MSC 86A10 Research Article

Fractal structure effect on electric field within thunderclouds

T. S. Kumykov

Institute of Applied Mathematics and Automation of Kabardin-Balkar Scientific Centre of RAS, 360000, KBR, Nalchik, Shortanov street, 89 A, Russia
E-mail: macist20@mail.ru

The paper considers the degree of fractal structure impact on the electric field intensity inside thunderstorm clouds using the apparatus of fractional integro-differentiation. We propose a mathematical model of intensity dynamics of a static electric field in the thunderstorm clouds, taking into account media with fractal dimension. The results obtained confirm the close connection of electrophysical processes in thunderclouds with the fractal medium itself.

Key words: thundercloud, mathematical model, fractal medium, intensity, electric field.

DOI: 10.26117/2079-6641-2021-37-4-84-91

Original article submitted: 09.10.2021 Revision submitted: 10.11.2021

For citation. Кумыков Т. С. Fractal structure effect on electric field within thunderclouds. *Vestnik KRAUNC. Fiz.-mat. nauki.* 2021, **37**: 4, 84-91. DOI: 10.26117/2079-6641-2021-37-4-84-91

Introduction

Both in the classical works of the last century [1]–[6], and in the works of modern geophysicists, special attention is paid to the increasing dependence on atmospheric phenomena powered by thunderstorms [7]–[11]. The research for the electrical state and the thunderstorm activity forecasting has been more and more pressing year from year. The available knowledge of physical and, especially, electrical, processes inside the thunderclouds does not yet respond to challenges in line with global trends.

To draw an objective, in physics sense, picture of the thunderstorm growth is impossible largely on the account of the limited number of the experimental data related to electrical parameters of the thunderstorm development at various stages. Therefore, investigations aimed at getting better understanding of thunderstorm electrophysical processes employing various mathematical modelling techniques that provide appropriate description are very much to the point.

Funding. The study was carried out without financial support from foundations.

In recent decades, thunderclouds physical processes studies in view of the fractal structure of the clouds have been expanded significantly. Clouds belong to irregular self-affine fractals, and the processes occurring in such a medium are described by differential equations involving fractional derivatives. This approach allows to implicitly include additional factors for physical system interaction [12]. One of these factors is the fractal structure of the cloud environment. A fractal medium is a medium distributed in space, the mass dimension of which is less than the dimension of the space being filled. The mentioned factor together with the apparatus of fractional integro-differentiation basically transform the considered equations of atmospheric dynamics into differential equations of fractional order.

Developing mathematical models taking into account effect of the fractal structure on various geophysical processes is of great interest nowadays, that contribute to the general picture of cloud physics. The fractal dimension of cumulonimbus with particularly powerful convection currents is equal to 1.36$\pm$ 0.1 [13]. Atmospheric processes in view of fractal structures are investigated in [14]–[17]. Fractal structure is a consequence of many processes and phenomena of irreversible growth such as diffusion, aggregation, destruction, percolation, dynamic chaos, dissolution; and the fractional calculus is an excellent tool for describing related to them processes.

This study is devoted to thunderclouds and processes inside, and significant importance is attached to the electric field. We try to investigate the degree of the fractal structure influence on the static electric field strength using the apparatus of fractional integro-differentiation.

Statement and solution of the problem

As is known, the electric field produced by thunderclouds depends on the charge distribution within them [2]. In convective clouds, usually in most cases, the negative charge predominates in the lower part portion of the cloud, while the positive charge is concentrated in the upper one.

Consider an elementary electrical field generation scheme in a fine-droplet cloud, assuming that the small droplets of water of same size and same charge determined with respect to Frenkel [18] in the following form [19]

$$q(x,t) = 4\pi\varepsilon_0\zeta R(x,t), \tag{1}$$

where ε_0 is the electrical constant, $R(x,t)$ – is the bubble radius, ζ – is the electrokinetic potential. Note that the space charge of a cloud in any of its part is a collection of the charges on cloud particles, precipitation particles and the charge carriers (ions) [2]. The cloud can be divided into parts with the total charge of each part is zero. Under such scheme, the highest field strength should be at the border between space charges of opposite signs. The direction of the electric field in this case is positive.

