pm}{Pr} \nabla^2 T, \\ \frac{\partial \mathbf{B}}{\partial t} &= \mathbf{rot} (\mathbf{v} \times \mathbf{B}) + R_\alpha \mathbf{rot} (\alpha \mathbf{B}) + \nabla^2 \mathbf{B}, \\ \nabla \cdot \mathbf{v} &= 0, \\ \nabla \cdot \mathbf{B} &= 0. \end{aligned} \tag{1}$$

The basic dimensionless parameters (similarity number) are Ekman number  $E = \nu / \Omega r_o^2$ , Rayleigh number  $Ra = \beta g_0 \delta T r_o / \nu \Omega$ , Prandtl number  $Pr = \nu / \kappa$ , Prandtl magnetic number  $Pm = \nu / \eta$  and  $\alpha$ -effect amplitude  $R_\alpha$ . In these expressions,  $\nu$  denotes kinematic viscosity,  $\beta$  is the coefficient of volumetric expansion,  $g_0$  is free fall acceleration at the inner boundary,  $\delta T = T_i - T_o$  is the temperature difference at the inner and outer

boundaries,  $\kappa$  is the thermal conductivity coefficient, and  $\eta$  is the omnic dissipation coefficient. Such a set of parameters arises if  $r_o$ ,  $r_o^2/\eta$ ,  $\delta T$  and  $\sqrt{\mu_0 \rho \omega \eta}$ , where  $\rho$  is density, and  $\mu$  is magnetic permeability, are the units of length, time, temperature and the magnetic field, respectively. Inner core radius expressed by such a unit of length is  $r_i = 0.35$ .

In equations (1), the variable  $T$  denotes deviation of temperature from the equilibrium hyperbolic  $r$ -profile  $T_s = \frac{1/r - 1}{1/r_i - 1} + T_i - 1$ .

We consider the turbulence in the core to be isotropic and apply the scalar parametrization of  $\alpha$ -effect in the form  $\alpha = a(r) \cos \theta$ , where  $\max |a(r)| = 1$ .

For the velocity  $\mathbf{v}$  we assume adhesion boundary conditions. We also think that the magnetic permeability of the inner and outer cores are similar and the medium outside the core ( $r > r_o$ ) is nonconducting. Thus, for the magnetic field we assume vacuum boundary conditions at the outer boundary, and boundedness conditions in the center of the Earth. At last, temperature deviation  $T$  at the outer core boundaries is considered to be zero.

Further we will use special representations for velocity, temperature and magnetic field based on some spectral decompositions. Applying the decompositions, we obtain equations for our model by Galerkin method.

## Convective part of the model

We proceed from the main requirement that the hydrodynamic part of our model should coincide with the classical Lorentz system for convection in a plane layer. This coincidence should be not only formal but should also correspond to the idea of development of Lorentz system.

We remind that to derive the Lorentz system, we use a one-mode approximation for velocity and a two-mode approximation for temperature in the problem of free convection in a plane layer [6]. As a velocity mode we apply one of the eigenmodes of liquid free oscillations in the layer. The mode vertical component does not have zeros between the layer boundaries, i.e. it provides liquid transfer from the lower boundary to the upper one. Two eigenmodes of free attenuating oscillations of temperature deviations from the stationary profile are used as temperature modes. One of them is spatially compatible with the velocity mode, the other changes only in vertical direction.

We apply similar modes in our model. It is important that during any one-mode approximation of the velocity, the Galerkin procedure removes the Coriolis term from the Navier-Stokes equation. So, it is necessary to apply the modes which spatial structure contains information on layer rotation. One of the possible ways is the application of eigenmodes of free oscillations of a viscous rotating liquid, i.e. solution of a spectral problem

$$\begin{aligned} \frac{E}{\text{Pm}} \mu \mathbf{v} + E \nabla^2 \mathbf{v} - 2 \mathbf{e}_z \times \mathbf{v} - \nabla p &= \mathbf{0}, \\ \nabla \cdot \mathbf{v} &= 0, \\ \mathbf{v}(r = r_i) &= \mathbf{v}(r = r_o) = \mathbf{0}. \end{aligned} \tag{2}$$

For convenience, we use the same dimensionless parameters as for equations (1).

