Вестник КРАУНЦ. Физ.-мат. науки. 2020. Т. 30. № 1. С. 42-58. ISSN 2079-6641

DOI: 10.26117/2079-6641-2020-30-1-42-58

MSC 32A07, 432A10, 32A07

ON SOME NEW DECOMPOSITION THEOREMS IN MULTIFUNCTIONAL HERZ AND BERGMAN ANALYTIC FUNCTION SPACES IN BOUNDED PSEUDOCONVEX DOMAINS

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Under certain integral condition which vanishes in onefunctional case we provide new sharp decomposition theorems for multifunctional Herz and Bergman spaces in the unit ball and pseudoconvex domains expanding known results from the unit ball. Our theorems extend also in various directions some known theorems on atomic decompositions of onefunctional Bergman spaces in the unit ball and in bounded pseudoconvex domains.

Keywords: Herz spaces, Bergman spaces, analytic functions, pseudoconvex domains, unit ball

Introduction

The problem we consider is well-known for functional spaces in $\mathbb{R}^n$ (the problem of equivalent norms) (see, for example [4]). Let $X, (X_j)$ be a function space in a fixed product domain and(or) in $\mathbb{C}^n$ (normed or quazinormed) we wish to find equivalent expression for $\|f_1 ... f_m\|_X; m \in \mathbb{N}$ (Note these are closely connected with spaces on product domains since often if $f(z_1 ... z_m) = \prod_{j=1}^{m} f_j(z_j)$, then $\|f\|_X = \prod_{j=1}^{m} \|f_j\|_{X_j}$). These results also as we'll see extend some well-known assertions on atomic decomposition of A^p_α type spaces.

To study such group of functions it is natural ,for example, to ask about structure of each $\{f_j\}_{j=1}^{m}$ of this group.

This can be done for example if we turn to the following question find conditions on $\{f_1, ..., f_m\}$; so that $\|f_1, ..., f_m\|_X \asymp \prod_{i=1}^{m} \|f_j\|_{X_j}$ decomposition is valid. In this case

we have if for some positive constant c; $\prod_{i=1}^{m} \|f_j\|_{X_j} \leq c\|f_1...f_m\|_X$; then we have each $f_j, f_j \in X_j, j = 1...m$, where X_j is a new normed (or quazinormed) function space in $\mathbb{D}$ domain and we can easily now provide properties of $\{f_j\}$ based on facts of already known one functional function space theory. (For example to use known theorems for each $f_j \in X_j, j = 1...m$, on atomic decompositions). This idea was used for Bergman spaces in the unit ball and then in bounded pseudoconvex domains with smooth boundary in recent papers [5]. In this paper we extend these results in various directions using modification of known proof.

We provide a complete proof of basic known case then show in details how to modify it to get new results. The old known proof is simple and very flexible as it turns out and we can easily get, as we can see below, various new interesting results from it directly. This remark is leading us to provide only some sketchy arguments sometimes below of proofs when we deal with new theorems, since the core of all proofs is basically the same in all our theorems. Here is the transparent proof of the classical case of the Bergman space (A^p_α) case in the unit ball $\mathbb{C}^n$. The case of Bergman space in more general pseudoconvex domain can be seen in [5].

Main results

We define A^p_α space as usual

$$A^p_\alpha = \left\{ f \in H(B) : ||f||^p_{A^p_\alpha} = \int_B \left| f(z) \right|^p (1-|z|)^\alpha dV(z) < \infty \right\},$$

$dV(z)$ is a Lebesgue measure on B, f_j is analytic in B, $0 < p < \infty, \alpha > -1$, $j = 1, \ldots, m$. Where $H(B)$ is a space of all analytic functions in the unit ball B.

We show now that $\|f_1...f_m\|_{A^p_\tau} \asymp \prod_{i=1}^{m} \|f_j\|_{A^p_{\alpha_j}}$ is valid under certain integral (A) condition (see below) if $p \leq 1$ and if $\tau = \tau(p, \alpha_1, ...\alpha_m, m)$.

Note from our discussion above the only interesting part is to show that

$$\prod_{i=1}^{m} \|f_j\|_{A^p_{\alpha_j}(B)} \leq c_1 \|f_1...f_m\|_{A^p_\tau(B)},$$

since the reverse follows directly from the uniform estimate (see [3])

$$|f(z)|(1-|z|)^{\frac{\alpha_j+n+1}{p}} \leq c\|f\|_{A^p_{\alpha_j}}, 0 \leq p < \infty, \alpha_j > -1, j = 1, ..., m,$$

and ordinary induction. This lead easily to the fact that τ can be calculated

$$\tau = (n+1)(m-1) + \sum_{j=1}^{m} \alpha_j, \alpha_j > -1, 0 \leq p < \infty.$$

Note similar very simple proof based only on various known uniform estimates can be used in all our proofs below for similar inequalities for various spaces. So we mainly concentrate on reverse estimates (see [6]).

Note this argument also allows easily to obtain even more general version with $|f_1|^{p_1}...|f_m|^{p_m}$ instead of $|f_1|^p...|f_m|^p$ (which was discussed above where $0 < p_j < \infty$, $j = 1, ...m$).

Let us now return to the proof of the reverse estimate.

We denote by dV (or $d\delta$) Lebesgue measure on unit ball B and by C, C_α, C_1, C_2 various positive constants below. We also denote by $D(a_k, r)$ or $B(z, r)$ Bergman ball in $\mathbb{B}$ (see [3]).