When water drops fall, a current is produced inside the cloud. If this current is compensated by the conduction current within the electric field resulted from the space charges division, then a stationary electric state of the cloud is achieved.

In this case, the equation that describes the stationary state of a homogeneous cloud has the form

$$qnv = \lambda E, \tag{2}$$

where n – is the number of drops, v – is the droplet falling velocity with respect to the air, λ – is the air conductivity in the cloud, E – is the electric field strength.

By (2) express the electric field strength

$$E = \frac{nv}{\lambda}q, \tag{3}$$

which describes the electrostatic potential without taking into account the fractal structure of the cloud medium.

First, considering fractal structure of the cloud write out the relationship between the electric field intensity and the charge of the drops with time

$$\frac{dE(x,t)}{dt} = \frac{nv}{\lambda}\frac{dq(x,t)}{dt}. \tag{4}$$

Equation (4) is a differential equation of the relationship between the cloud electric field strength and the charge on the cloud droplets. On the right side of (4), the change in the charge can be written as [19]

$$\frac{dq(x,t)}{dt} = 4\pi\varepsilon_0\zeta\frac{d}{dt}R(x,t). \tag{5}$$

The charge growth is observed with the droplet size increases. Suppose that any droplet in contact with a surface of another droplet unite. In this case, the concentration of droplets on the spherical surface is maintained zero. Thus, the change in the charge on the droplet results from the diffusion fluxes that correspond to the concentration gradients near the droplet surface. Hence, the amount of change in charge $\frac{dq(x,t)}{dt}$ acquired by the droplet can be represented by the droplet stream$j(x,t)$, i.e.

$$\frac{dq(x,t)}{dt} = j(x,t). \tag{6}$$

In [20], an equation is obtained allowing to calculate the the flow of the physical quantity$j(x,t)$ (in order to avoid confusions) for a given value of its fractional derivative at point x at time t, in the form

$$j(x,t) = \lambda\partial_{0t}^{\alpha}u(x,\tau), \tag{7}$$

where $\lambda = const > 0$, $u(x,\tau)$ – is the force that causes the flux $j(x,t)$, $\partial_{at}^{\alpha}u(x,t) = sign^n(a-t)\ D_{at}^{\alpha-n}\frac{\partial^n u(x,t)}{\partial t^n}$, – is the regularized derivative of fractional order α of the function $u(x,t)$ with the starting and ending point 0 and t (the Caputo derivative), where D_{at}^{α} – is the Riemann-Liouville fractional integro-differentiation operator determined as follows:

$$D_{at}^{\alpha}u(t) = \begin{cases} \frac{sign(t-a)}{\Gamma(-\alpha)}\int\limits_a^t \frac{u(s)ds}{|t-s|^{\alpha+1}}, & \alpha < 0, \\ u(t), & \alpha = 0, \\ sign^n(t-a)\frac{\partial^n}{\partial t^n}D_{at}^{\alpha-n}u(t), & n-1 < \alpha \le n, n \in N, \end{cases}$$

where (z) – is the Euler's gamma function, α – is the order for integro-differentiation.

Equation (7) allows calculating the flux of the physical quantity for the media with fractal structure. In nature, such media with fractal structure include clouds, which in general, is a fractal. Physical processes in environments of such fractal in time nature, are well described using the apparatus of fractional integro-differentiation.

A fractal environment, as a rule, is a medium with memory and there is a relation between the flow of the physical quantity and the fractal structure of the environment.

In the case under consideration, the flow is caused by increase in the droplet as a consequence of a diffusion flow therefore, (7) can be rewritten as

$$j(x,t) = \lambda \partial_{0t}^{\alpha} R(x,t). \tag{8}$$

Hence (5) in view of (8) takes the form

$$\partial_{0t}^{\alpha} R(x,t) - kR(x,t) = 0, \tag{9}$$

where $k = \frac{4\pi\varepsilon_0\zeta}{\lambda}$. To equation (9) add the initial condition

$$R(x,0) = R_0. \tag{10}$$

Since$f(x) = 0$ the solution to problem (10) for equation (9) has the following general form