A nonviscous analogue of this problem is known as Poincare problem and is skew-hermitian [7]. Further we will call the solutions of problem (2) the Poincare modes. In the viscous case, the problem operator does not have hermitian or skew-hermitian symmetry which makes significant difficulties. The exact solutions are unknown at present. There are only some estimates of the spectrum, and the system completeness of eigen- and adjoint fields of this problem is proved [8]

We will apply a simple approximation of solution of problem (2) based on the well known modes of nonrotating shell free oscillations. Formally they are determined as solutions of a spectral problem

$$\begin{aligned} \frac{E}{\rho m} \mu \mathbf{v} + \nabla^2 \mathbf{v} - \nabla p = \mathbf{0}, \quad \nabla \cdot \mathbf{v} = 0, \\ \mathbf{v}(r = r_i) = \mathbf{v}(r = r_o) = \mathbf{0}. \end{aligned} \quad (3)$$

This problem is a Hermitian one. Its solutions form a complete orthogonal system and fall into the classes of toroidal and poloidal modes.

The toroidal and poloidal modes have the following form, respectively:

$$\begin{aligned} \mathbf{v}_{k,n,m}^T &= R_{k,n}^T(r) \left( \frac{1}{\sin \theta} \mathbf{e}_\theta \frac{\partial}{\partial \varphi} - \mathbf{e}_\varphi \frac{\partial}{\partial \theta} \right) Y_n^m(\theta, \varphi), \\ \mathbf{v}_{k,n,m}^P &= n(n+1) \frac{R_{k,n}^P(r)}{r} Y_n^m(\theta, \varphi) \mathbf{e}_r + \\ &+ \left( \frac{d}{dr} + \frac{1}{r} \right) R_{k,n}^P(r) \left( \mathbf{e}_\theta \frac{\partial}{\partial \theta} + \frac{1}{\sin \theta} \mathbf{e}_\varphi \frac{\partial}{\partial \varphi} \right) Y_n^m(\theta, \varphi), \\ &k = 0, 1, \dots, \quad n = 1, 2, \dots, \quad m = -n, \dots, n, \end{aligned} \quad (4)$$

where  $R_{k,n}^T(r)$  and  $R_{k,n}^P(r)$  are some combinations of power functions and Bessel spherical functions,  $Y_n^m(\theta, \varphi)$  are spherical harmonics.

The following sets of solenoidal field equal to zero at the boundaries form the decomposition of solenoidal field space into a direct sum of orthogonal subspaces:

$$\begin{aligned} H_0^T &= \{ \mathbf{v}_{k_1,1,0}^T, \mathbf{v}_{k_2,2,0}^P, \mathbf{v}_{k_3,3,0}^T, \mathbf{v}_{k_4,4,0}^P, \dots \}, \\ H_0^P &= \{ \mathbf{v}_{k_1,1,0}^P, \mathbf{v}_{k_2,2,0}^T, \mathbf{v}_{k_3,3,0}^P, \mathbf{v}_{k_4,4,0}^T, \dots \}, \\ H_m^T &= \{ \mathbf{v}_{k_0,m,\pm m}^T, \mathbf{v}_{k_1,m+1,\pm m}^P, \mathbf{v}_{k_2,m+2,\pm m}^T, \mathbf{v}_{k_3,m+3,\pm m}^P, \dots \}, \\ H_m^P &= \{ \mathbf{v}_{k_0,m,\pm m}^P, \mathbf{v}_{k_1,m+1,\pm m}^T, \mathbf{v}_{k_2,m+2,\pm m}^P, \mathbf{v}_{k_3,m+3,\pm m}^T, \dots \}, \\ &k_i = 0, 1, 2, \dots, \quad m = 1, 2, 3, \dots \end{aligned} \quad (5)$$

Each of these subspaces is invariant in regard to problem (2) operator. It is easy to check that if we calculate the operator matrix in basis (4). Thus, each of Poincare modes lie completely in one of the subspaces (5).