Assuming that

$$f_1(w_1)...f_m(w_m) = c_b \int_B \frac{(f_1(z)...f_m(z))(1-|z|)^\alpha dV(z)}{\prod_{j=1}^m (1- < z, w_j >)^{\frac{n+1+\alpha}{m}}} \tag{A},$$

$$\alpha > \alpha_0, w_j \in B, j = 1, ..., m.$$

Using Fubini's theorem and extended version of the following estimate (see [5])

$$\int_B |f(z)|(1-|z|^2)^{\frac{n+1+\alpha}{p}-(n+1)} dV(z) \le c\|f\|_{A^p_\alpha}, \tag{$\tilde{A}$}$$

$$\alpha > -1, 0 < p \le 1, f \in H(B_n);$$

$$f(z) = \frac{\tilde{f}(z)}{(1-wz)^v}, v > 0, w \in B, \tilde{f} \in H(B).$$

We get for $\tau = (n+1+\alpha)p - (n+1), \tau > -1,$

$$\prod_{k=1}^m \int_B |f_k(z_k)|^p \times (1-|z_k|)^{\alpha_k} d\delta(z_k) =$$

$$= \int_B ... \int_B \prod_{k=1}^m |f_k(z_k)|^p \prod_{k=1}^m (1-|z_k|^2)^{\alpha_k} d\delta(z_1)...d\delta(z_m) \le$$

$$\le c \int_B ... \int_B \int_B \frac{|f_1(z)|^p ... |f_m(z)|^p \times (1-|z|^2)^\tau}{\prod_{k=1}^m |1- < z, z_k > |^{\frac{n+1+\alpha}{m} p}} d\delta(z) \times$$

$$\times \prod_{k=1}^m (1-|z_k|^2)^{\alpha_k} \times d\delta(z_1)...d\delta(z_m) \le \bar{c} \int_B \prod_{k=1}^m |f_k(z)|^p \times$$

$$\times (1-|z|^2)^\tau d\delta(z) \int_B ... \int_B \prod_{k=1}^m (1-|z_k^2|)^{\alpha_k} \times \frac{1}{\prod_{k=1}^m |1- < z, z_k > |^{\frac{n+1+\alpha}{m} p}} d\delta(z_1)...d\delta(z_m);$$

where $\alpha_k > -1, k = 1, ..., m, \tau = (n+1+\alpha)p - (n+1), \tau > -1; \alpha > \alpha_0; \alpha_0 = \alpha_0(n, m, \alpha_1, ..., \alpha_m)$.

Using the estimate

$$\int_B \frac{(1-|z|^2)^t dV(z)}{|1- < z, w > |^{n+1+t+S}} \le \frac{c}{(1-|z|^2)^S}; \quad t > -1, S > 0, z \in B;$$

we get finally from (A)

$$\prod_{k=1}^m \int_B |f_k(z)|^p (1-|z_k|^2)^{\alpha_k} dV(z_k) \le c \int_B \prod_{k=1}^m |f_k(z)|^p (1-|z|^2)^{\tau_1} dV(z) < \infty,$$

where $0 < p \leq 1, \tau_1 = (m-1)(n+1) + \sum\limits_{k=1}^{m} \alpha_k, \tau_1 > -1$.

A careful analysis of this proof shows that various extensions can be provided by small modification of these arguments. We first formulate our results then provide detailed comments needed for proofs of these results in the unit ball of $\mathbb{C}^n$.

We first define some direct extensions of classical Bergman A^p_α function spaces in the unit ball Herz spaces.

We fix an r-lattice $\{a_k\}$ in the ball (see [3]) till the end of paper.

Let

$$K^{p,q}_\alpha = \left\{ f \in H(B) : \int\limits_B \left(\int\limits_{B(z,r)} |f(\tilde{z})|^p (1-|\tilde{z}|)^\alpha dV(\tilde{z}) \right)^{\frac{q}{p}} dV(z) < \infty \right\};$$

$$M^{p,q}_\alpha = \left\{ f \in H(B) : \sum_{k \geq 0} \left(\int\limits_{D(a_k,r)} |f(w)|^p (1-|w|)^\alpha dV(w) \right)^{\frac{q}{p}} < \infty \right\};$$

$$0 < p,q < \infty, \alpha > -1;$$

(Note $M^{p,p}_\alpha = A^p_\alpha, 0 < p < \infty, \alpha > -1$),

$$K^{p,\infty}_\alpha = \left\{ f \in H(B) : \int\limits_B \left(\sup_{z \in B(w,r)} \right) |f(z)|^p (1-|z|)^\alpha dV(w) < \infty \right\};$$

$$M^{p,\infty}_\alpha = \left\{ f \in H(B) : \sum_{k \geq 0} \left(\sup_{z \in D(a_k,r)} \right) |f(z)|^p (1-|z|)^\alpha < \infty \right\}.$$

$$0 < p,q < \infty, \alpha \geq 0;$$

These are Banach space for $(p,q) \geq 1$ and complete metric spaces for other values.

Theorem 1. *Let X be one of these spaces and $0 < q \leq p \leq 1$; (or $0 < p \leq 1$; $q = \infty$). Then we have for $f_1, ..., f_m \in H(B); \alpha_j > -1$, (or $\alpha_j \geq 0$) ; $j = 1, ..., m$, $\|f_1 ... f_m\|_{X^{p,q}_\tau} \asymp$ $\asymp \prod\limits_{i=1}^{m} \|f_i\|_{X^{p,q}_\alpha}$, for some $\tau, \tau > -1$(or $\tau \geq 0$), if some $\beta, \beta > \beta_0$;*

$$f_1(w_1) ... f_m(w_m) = \int\limits_B \frac{(f_1(z) ... f_m(z)) \times (1-|z|)^\beta}{\prod\limits_{j=1}^{m} (1 - < z, w_j >)^{\frac{\beta+n+1}{m}}} dV(z); w_j \in B, j = 1, ..., m; \qquad (S)$$

where $\tau = \tau(p,q,n,m,\alpha_1, ... \alpha_m)$.

Our theorem extend a known result on atomic decomposition of Bergman multifunctional space A^p_α (see [5]). For $p = q$ in the unit disk, ball we have $M^{p,p}_\alpha = A^p_\alpha, K^{p,p}_\alpha = A^p_\beta, 0 < p < \infty$ for some $\beta = \beta(p,q)$ (see [1]). If in addition $m = 1$ then integral condition(s) vanishes and we can apply know atomic decomposition theorem for A^p_α class in the ball, disk (see [3]).

Remark 1. For each space τ is a special number.