$$R(x,t) = \frac{R_0 t^{-\alpha}}{\Gamma(1-\alpha)} * t^{\alpha-1} E_{\alpha,\alpha}(kt^{\alpha}) + \sum_{k=1}^{n} b_k t^{\alpha-k} E_{\alpha,\alpha-k+1}(kt^{\alpha}) \tag{11}$$

where $b_k = \lim\limits_{t\to 0} D_{0t}^{\alpha-k} r(x,t)$, $E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^n}{\Gamma(\alpha k+\beta)}$ is the Mittag-Leffler type function [21]. Since $R(x,t)$ is the solution to (9), $b_k = 0$ and formula (11) can be rewritten as

$$R(x,t) = R_0 E_{\alpha,1}(kt^{\alpha}). \tag{12}$$

Substitute (12) into (1) and obtain

$$q(x,t) = 4\pi\varepsilon_0 q_0 E_{\alpha,1}(kt^{\alpha}), \tag{13}$$

Taking into account (13), integrate (4) and get

$$E(x,t) = K\int_0^t E_{\alpha,1}(kt^{\alpha})\,dt = KtE_{\alpha,2}(kt^{\alpha}) + C. \tag{14}$$

where $K = \frac{4\pi\varepsilon_0 v}{\lambda} q_0$, $q_0 = \zeta R_0$.

At the initial moment of time set the value for the electric field $E(x,0) = E_0$, thus $C = E(x,0) = E_0$ and formula (14) takes the final form

$$E(x,t) = \frac{4\pi\varepsilon_0 v}{\lambda} q_0 t E_{\alpha,2}(kt^{\alpha}) + E_0. \tag{15}$$

Expression (15) is a generalized equation for the strength dynamics in the static electric field, taking into account fractal structure of clouds.

Calculation results of the model

The stationary electric field strength in thunderstorm clouds are calculated using the following values $\varepsilon_0 = 8.85\cdot 10^{-12} F/m$, $p = 50 m^2/s$, $\lambda = 10^{-12} \div 10^{-14} Sm/m$, the drop size $a = 10 \div 20$ mk [3]. The graphs in Figure show the electric field intensity change effected by different values of the fractional parameter of the clouds. It is obvious for small values of the fractional parameter α a linear increase for the charge is observed however with an increase in the fractional parameter the linearity is violated. You can also notice that the origin of the graphs does not coincide with the origin of coordinates, this indicates that after a thunderstorm breakdown, the electric field may still exist, it does not reset. Once a breakdown occurs fractional parameter increases, and after some relaxation time the electric field intensity begin to be restored.

The stationary electric field intensity for the cloud droplet of 10 microns in diameter substantially lower compared for droplets of 20 microns (Fig.).

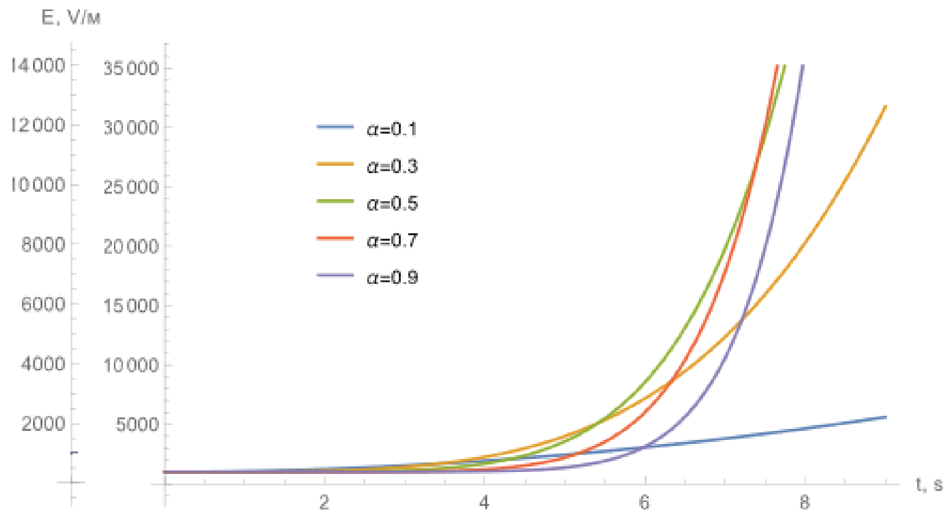

Figure. Graphs for the stationary electric field intensity in thunderclouds at different α with drops of 10-20 microns in diameter.