To represent the 6-cell convection, the subspace  $H_2^P$  containing a poloidal mode  $\mathbf{v}_{0,4,\pm 2}^P$  is important. It is clear from equations (4) that the radial component of this mode in  $(\theta, \varphi)$ -direction is defined by the spherical harmonic  $Y_4^{\pm 2}$ . In our model we apply only one velocity mode  $\mathbf{v}_0$  which is the simplest approximation of Poincare modes from the subspace  $H_2^P$ . We determine  $\mathbf{v}_0$  as follows:

$$\mathbf{v}_0 = \beta_1 \mathbf{v}_{1,3,-2}^T + \beta_2 \mathbf{v}_{1,3,2}^T + \beta_3 \mathbf{v}_{0,4,-2}^P + \beta_4 \mathbf{v}_{0,4,2}^P + \beta_5 \mathbf{v}_{0,5,-2}^T + \beta_6 \mathbf{v}_{0,5,2}^T, \quad (6)$$

where the coefficients  $\beta_i$  are found from problem (2) by Galerkin method.

It is clear that the radial component  $\mathbf{v}_0$  has the structure of the harmonic  $Y_4^{\pm 2}$  in  $(\theta, \varphi)$ -direction.

Now we consider the eigenmodes of thermal diffusion in the outer core. They are defined as  $T_{k,n,m} = X_{k,n}(r)Y_n^m$ , where  $X_{k,n}(r)$  are the linear combinations of Bessel spherical functions of the first and the second types.

In our model, the main temperature mode  $T_0$  should be consistent with the velocity radial component. Thus we determine it by the expression

$$T_0 = X_{0,4}(r) [\beta_3 Y_4^{-2} + \beta_4 Y_4^2] = \beta_3 T_{0,4,-2} + \beta_4 T_{0,4,2}, \quad (7)$$

where  $\beta_3$  и  $\beta_4$  are taken from formula (6).

The following temperature mode  $T_1$  is chosen in the form  $T_1 = T_{1,0,0} = X_{1,0}Y_0^0$  similar to the mode from the Lorentz system.

Finally, we obtain the following representation for velocity and temperature in our model

$$\mathbf{v}(\mathbf{r}, t) = u(t)\mathbf{v}_0(\mathbf{r}), \quad T(\mathbf{r}, t) = \theta_0(t)T_0(\mathbf{r}) + \theta_1(t)T_1(\mathbf{r}). \quad (8)$$

## Magnetic part of the model

For representation of the magnetic field we apply some modes of omnic dissipation, i.e. solutions of a spectral problem

$$\begin{aligned} \eta \mathbf{B} + \nabla^2 \mathbf{B} &= 0, \quad \nabla \cdot \mathbf{B} = 0, \\ \mathbf{B}(r=0) &\neq \infty, \quad \mathbf{B}^T(r \geq 1) = 0, \quad \mathbf{rot} \mathbf{B}^P(r \geq 1) = 0, \end{aligned} \quad (9)$$

where  $\mathbf{B}^T$  and  $\mathbf{B}^P$  are the toroidal and the poloidal components of the magnetic field, respectively.

The toroidal and the poloidal solutions of the problem are:

$$\begin{aligned} \mathbf{T}_{kn}^m &= a_{kn}^T \mathbf{rot} \left( j_n \left( \sqrt{\eta_{kn}^T} r \right) Y_n^m(\theta, \varphi) \mathbf{r} \right), \\ \mathbf{P}_{kn}^m &= a_{kn}^P \mathbf{rotrot} \left( j_n \left( \sqrt{\eta_{kn}^P} r \right) Y_n^m(\theta, \varphi) \mathbf{r} \right), \end{aligned} \quad (10)$$

where  $j_n(\cdot)$  are Bessel spherical functions of the first type. The eigenvalues  $\eta_{kn}^T$  and  $\eta_{kn}^P$  are determined from the boundary conditions, and  $a_{kn}^T$  and  $a_{kn}^P$  are normalizing factors. Further, we will consider these factors to be chosen so that the mean-square value of modes is equal to a unit. All these modes together form an orthogonal system.

To choose the magnetic modes, we used the following scheme suggested in the paper [9].