Remark 2. For mixed norm $A^{p,q}_{\alpha}, F^{p,q}_{\alpha}$ spaces we found almost sharp results, where

$$A^{p,q}_{\alpha}=\left\{f\in H(B):\int\limits_0^1\left(\int\limits_S|f(z)|^p d\sigma(\xi)\right)^{\frac{q}{p}}(1-|z|)^{\alpha}d|z|<\infty\right\};$$

$$0<p,q<\infty,\alpha>-,;$$

where $S=|z|=1, d\sigma$ is a Lebesgue measure on S.

$$F^{p,q}_{\alpha}=\left\{f\in H(B):\int\limits_S\left(\int\limits_0^1|f(z)|^p(1-|z|)^{\alpha}d|z|\right)^{\frac{q}{p}}d\sigma(\xi)<\infty\right\};$$

$$F^{p,\infty}_{\beta}=\left\{f\in H(B):\int\limits_S\left(\sup_{0<r<1}\right)|f(r\xi)|^p(1-r)^{\beta}d\sigma(\xi)<\infty\right\};$$

$$A^{p,\infty}_{\alpha}=\left\{f\in H(B):\int\limits_0^1(M_{\infty}(f,r)^p)\,(1-r)^{\alpha}dr<\infty\right\};$$

$$0<q<\infty, 0<p<\infty, \alpha>-1, \beta\geq 0.$$

The proof of Theorem 1.

First we put sketches of proofs then we add details. For starters, we consider Herz spaces with finite indexes, then Herz spaces with infinite indexes. As in A^p_{α} space case we have

$$|f_1(z_1)^p|...|f_m(z_m)|^p\leq c\int\limits_B\frac{\prod\limits_{j=1}^m|f_j(w)|^p\times(1-|w|)^{\delta}dV(w)}{\left|\prod\limits_{k=1}^m<1-z_k,\overline{w}>\right|^{\frac{\alpha+1+n}{m}p}},\qquad(B)$$

$$p\leq 1,\delta=\alpha p+(n+1)(p-1).$$

Then we have to use for example one of these known estimates (see [5])

$$\int\limits_{B(z,r)}\frac{(1-|w|)^{\alpha}}{|1-\overline{w}t|^S}dV(W)\leq\frac{C_1}{|1-\overline{z}t|^{s-\alpha-(1+n)}};S>\alpha+(1+n),z,t\in B,\alpha\geq 0;$$

$$\sum_{k\geq 0}\frac{(1-|a_k|)^{\alpha}}{|1-\overline{a}_k w|^V}\leq C_2\sum_{k\geq 0}\int\limits_{D(a_k,r)}\frac{(1-|z|)^{\tilde{\alpha}}dV(z)}{|1-\tilde{w}z|^V}\leq C_3\frac{1}{(1-|w|)^{V-(\tilde{\alpha}+n+1)}};$$

$$a_k,w\in B;\alpha\geq -1;\tilde{\alpha}=\alpha-(n+1),\tilde{\alpha}>-1,V>(\tilde{\alpha}+n+1);$$

$$\int\limits_S\frac{d\sigma(\xi)}{|1-\overline{\xi}w|^S}\leq\widetilde{C_3}\frac{1}{(1-|w|)^{S-n}};S>n,S>0;w\in B;$$

$$\int\limits_0^1\frac{(1-r)^{\alpha}dr}{|1-rw|^V}\leq C_4\frac{1}{|1-w|^{V-\alpha-1}};V>(\alpha+1);\alpha>-1;w\in B.$$

Second and third are valid pseudoconvex domain (see [1], [2], [5]).

Then one more time $(\tilde{A}),(B)$, then again estimates which are given above in appropriate order for each space. We omit easy details the proof of the one side is complete for Herz $K_{\alpha}^{p,q}, M_{\alpha}^{p,q}$ spaces. To show the reverse we have to use well known uniform estimates for $A_{\alpha}^{p,q}, F_{\alpha}^{p,q}, M_{\alpha}^{p,q}, K_{\alpha}^{p,q}$ function spaces (see, for example [9], and various references these also).

There are, $\tau_0, \tau_1, \tau_2, \tau_3$ so that

$$|f(z)|(1-|z|)^{\tau_0} \leq \overline{C}_1 |f|_{A_{\alpha}^{p,q}};$$

$$|f(z)|(1-|z|)^{\tau_1} \leq \overline{C}_2 |f|_{F_{\alpha}^{p,q}};$$

$$|f(z)|(1-|z|)^{\tau_2} \leq \overline{C}_3 |f|_{M_{\alpha}^{p,q}};$$

$$|f(z)|(1-|z|)^{\tau_3} \leq \overline{C'}_3 |f|_{K_{\alpha}^{p,q}};$$

$\tau_j = \tau_j(p,q,\alpha); j = 0,1,2,3$ (see [6]).

For last two estimates we have to use the elementary estimate only

$$|f(z)|^p \leq C \left(\int\limits_{B(z,r)} |f(w)|^p V_{\alpha}(w) \right) (1-|z|)^{-(\alpha+n+1)},$$

$\alpha > -1, 0 < p < \infty, z \in B$ (see [3], [9], [10]). Values of τ_j can be fixed easily by interested readers.

To finish the proof of Theorem 1 we must finally add some remarks concerning other function spaces. Namely we have to use the following estimates for spaces with infinite index and keep all ingredients of (A_{α}^{p}) spaces proof which we provided above.

For $M_{\alpha}^{p,\infty}, K_{\alpha}^{p,\infty}$ spaces we have to follow again the proof of A_{α}^{p} spaces we provided above and the known estimate

$$\sup_{w \in D(a_{k,r})} \frac{1}{|1-wz|} \leq C' \frac{1}{|1-a_k z|}; \qquad (\widetilde{X})$$

for any $a_k \in B, z \in B$ for some constant C', since $|1-\widetilde{w}z| \asymp |1-\widetilde{w}z|$ for any $z \in B, \widetilde{w} \in B(z,r)$ and $\tilde{w}, z \in B$ (see [3], [9], [10]) and for every Bergman ball $B(z,r); z \in B, r > 0$.

For $F_{\alpha}^{p,\infty}$ spaces we must simply replace $(\widetilde{X})$ by the following known estimates

$$\left(\sup_{0<r<1} \right) \frac{(1-r)^{\tilde{\alpha}}}{|1-rw|^{\alpha}} \leq \left(\sup_{0<r<1} \right) \frac{1}{|1-rw|^{\alpha-\tilde{\alpha}}} \leq \frac{1}{|1-w|^{\alpha-\tilde{\alpha}}},$$

$$\alpha > \tilde{\alpha}, r \in (0,1), w \in B, \tilde{\alpha} \geq 0, \xi \in S.$$

For $A_{\alpha}^{p,\infty}$

$$\frac{1}{|1-wz|^{\alpha}} \leq \frac{1}{(1-|w||z|)^{\alpha}}, \alpha \geq 0, w, z \in B;$$

.