This demonstrates the important role the cloud droplet parameters and fractional parameter play. It can be noted that the most favorable conditions for a rapid restoration of the stationary electric field intensity is between the fractional parameter $\alpha = 0.3$ and $\alpha = 0.7$. However, what are the fractal properties of cloud structures, how do they depend on the state of the environment, and how the dimension of the fractal is related to the fractional derivative index in equation for the stationary electric field intensity in thunderstorm clouds; all these issues require separate investigation.

Conclusion

This paper demonstrates a universal approach to describing the strength dynamics of the electric field for a thundercloud at its early stage. This approach is widely used in the study of irregular structures featuring time fractal characteristics.

In our case, the versatility of the approach is justified by the fact that many physical processes in clouds can be simulated using the fractional integro-differentiation apparatus, which allows describing the evolution of a certain system with losses, and, as noted in [22], the fractional derivative index indicates a fraction of the system states that persist over the entire evolutionary time t.

For simulation the strength dynamics in the static electric field in accordance with the above technique we employ the fractional differential equation and the fractal properties. Based on the considered model, it can be concluded that the electrophysical processes are closely connected with fractal structure that in this work is presented by thunderstorm clouds and that the proposed semi-empirical model quite satisfactorily describes the main concepts of the process under study. Valuable knowledge on the physical processes in the cloud is contained in spatio-temporal dynamics of the static electric field, obtained straightforwardly with numerical simulation, and can be aligned with experimental data. The present work was not focused on a detailed comparison of the claimed model with experimental data since this requires three-dimensional modeling at least.

Competing interests. The author declares that there are no conflicts of interest regarding authorship and publication.

Contribution and Responsibility. The author contributed to this article. The author is solely responsible for providing the final version of the article in print. The final version of the manuscript was approved by the author.

References

1. Muchnik V. M. *Fizika grozy*. L.: Gidrometeoizdat, 1974. 351 pp. (In Russian)
2. Shishkin N. S. *Oblaka, osadki i grozovoe ehlektrichestvo*. L.: Gidrometeoizdat, 1964. 402 pp. (In Russian)
3. Mason B. J. *Fizika oblakov*. L.: Gidrometeoizdat, 1961. 542 pp. (In Russian)
4. Chalmers J. A. *Atmosfernoe elektrichestvo*. L.: Gidrometeoizdat, 1974. 420 pp. (In Russian)
5. Yuman M. *Molniya*. M.: Publishing house Mir, 1972. 328 pp. (In Russian)
6. Imyanitov I. M. Elektricheskaya struktura konvektivnykh oblakov i ee svyaz' s dvizheniem vozdukha v oblakakh / *Issledovanie oblakov, osadkov i grozovogo elektrichestva*. M., Gidrometeoizdat, 1961, pp. 225–238 (In Russian).
7. Mareev E. A., et al. Sovremennye problemy issledovaniya grozovogo elektrichestva // *Proectirivanie i tehnologia electronnih sredstv*, 2004, pp. 7–16 (In Russian).
8. Trakhtengerts V. Yu., Iudin I. D. Aktual'nye problemy elektrodinamiki grozovogo oblaka // *Izvestiya vuzov. Radiophysics*, 2005. vol. 48, no. 9, pp. 810–821 (In Russian).
9. Adzhiev A. Kh., Kupovykh G. V. *Atmosferno-elektricheskie yavleniya na Severnom Kavkaze*. Taganrog: Taganrog State Radio Engineering University, 2004. 122 pp. (In Russian)
10. Mareev E. A., Anisimov S. V. Global electric circuit as an open dissipative system // *Proc. 12th Int. Conf. on Atmospheric Electricity. Versailles*, 2003, pp. 797–800.
11. Kumykov T. S. Zhekamukhov M. K., Karov B. G. Elektrizatsiya i prostranstvennoe razdelenie zaryadov pri vydelenii puzyr'kov vozdukha v protsesse koagulyatsionnogo rosta gradin v oblake II. Generirovanie grozovogo elektrichestva za schet vydeleniya zaryazhennykh puzyr'kov pri namerzanii pereokhlazhdennykh oblachnykh kapel' na poverkhnosti gradin // *Meteorology and Hydrology*, 2008. vol. 12, pp. 15-24 (In Russian).
12. Shogenov V. Kh., Shkhanukov-Lafishev M. Kh., Beshtoev Kh. M. *Drobnye proizvodnye: interpretatsiya i nekotorye primeneniya v fizike*. Dubna.: Communications of the Joint Institute for Nuclear Research, 1997. 20 pp. (In Russian)
13. Rys F. S., Waldvogel A. Fractal shape of hail clouds // *Phys. Rev. Lett.*, 1986. vol. 56, pp. 784—787.
14. Nakhushev A. M. *Drobnoe ischislenie i ego primenenie*. M.: Fizmatlit, 2003. 272 pp. (In Russian)
15. Iudin D. I., Trakhtengerts V Y., Hayakawa M. Fractal dynamics of electric discharges in a thundercloud // *Phys. Rev. E.*, 2003. vol. 68, pp. 016601.
16. Kumykov T. S. Charge accumulation in thunderstorm clouds: fractal dynamic model // *E3S Web of Conferences*, 2019. vol. 127, pp. 01001.
17. Kumykov T. S. Charging mechanisms for cloud particles in view of fractal medium // *IOP Conf. Ser. Eng.*, 2019. vol. 698, pp. 044029.
18. Frenkel Ya. I. The theory of basic phenomena of atmospheric electricity / *Collection of selected works*, vol. 2. M., Science, 1958, pp. 538-567.