First of all, we arrange the modes according to the increase of eigenvalues, i.e. according to the dissipation rate. We obtain the following sequence where the boxes indicate doublets, mode pairs with similar eigenvalues:  $\mathbf{P}_{01}^{-1..1}$ ,  $\boxed{\mathbf{T}_{01}^{-1..1}, \mathbf{P}_{02}^{-2..2}}$ ,  $\boxed{\mathbf{T}_{02}^{-2..2}, \mathbf{P}_{03}^{-3..3}}$ ,

$$\mathbf{P}_{11}^{-1..1}, \boxed{\mathbf{T}_{03}^{-3..3}, \mathbf{P}_{04}^{-4..4}}, \boxed{\mathbf{T}_{11}^{-1..1}, \mathbf{P}_{12}^{-2..2}}, \boxed{\mathbf{T}_{04}^{-4..4}, \mathbf{P}_{05}^{-5..5}}, \dots$$

Then we choose several modes with the least eigenvalues denoting them as  $\mathbf{B}_k(\mathbf{r})$ .

Now we write the magnetic field as  $\mathbf{B}(\mathbf{r}, t) = \sum_k g_k(t) \mathbf{B}_k(\mathbf{r})$  and substitute this expression into the induction equation (the third one from equations (1)). We define the velocity as (8), where we temporally think that  $u(t) = u_0 = \text{const} > 0$ . The velocity mode  $\mathbf{v}_0$  is considered to be normalized so that its mean-square value is equal to a unit. Then  $u_0$  may be interpreted as Reynolds magnetic number  $\text{Re}_m$ . Next we apply the Galerkin method to the induction equation and obtain the system

$$\begin{aligned} \frac{dg_k}{dt} &= \text{Re}_m \sum_i W_{ki} g_i + R_\alpha \sum_i A_{ki} g_i - \eta_i g_i, \\ W_{ki} &= \int \mathbf{rot}(\mathbf{v}_0 \times \mathbf{B}_i) \mathbf{B}_k dV, \\ A_{ki} &= \int \mathbf{rot}(a(r) \cos \theta \mathbf{B}_i) \mathbf{B}_k dV, \end{aligned} \quad (11)$$

where  $\eta_i$  is the eigenvalue for  $\mathbf{B}_i$ .

Practically we obtain a low mode approximation for the problem of kinematic dynamo with six convective cells. We assume that  $\lambda_i$  denotes the eigenvalues of the matrix of system (11). The kinematic dynamo generates a field if  $\max \Re \lambda_i > 0$ . The mode with such an eigenvalue is the leading one growing faster than the others. If the eigenvalue of the leading mode is complex, the magnetic field oscillates.

Further, we gradually increase the number of magnetic modes until we get an oscillating dynamo.

That is the scheme of selection of magnetic modes.

Of course, the eigenvalues depend on the parameters  $\text{Re}_m$  and  $R_\alpha$ , and on the form  $a(r)$  of a radial profile of  $\alpha$ -effect. We made a series of calculations applying two expressions for the profiles  $a(r) = 1$  and  $a(r) = r$ . It turned out that the results differ insignificantly. In the calculation, the parameters  $\text{Re}_m$  and  $R_\alpha$  varied uniformly within the logarithmic scale in the range of  $[10^{-1}; 10^3]$ . Having obtained the oscillating dynamo, we cast out some modes if it did not change qualitatively the result or did not change significantly the oscillation threshold.

In the result, we decided upon the magnetic modes  $\mathbf{P}_{01}^0, \mathbf{P}_{03}^{\pm 2}, \mathbf{P}_{11}^0, \mathbf{T}_{04}^{\pm 2}, \mathbf{P}_{05}^{\pm 2}$ .

The obtained scheme of generation of magnetic modes by a large-scale convection and  $\alpha$ -effect is shown in Fig. 1, and the regions of oscillating and nonoscillating dynamo on the plane of the parameters  $(\text{Re}_m, R_\alpha)$  are illustrated in Fig. 2.

It is clear from the figures that for quite large  $R_\alpha$  the generation is possible only due to the  $\alpha$ -effect, but generation of a dipole component does not occur. In its turn, at quite large  $\text{Re}_m$  field generation without  $\alpha$ -effect is possible, but there will be no any reversals.

## Nonlinear model of MGD-convection

In the kinematic dynamo described above, only unfinite growth of the field is possible. To obtain a stable generation, it is necessary to introduce a suppression mechanism. In connection with that, pay attention to the regions marked by ellipses in Fig. 2. We hold fixed  $R_\alpha \sim 15$ . If the point  $(\text{Re}_m, R_\alpha)$  changes along the ellipse major axis, the value  $\text{Re}_m$  also varies. In this case, dynamo does not function at extreme positions. In such a way, we can obtain generation of a bounded quantity field.