To prove the reverse note that Uniform estimates from below are almost obvious (see [9], [10] for $F_{\alpha}^{p,\infty}$) .

Note now if each (f_i) from one functional (X_i) space can be decomposed into atoms then, since $\|f_1...f_m\|_X \asymp \prod_{i=1}^{m} \|f_i\|_{X_i}$; we can also decompose each (f_i) also as soon as $\|f_1...f_m\|_X < \infty, m > 1$, as soon as integral condition we posed is valid for spaces with infinite or finite indexes.

Now we turn to the case of more general spaces on bounded pseudoconvex domains with smooth boundary on Ω, using Kobayashi balls $B(z,r)$.

First we define spaces, then we formulate our theorems, sketchy proofs will be also provided below. The reader may recover them easily following our remarks, below related with proofs and proof of unit ball case.

We refer for basic definitions of function theory in Ω to [2], [5], [7], [9].

Let further (see, for example, [9] for some of these spaces)

$$(A_\alpha^{p,q})(\Omega) = \left\{ f \in H(\Omega) : \int_0^\rho \left(\int_{\partial D_r} |f(\omega)|^p d\sigma(\omega) \right)^{\frac{q}{p}} \times r^\alpha dr < \infty \right\};$$

$$\alpha > -1; 0 < p, q \leq \infty.$$

We refer to [5] for ∂D_r domain and $d\sigma$ is a Lebesgue measure on ∂D_r where $H(\Omega)$ is a space of all analytic functions on Ω, $\delta(w) = dist(w, \partial\Omega)$ (for these $A_\alpha^{p,q}$ spaces our result is almost sharp).

Let also

$$(M_\alpha^{p,q})(\Omega) = \left\{ f \in H(\Omega) : \int_\Omega \left(\int_{B(z,r)} |f(\omega)|^p \delta^\alpha(\omega) dV(\omega) \right)^{\frac{q}{p}} dV(z) < \infty \right\};$$

$$\alpha > -1, 0 < p, q < \infty;$$

$$(K_\alpha^{p,q})(\Omega) = \left\{ f \in H(\Omega) : \sum_{k \geq 0} \left(\int_{D(a_k,r)} |f(\omega)|^p \delta^\alpha(\omega) dV(\omega) \right) < \infty \right\};$$

$$\alpha > -1, 0 < p, q < \infty;$$

$$(K_\alpha^{p,\infty})(\Omega) = \left\{ f \in H(\Omega) : \int_\Omega \left(\sup_{z \in B(w,r)} |f(z)|^p \right) \delta^\alpha(z) dV(\omega) < \infty \right\};$$

$$0 < p \leq \infty;$$

$M^{p,\infty}$ can be defined similarly as in the ball .

We in case of this general spaces and domains however put one additional condition on Bergman Kernel $K(z,\omega)$ in Ω domain to get a new sharp result.

Theorem 2. *(for pseudoconvex domains)*

Let $f_i \in H(\Omega), i = 1, ..., m.$

Let X be one of $K_\alpha^{p,q}$, *or* $M_\alpha^{p,q}$ *type spaces defined above. Then we have for some* τ

$$\|f_1, ...f_m\|_{X_\tau^{p,q}} \asymp \prod_{i=1}^{m} \|f_i\|_{X_{\alpha_i}^{p,q}},$$

for $0<q\leq p\leq 1$ *or* $p\leq 1, q=\infty, \alpha_i>-1$ *or* $\alpha_i\geq 0$ *for* $i=1,...,m$, *if*

$$(f_1(z_1),...f_m(z_m))=C\int\limits_{\Omega}(f_1(\omega),...,f_m(\omega))\left(\prod_{j=1}^{m}K_{\frac{s+n+1}{m}}(z_j,\omega)\right)\delta^{\tau}(\omega)dV(\omega)$$

$$s>\tau_0, z_i\in\Omega, i=1,...,m$$

under one additional condition on Bergman Kernel of t type $K_t(z,w)$

$$\int\limits_{B(\tilde{z},r)}|K_t(z,\omega)|^p\,\delta^{\tilde{\alpha}}(z)dv(z)\leq\widehat{C}\left|K_{tp+\tilde{\alpha}+n+1}(w,\tilde{z})\right|, \tilde{z},w\in\Omega$$

for every Kobayashi ball $B(\tilde{z},r), \tilde{z}\in B, \tilde{\alpha}>-1, t>0, r>0$ *(with modification for* $p=\infty$*).*

Theorems 1,2 are probably valid for $p,q\geq 1$ we will turn to this problem in other paper.

Remark 3.

Similar results with very similar proofs are valid for analytic spaces on tubular domains over symmetric cones. Such type spaces on unbounded domains were studied recently by many authors. (see, for example [6]- [8], and various references there).

Proofs essentially are the same and we will present them in other separate paper devoted mainly on spaces on such type general unbounded domains in $\mathbb{C}^n$.

For example for $(A^{p,q}_{\tau})$ spaces in tubular domain T_{Ω} $\|f_1...f_m\|_{A^p_S(T_{\Omega})}\asymp\prod\limits_{i=1}^{m}\|f_i\|_{A^p_{\tau_i}(T_{\Omega})}$ is valid for $1<p<\infty;\tau_i>-1; S=S(\tau_1...\tau_m,p,q,m)$; if

$$f_1(w_1)...f_m(w_m)=\int\limits_{T_{\Omega}}\frac{f_1(z)...f_m(z)\triangle^{\tau}I_m(z)}{\prod\limits_{i=1}^{m}\triangle^{\frac{\tau+\frac{2n}{r}}{m}}\left(\frac{z-w_i}{i}\right)}dV(z)$$

for $w_j\in T_{\Omega}, \tau>\tau_0, \tau_0$ is large enough $j=1,...,m$, where $\triangle^{\tau}$ is a determinant function of T_{Ω} (see [6]- [8]), dV is a Lebegues measure on T_{Ω}.

The proof of theorem 2.