19. Kumykov T. S. Matematicheskoe modelirovanie formirovaniya raznosti potentsialov pri kristallizatsii oblachnykh kapel' s uchetom fraktal'nosti sredy // *Bulletin of the South Ural State University. Series: Mathematical Modeling and Programming*, 2017. vol. 10, no. 3, pp. 16–24 (In Russian).

20. Kumykov T. S. Dinamika zaryada oblachnyh kapel' vo fraktal'noj srede // *Mathematical modeling.*, 2016. vol. 12, pp. 56–62 (In Russian).

21. Pshu A. V. Kraevye zadachi dlya differencial'nyh uravnenij s chastnymi proizvodnymi drobnogo i kontinual'nogo poryadka // *Nalchik: Publishing house KBSC RAS.*, 2005, pp. 185 (In Russian).

22. Nigmatullin R. R. Drobnyy integral i ego fizicheskaya interpretatsiya // *Teor. Mat. Fiz.*, 1992. vol. 90, no. 3, pp. 354–368 (In Russian).

УДК 517.958+550.3+551.5 Научная статья

Влияние фрактальной структуры на электрическое поле в грозовых облаках

Т. С. Кумыков

Институт прикладной математики и автоматизации КБНЦ РАН, 360000, г. Нальчик, ул. Шортанова, 89А

E-mail: macist20@mail.ru

В статье с помощью аппарата дробного интегродифференцирования рассматривается степень влияния фрактальной структуры на напряженность электрического поля внутри грозовых облаков. Предлагается математическая модель динамики напряженности статического электрического поля в грозовых облаках с учетом сред с фрактальной размерностью. Полученные результаты подтверждают тесную связь электрофизических процессов в грозовых облаках с самой фрактальной средой.

Ключевые слова: грозовое облако, математическая модель, фрактальная среда, напряженность, электрическое поле.

DOI: 10.26117/2079-6641-2021-37-4-84-91

Поступила в редакцию: 09.10.2021 В окончательном варианте: 10.11.2021

Для цитирования. Kumykov T. S. Fractal structure effect on electric field within thunderclouds // *Вестник КРАУНЦ. Физ.-мат. науки*. 2021. Т. 37. № 4. С. 84-91. DOI: 10.26117/2079-6641-2021-37-4-84-91

Конкурирующие интересы. Конфликтов интересов в отношении авторства и публикации нет.

Авторский вклад и ответственность. Автор участвовал в написании статьи и полностью несет ответственность за предоставление окончательной версии статьи в печать.

Финансирование. Исследование выполнялось без финансовой поддержки фондов.