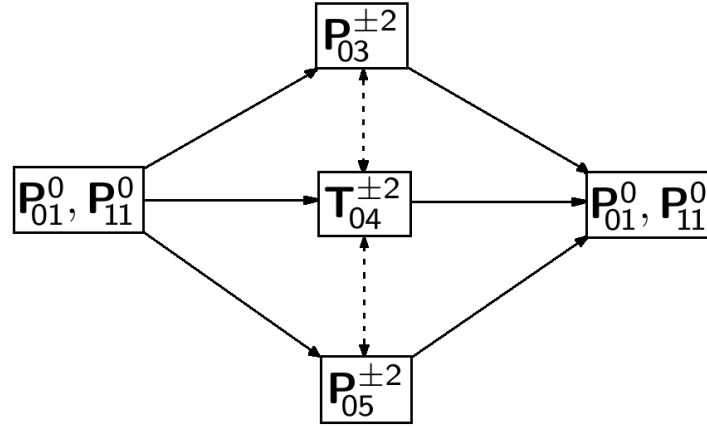


Fig. 1. Dynamo scheme. Solid arrow are large-scale generators; dashed arrows are  $\alpha$ -generators.

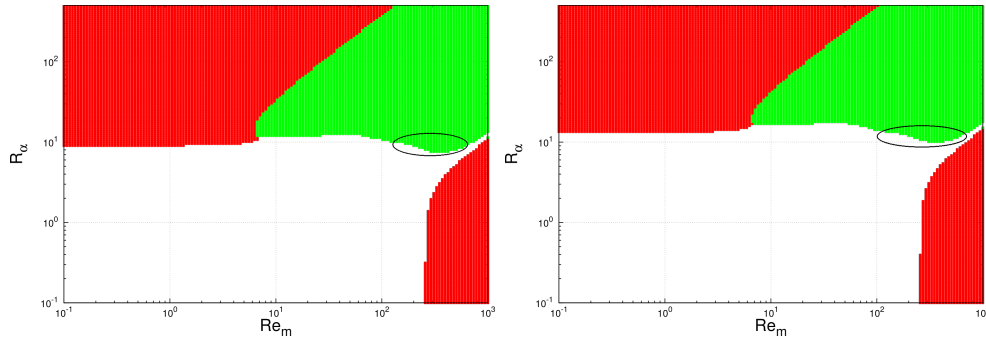


Fig. 2. Regions of field generation. Red points are nonoscillating dynamo; green points are oscillating dynamo.  $a(r) = 1$  (at the top) and  $a(r) = r$  (at the bottom).

We should remind that in our construction,  $Re_m$  is a fixed level of the amplitude  $u(t)$  of velocity mode  $\mathbf{v}_0$ . Now we can make a combined model of MGD-convection with velocity variable amplitude. To obtain it, we substitute the expressions for velocity, temperature and magnetic field into the first three expressions of (1) and apply the Galerkin method.

We obtain a nonlinear dynamic system describing the 6-cell MHD-convection in the Earth's core:

$$\begin{aligned}
 \frac{E}{Pm} \frac{du(t)}{dt} &= -E\mu u(t) + RaPmS\theta_0(t) + \sum_{i,j=1}^8 L_{ij}g_i(t)g_j(t), \\
 \frac{d\theta_0(t)}{dt} &= -Fu(t)\theta_1(t) + Hu(t) - \frac{Pm}{Pr}\zeta_0\theta_0(t), \\
 \frac{d\theta_1(t)}{dt} &= Fu(t)\theta_0(t) - \frac{Pm}{Pr}\zeta_1\theta_1(t), \\
 \frac{dg_k(t)}{dt} &= u(t) \sum_{i=1}^8 W_{ki}g_i(t) + R_\alpha \sum_{i=1}^8 A_{ki}g_i(t) - \eta_i g_i(t), \\
 k &= 1, \dots, 8,
 \end{aligned} \tag{12}$$

where  $\zeta_i$  are eigenvalues of temperature modes  $T_i$ , and  $\mu > 0$ ,  $S$ ,  $L_{ij}$ ,  $F$ ,  $H$  are Galerkin coefficients.