The proof of this theorem is a repetition of the proof of the unit ball case we provided above with accurate appropriate replacement of estimates that we indicated and used in case of the unit ball for case of bounded strongly pseudoconvex domain with smooth boundary. We provide these estimates below indicated with references which are needed.

The core of the proof is the following estimate which can be seen in [5] for $p\leq 1$; $s_1, s\geq 0$.

$$Let f\in H(\Omega),\left(\int\limits_{\Omega}|f(w)|^{S_1}\times|\prod_{j=1}^{m}K_{\tau_1}(w,z)|^S\times\delta^V(w)dV(w)\right)^p\leq$$

$$\leq C\int\limits_{\Omega}|f(w)|^{ps_1}\times|\prod_{j>1}^{m}K_{\tau_j}(w,z_j)|^{ps}\times\delta^{p(n+1)V+(n+1)p-(n+1)}dV(w),$$

which actually during the proof will be used twice for $p\leq 1$ and $\frac{q}{p}\leq 1$ with different values of V then in between the addition condition on kernel for $K^{p,q}_{\tau}, M^{p,q}_{\tau}$ spaces must be used with it is obvious modification for $(K^{p,\infty})$ spaces.

Next a known Forelly-Rudin estimate at final step must be used namely

$$\int\limits_{\tilde{D}} |K_t(z,w)|^P \delta^S(w) dV(w) \leq \tilde{c}(\delta^{-tp+S+n+1}(z));$$

$$z \in \tilde{D}, -tp+S+n+1<0, s>-1, t>0, 0<p<\infty;$$

and finally for $A_S^{p,q}$ spaces we have to use the estimate from [5]

$$\int\limits_{\partial D_\varepsilon} |K_t(z,\xi)|^P d\sigma(\xi) \leq c(\delta(z)+t)^{-tp+n}; t>0, p>0, z \in \tilde{D}, \varepsilon>0, n<tp.$$

And for the proof of theorem 2 we leave remaining easy calculations to readers since it is a copy of the unit ball case.

Remark 4.

Note similar results are valid also in analytic spaces in product domains. In the unit ball the most typical example is the following function space on $B \times ... \times B = B^m$

$$A^{p_1,...p_m}_{\tau_1,...\tau_m}(B^m) =$$

$$= \left\{ f \in H(B^m : \left(\int\limits_B ... \left(\int\limits_B |f(z_1,...,z_m)|^{p_1} dV_{\tau_1}(z_1) \right)^{\frac{p_2}{p_1}} dV_{\tau_2}(z_2) dV_{\tau_m}(z_m) \right)^{\frac{1}{p_m}} < \infty \right\};$$

$$\tau_i > -1, p_i \in (0,\infty), i = 1,...,m.$$

For $0 < p_m \leq ... \leq p_2 \leq p_1 \leq 1$ there is a similar assertion as in Theorem 1. The uniform estimate for this space can be seen for example in [6]. We leave this easy task to readers.

Remark 5.

It will be interesting to study similar type problems in such type function spaces but, with fractional derivative involved (Hardy-Sobolev, Bergman-Sobolev) in the unit ball or bounded strongly pseudoconvex domains or convex domains of type in $\mathbb{C}^n$.

Remark 6.

It will be also nice to obtain various versions of results of this note for various new, so-called, weighted function spaces in the unit ball in bounded pseudoconvex domains and in tubular domains over symmetric cones in $\mathbb{C}^n$ (see, for example, for such type new function spaces [8] and various references there also.)

The proof of theorem 1 (continuation).

We provide some easy details for $K_\alpha^{p,q}$ functional spaces. We have first the following obvious estimate:

$$|f(z)|^q (1-|z|)^\tau \leq C \left(\int\limits_D \left(\int\limits_{B(z,r)} |f(w)|^P dV_\alpha(w) \right)^{\frac{q}{p}} dV(z) \right) \qquad (G)$$

for $\tau = \frac{\alpha q}{p} = (n+1)(\frac{1}{p}+\frac{1}{q})q, 0<p,q<\infty, z \in D.$

This estimate is almost obvious (see the proof above). This gives us one part (see below).

For $q \le p \le 1$ we show the reverse now. We have the following simple chain of estimates following the proofs we presented above and estimates there

$$\int\limits_{B(\bar{z},r)} \dots \int\limits_{B(\tilde{z},r)} \prod_{k=1}^{m} |f_k(z_k)|^p \prod_{k=1}^{m} (1-|z_k|^2)^{\alpha_k} d\delta(z_1)\dots d\delta(z_m) \le$$

$$\le \int\limits_B \prod_{k=1}^{m} |f_k(z)|^p (1-|z|^2)^{\tau} \times$$

$$\times \int\limits_{B(\tilde{z},r)} \dots \int\limits_{B(\tilde{z},r)} \frac{\prod\limits_{k=1}^{m} (1-|z_k|^2)^{\alpha_k}}{\prod\limits_{k=1}^{m} |1- < z, z_k > |^{\frac{(n+1+\alpha)p}{m}}} \prod_{k=1}^{m} d\delta(z_k) d\delta(z_k);$$

$$(\tau = (n+1+\alpha)p - (n+1)) \le c \int\limits_B \prod_{k=1}^{m} |f_k(z)|^p \times (1-|z|^2)^{\tau} d\delta(z) \times$$

$$\times \frac{d\delta(z)}{\prod\limits_{k=1}^{m} |1- < z_k, \bar{z} > |^{\frac{(n+1+\alpha)p}{m} - (\alpha_k + n + 1)}}.$$

Hence

$$\prod_{k=1}^{m} \|f_k\|_{K^{p,q}_{\alpha_k}} \le c_1 \int\limits_B \prod_{k=1}^{m} |f_k(z)|^p (1-|z|^2)^{\tau \frac{q}{p} + \frac{q}{p}(n+1) - (n+1)} \times$$

$$\times \int\limits_B \dots \int\limits_B \frac{\prod\limits_{k=1}^{m} d\delta(z_k) dV(z)}{\prod\limits_{k=1}^{m} |1- < z_k, \bar{z} > |^{\frac{(n+1+\alpha)q}{m} - (\alpha_k + n + 1)\frac{q}{p}}} \le c_2 \|f_1 \dots f_k\|_{A^q_S} \le c_3 \|f_1 \dots f_k\|^q_{K^{p,q}_{S_1}};$$

where

$$S = \tau \frac{q}{p} + \frac{q}{p}(n+1) - (n+1) - (n+1+\alpha)q + m(\alpha_k + n + 1)\frac{q}{p} + (n+1)m =$$

$$= (n+1+\alpha)q - (n+1+\alpha)q - (n+1) + m(\alpha_k + n + 1)\frac{q}{p} + (n+1)m =$$

$$= (m-1)(n+1) + \left(\sum_{k=1}^{m} \alpha_k \right) \frac{q}{p} + (n+1)\frac{q}{p} m.$$

Using (G) and induction $\|f_1 \dots f_m\|^q_{M^{p,q}_{S_1}} \le c_4 \prod\limits_{k=1}^{m} \|f_k\|^q_{M^{p,q}_{\alpha_k}}$ we get what we need.

Indeed we have

$$\int\limits_B |G(z)|^q \times (1-|z|)^S dV(z) \le \tilde{c} \int\limits_B \left(\int\limits_{B(z,r)} |G(\tilde{s})|^P dV_{S_1}(\tilde{s}) \right)^{\frac{q}{p}} \times dV(z);$$

$$G \in H(B), s_1 = \frac{p}{q}S - (n+1); S_1 = \sum_{k=1}^{m} \alpha_k + \left(\frac{p}{q}\right)(m-1)(n+1) + (n+1)(m-1).$$

Now we show some details for $A_\alpha^{p,q}$ functional spaces below (for $F^{p,q}$ class it is the same), following the scheme of the proof which was provided above.

First (see [9], [10])

$$|f(z)|^q (1-|z|)^\tau \leq \widetilde{c}\|f\|^q_{A_\alpha^{p,q}}, z \in B \qquad (H)$$

$$\tau = (\frac{\alpha}{q} + \frac{n}{p} + \frac{1}{q})q, 0 < p, q < \infty$$

(and similarly for $F_\alpha^{p,q}$ (see [9], [10]) an analogue of (H) is valid.

For $q \leq p \leq 1$ we show the reverse now we have the following simple chain of estimates following the scheme of proof we presented above.

Let $\tau = (n+1+\alpha)p - (n+1)$ and

$$\int_0^1 \dots \int_0^1 \left(\int_S \dots \int_S \prod_{k=1}^m |f_k(z_k)|^p \prod_{k=1}^m dm_n(\vec{\xi}_k) \prod_{k=1}^m (1-|z_k|^2)^{\alpha_k} d|z_k| \leq (q \leq p, p \leq 1) \leq \right.$$

$$\leq c \int_0^1 \dots \int_0^1 \left(\int_B \frac{\prod_{k=1}^m |f_k(z)|^p \times (1-|z|^2)^\tau dV(z)}{\prod_{k=1}^m (1-|z||z_k|)^{\frac{(n+1+\alpha)}{m}p-n}} \right)^{\frac{q}{p}} \prod_{k=1}^m (1-|z_k|^2)^{\alpha_k} d|z_k| \leq$$

$$\leq \tilde{c} \int_B \int_0^1 \int_0^1 \frac{\prod_{k=1}^m |f_k(z)|^q \times (1-|z|^2)^{\tau\frac{q}{p} + \frac{q}{p}(n+1) - (n+1)} dV(z) \prod_{k=1}^m (1-|z_k|^2)^{\alpha_k} d|z_k|}{\prod_{k=1}^m (1-|z||z_k|)^{\frac{(n+1+\alpha)}{m}p - n + \frac{q}{p}}} \leq$$

$$\leq \tilde{c} \prod_{k=1}^m |f_k(z)|^q \times (1-|z|)^V dV(z) \leq c_3 \prod_{k=1}^m \|f_k\|_{A_v^{p,q}(B)},$$

where

$$v = v(p,q,n,\vec{\alpha},m) = \sum_{k=1}^m \alpha_k + (m-1) - n + n\frac{q}{p}m;$$

and

$$V_1 = \sum_{k=1}^m \alpha_k + n\frac{q}{p}(m-1) + (m-1),$$

(see for last the embedding [9], [10]) $A_v^{p,q} \subset A_v^{q,q}, q \leq p$.

On the other hand using (H) and ordinary induction we have also the reverse estimate, namely

$$\|f_1 \dots f_k\|^q_{A^{p,q}_{v_1}} \leq C \prod_{k=1}^m \|f_k(z)\|^q_{A^{p,q}_{\alpha_k}}; v_1 = \sum_{k=1}^m \alpha_k + \frac{nq}{p}(m-1) + (m-1)$$

So our (not sharp) theorem is proved also in this case of $A_\alpha^{p,q}$ spaces. The case of pseudoconvex domain of $A_\alpha^{p,q}(\Omega)$ spaces is very similar (we follow embeddings from [9], [10]). If $\frac{q}{p} \to 1$ then we have "ε- sharpness" for these spaces.

We finally turn to the case of $M^{p,q}_{\alpha}$ type spaces.

First we have the following estimate.

$$|f(a_k)|^q(1-|a_k|)^{\tau} \leq \tilde{c}\sum_{k\geq 0}|f(a_k)|^q(1-|a_k|)^{\tau} \leq$$

$$\leq c\sum_{k\geq 0}\left(\int\limits_{D(a_k,r)}|f(z)|^p(1-|z|)^{\alpha}dV(z)\right)^{\frac{q}{p}};$$

$\tau > 0, \tau = \frac{q}{p}\alpha + (n+1)\frac{q}{p}, \alpha > -1, 0 < p, q < \infty,$

or

$$|f(z)|^q(1-|z|)^{q\left(\frac{n+1+\alpha}{p}\right)} \leq \hat{C}_1\|f\|^q_{A^q_{\alpha}} \leq$$

$$\leq \hat{C}_2\sum_{k\geq 0}\left(\int\limits_{D(a_k,r)}|f(z)|^p(1-|z|)^{\alpha}dV(z)\right)^{\frac{q}{p}}; q \leq p < \infty.$$

And as a corollary of this estimate by induction we get the following estimate directly

$$\sum_{k\geq 0}\left(\int\limits_{D(a_k,r)}|\tilde{f}(z)|^p(1-|z|)^{\tilde{\alpha}}dV(z)\right)^{\frac{q}{p}} \leq \tilde{c}\prod_{k=1}^{m}\|f_k\|^q_{M^{p,q}_{\alpha_k}}, \tilde{f} = f_1 \ldots f_m;$$

where $\tilde{\alpha} = \sum\limits_{k\geq 0}^{m}\alpha_k + (n+1)(m-1); \alpha_k > -1, k = 1, \ldots, m, 0 < p, q < \infty.$

Now we show the reverse estimate following proofs we provided above for other spaces for $q \leq p \leq 1$. (Note it is easy to see all our arguments by repetition pass easily to the case of bounded strongly pseudoconvex domains with smooth boundary in $\mathbb{C}$ (see [1], [2], [5], [7] for all estimates which are needed)).