## Results of calculation experiments with the model

Now we describe the results of calculation experiments with model (12). We used the turbulent values for the dissipative coefficients : <http://www.usc.es/peace2/index.htm>  $\nu = 10^2$  m<sup>2</sup>/s,  $\kappa = 10^{-2}$  m<sup>2</sup>/s,  $\eta = 20$  m<sup>2</sup>/s [10]. The core outer radius is  $r_o = 3480$  km and the angular velocity is  $\Omega = 7.29 \times 10^{-5}$  rad/s. The corresponding values of model parameters are as follows:  $E = 10^{-7}$ ,  $Pr = 10^4$ ,  $Pm = 5$ . We also applied  $R_\alpha \sim 15$  according to the estimates from the paper [11] and different values of Rayleigh number from the range of  $Ra = 10^2 \div 10^4$ .

The equation system (12) was numerically solved applying standard procedures of SciLab package.

We found stable regimes of dynamo with reversals in the magnetic field for  $Ra \sim 10^3$ . Fig. 3 illustrates the velocity amplitudes and the dipole part of the magnetic field for  $Ra = 5000$ .

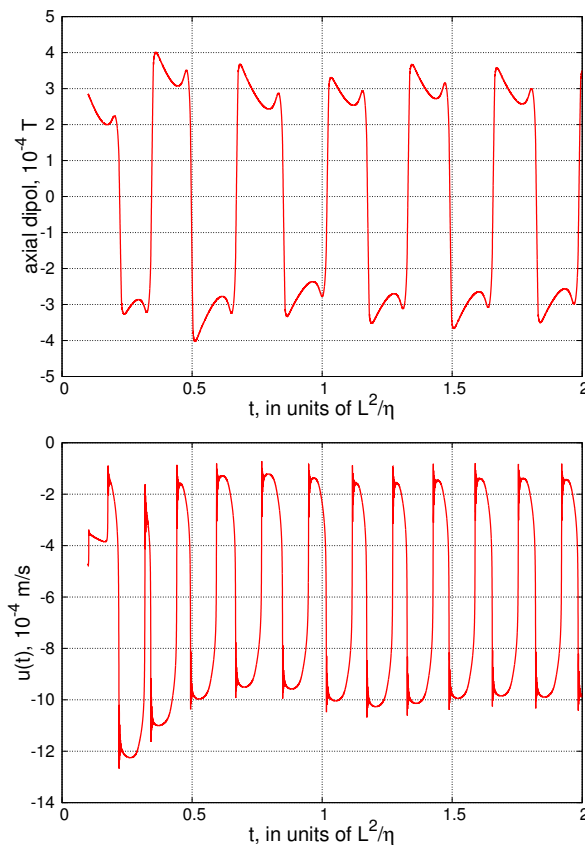


Fig. 3. Amplitudes of velocity modes and vertical dipole for  $Ra = 5000$

It is clear in the figure that the velocity and the magnetic field oscillate. Nevertheless, if the magnetic field changes its sign periodically, the velocity does not change its sign. Thus, reversal regime without convective structure rearrangement is realized in the model.



Of course, we should treat with caution the concrete numerical values obtained in such a simple model. Nevertheless, we should note that the characteristic velocity in Fig. 3 is  $5 \times 10^{-4}$  m/s. It agrees well with the known estimates of real convection velocity of  $\sim 10^{-4}$  m/m [1]. The typical value of the field of  $5 \times 10^{-4}$  Tl in the model also agrees with the extrapolation of the geomagnetic field real value on the core–mantle boundary [12]. The time interval between the reversals in the model is 3200 years that correlates with the average value of polarity for paleomagnetic scales of reversals [12].

## Conclusions

A low-mode dynamo model driven by a 6-cell convection in the Earth's core was developed. It takes into the consideration the turbulent mechanism of field generation. The model is base on indirect data on the large-scale structure of convection.

To describe the spatial structure of flows, a one-mode approximation of the velocity was suggested. This mode is the simplest approximation of one of the eigenmodes of liquid core free oscillations.

Numerical simulation showed that a stable regime of field generation accompanied by field reversals can be realized in the model.

The typical values of the velocity and the field in the model coincide by the value order with the known estimates of real velocity and field.

Field reversals in the model are regular while the real sequence of reversals is chaotic. However, the authors think that it is impossible to obtain chaotic reversals in such a simple model without introduction of noise disturbances of the model during the simulation.

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