We have the following chain of estimates:

$$|f_1(z_1)\ldots f_m(z_m)|^p \leq \tilde{c}\int\limits_B \frac{|f_1(w)\ldots f_m(w)|^p \times (1-|w|)^{\tau}dV(w)}{\left|\prod\limits_{k=1}^{m}(1-<\bar{z}_k, w>)^{\frac{\alpha+n+1}{m}p}\right|};$$

$p \leq 1, \tau = \alpha p + (n+1)(p-1); \alpha > \alpha_0; z_j \in B; j = 1, \ldots, m.$

$$\prod_{k=1}^{m}\|f_k\|^q_{M^{p,q}_{\alpha_k}} \leq c_6\sum_{k_1\geq 0}\ldots\sum_{k_m\geq 0}\int\limits_B |f_1(w)\ldots f_m(w)|^q \times (1-|w|)^{\tau\frac{q}{p}} \times$$

$$\times\left(\prod_{k=1}^{m}\frac{(1-|a_k|^{\alpha_k})}{|1-\bar{a}_k w|^{\frac{\alpha+n+1}{m}p-(n+1)\frac{q}{p}}}\right)dV(w) \leq (q \leq p, \alpha_k > n, k = 1, \ldots, m) \leq$$

$$\leq c_7\int\limits_B |f_1(w)\ldots f_m(w)|^q \times (1-|w|)^{\left(\sum\limits_{k=0}^{m}\alpha_k+(n+1)(m-1)\right)\frac{q}{p}}dV(w) \leq c_8\|f_1\ldots f_m\|^q_{M^{p,q}_{\tilde{\alpha}}}.$$

Since we have using known properties of r-lattices

$$\sum_{k\geq 0}\frac{(1-a_k|)^S}{|1-a_kw|^\tau}\leq\tilde{c}\sum_{k\geq 0}\int\limits_{D(a_k,r)}\frac{(1-|v|)^{\tilde{S}}dV(v)}{|1-vw|^\tau}\leq(s>-1,\tau>S)\leq\frac{c}{(1-|w|)^{\tau-S}},w\in B,$$

and we obtain what we need.

Now our proof is complete.

Since note

$$\|f\|_{A^q_{\tilde{A}}}=\sum_{k\geq 0}\int\limits_{D(a_k,r)}|f|^q(1-|w|)^{A\frac{q}{p}}dV(w)\leq\sum_{k\geq 0}\left(\max_{D(a_k,r)}|f(w)|^q\right)(1-|a_k|)\tau\leq c\|f\|^q_{M^{p,q}_A};$$

where

$$\tau=\left(A\frac{q}{p}+n+1\right);\tilde{A}=A\frac{q}{p};q\leq p;\tau\geq A\frac{q}{p}+(n+1)\frac{q}{p}.$$

Theorem 1 is proved completely.

We obtained new similar type results in other function spaces, proofs will be presented by us elsewhere.

We define Hardy spaces as follows

$$H^p(\Omega)=\left\{f\in H(\Omega):(\sup)_{t>0}\int\limits_{\partial\Omega_t}|f(\xi)|^pd\sigma(\xi)<\infty\right\},$$

$0<p<\infty,\partial\Omega_t=\{z:\rho(z)=t\}$ (see, for example, [2], [9]).

Theorem 3.

Let $f_i\in A^{p_i}_{\alpha_i},i=1,\dots,k;f_i\in H^{p_i};i=k+1,\dots,m,\ p_i\leq 1,i=1,\dots,m,\alpha_j>-1,\ j=1,\dots,k,\tau=n(m-k)+(n+1)(k-1)+\sum\limits_{j=1}^k\alpha_j,$

then

$$\int\limits_\Omega\prod_{j=1}^m\left|f_j\right|^{p_j}\delta(z)^{n(m-k)+\sum\limits_{j=1}^k\alpha_j+(n+1)(k-1)}dv(z)\leq$$

$$\leq C\prod_{j=k+1}^m\left|\left|f_i\right|\right|^{p_i}_{H^{p_i}}\prod_{j=1}^k\left|\left|f_i\right|\right|^{p_i}_{A^{p_i}_{\alpha_i}};\qquad(\widetilde{A})$$

and for cases when $p_i=p,j=1,\dots,m$ *the reverse is also true and we have a new sharp result*

$$I(\vec{f})=\int\limits_\Omega\prod_{j=1}^m\left|f_j(z)\right|^p\delta(z)^{n(m-k)+\sum\limits_{j=1}^k\alpha_j+(n+1)(k-1)}dv(z)\asymp$$

$$\asymp\prod_{j=k+1}^m\left|\left|f_i\right|\right|^p_{H^p}\prod_{j=1}^k\left|\left|f_j\right|\right|^p_{A^p_{\alpha_j}};\qquad(\widetilde{\widetilde{A}})$$

if

$$\prod_{j=1}^m f_i(w_i)=C_\beta\int\limits_\Omega\left(\prod_{j=1}^m f_j(z)\right)\times\prod_{j=1}^m K_{\frac{\beta+n+1}{m}}(z,w_j)\delta^\beta(z)dv(z);$$

$\beta > \beta_0, w_j \in \Omega_j, j = 1,\dots,k,$
$w_j \in \Omega, j = k+1,\dots,m.$ (T)

Theorem 4. *Let $f_i \in A^{\infty}_{\beta_i}, i = 1,\dots,k$ and $f_i \in A^{p_i}_{\alpha_i}, i = k+1,\dots,m.$ Let $\beta_j \geq 0, j = 1,\dots,m$, let also $p_i \leq 1$, let $\alpha_j > -1; j = 1,\dots,m;$ then we have*

$$\int_{\Omega} \prod_{j=1}^{m} \left|f_j\right|^{p_i} \cdot \delta(z)^{\sum_{j=1}^{k} \beta_j p_j + (n+1)(m-k-1) + \sum_{i=1}^{m} \alpha_i} dv(z) \leq$$

$$\leq C \prod_{i=1}^{k} \left|\left|f_i\right|\right|^{p_i}_{A^{p_i}_{\alpha_i}} \times \prod_{i=k+1}^{m} \left|\left|f_i\right|\right|^{p_i}_{A^{\infty}_{\beta_i}}; \qquad (K)$$

and if $p_i = p, i = 1,\dots,m$ we have a sharp result (the reverse of (K)) if

$$\prod_{i=1}^{m} f_i(w_i) = C_{\beta} \int_{\Omega} \prod_{j=1}^{m} f_j(z) \times K_{\frac{\beta+n+1}{m}}(z, w_j) \delta^{\beta}(z) dv(z); \beta > \beta_0; w_j \in \Omega, j = 1,\dots,m.$$

Put

$$BMOA^{p}_{t,v,s} = \left\{ f \in H(B) : \sup_{z \in B} \int_{B} \frac{\left|f(w)\right|^{p} (1-|w|)^{t}}{|1-\bar{z}w|^{v}} dv(w) \times (1-|z|)^{s} < \infty \right\},$$

$v, s \geq 0, p > 0, t > -1.$

For $p \geq 1$ this is a Banach space and complete metric space for $p < 1$. Obviously based on properties of r-lattices (the same result with same proof is valid in pseudoconvex domains) based on vital estimate from below of Bergman kernel on Bergman ball (see [2], [5]),

$$\left|\left|f\right|\right|_{BMOA^{p}_{t,vs}} \geq C \sup_{z \in B} \left|f(z)\right| (1-|z|)^{s+t-v+n+1},$$

$v, s \geq 0, p > 0, t > -1.$

Theorem 5. *Let $f_i \in A^{p}_{\alpha_i}, i = 1,\dots,m$ and $f_j \in BMOA^{p}_{t_j,s_j,v_j}$, $j = m+1,\dots,m+k.$*

Let $0 < p \leq 1, s_j \geq 0$ and also $t_j > -1, v_j \geq 0, j = m+1,\dots,m+k,$
$\alpha_k > -1, k = 1,\dots,m;$ *let $v_j - s_j - t_j < n+1;$*

$$\frac{\beta+n+1}{m+1} p < t_j + n + 1 < \frac{\beta+n+1}{m+1} p + v_j - s_j;$$

$j = m+1,\dots,m+k, \beta > \beta_0, n \in \mathbf{N}, m > 1, m \in \mathbf{N}.$

Then for $\delta(z) = 1-|z|, z \in B,$ we have

$$\int_{B} \prod_{j=1}^{m+k} \left|f_j(z)\right|^{p} \delta(z)^{\tau} dv(z) \asymp \prod_{k=1}^{m} \left|\left|f_k\right|\right|^{p}_{A^{p}_{\alpha_k}} \times \prod_{j=m+1}^{m+k} \left|\left|f_j\right|\right|^{p}_{BMOA^{p}_{t_j,s_j,v_j}}$$

if

$$\prod_{j=1}^{m+k} f_j(z_j) = C_{\beta} \int_{B} \prod_{j=1}^{m+k} f_j(w) \frac{1}{(1-z\bar{w})^{\frac{\beta+n+1}{m+k}}} \delta^{\beta}(w) dv(w)$$

$\beta > \beta_0, z_j \in B, j = 1, \dots, m+k;$
β_0 *is large enough*

$$\tau = (m-1)(n+1) + \sum_{k=1}^{m} \alpha_k + \sum_{j=m+1}^{m+k} (t_j + s_j - v_j) + (n+1)k.$$

Let

$$\left(BMOA^p_{\tau,v,s}\right) = \left\{ f \in H(\Omega) : \sup_{w \in \Omega} \int_{\Omega} \left|f(z)\right|^p \delta(z)^t \cdot \left|K_v(z,w)\right| dv(z) \cdot (\delta^s(w)) < \infty \right\},$$

$0 < p < \infty, v > 0, t > -1, s \geq 0$, is a BMOA-type space in a bounded pseudoconvex domain with smooth boundary in C^n.

We formulate a version of theorem 5 for bounded pseudoconvex domains

Theorem 6. *Let* $p \leq 1$, *let* $v_j - s_j < n+1$,

$$\frac{\beta+n+1}{m} p - t_j > n+1, -s_j + v_j + \frac{\beta+n+1}{m} p - t_j - (n+1) > 0,$$

$$j = m+1, \dots, k, \beta > \beta_0, n \in \mathbf{N}, m > 1, m \in \mathbf{N},$$

and $\alpha_j > -1, j = 1, \dots, m$, *then if* $s_j > 0, t_j > -1, v_j > 0, j = m+1, \dots, m+k$,

then the assertion of previous theorem is valid if we replace $\frac{1}{(1-zw)^\tau}$ *by* $K_\tau(z,w)$ *for* $\tau > 0$ *in pseudoconvex domains for* $\left(BMOA^p_{\tau,v,s}\right)(\Omega)$ *spaces and for Bergman* A^p_α *spaces in same type domains.*

We will present complete proofs of these interesting results in our another paper which is under preparation now.

These results also extend in various directions a known theorem on atomic decomposition of one functional classical Bergman spaces In the unit ball and bounded pseudoconvex domains.

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Для цитирования: Shamoyan R. F., Tomashevskaya E. B. On Hardy type spaces in some domains in $\mathbb{C}^n$ and related problems // *Вестник КРАУНЦ. Физ.-мат. науки*. 2020. Т. 30. № 1. С. 42-58. DOI: 10.26117/2079-6641-2020-30-1-42-58

For citation: Shamoyan R. F., Tomashevskaya E. B. On Hardy type spaces in some domains in $\mathbb{C}^n$ and related problems, *Vestnik KRAUNC. Fiz.-mat. nauki*. 2020, **30**: 1, 42-58. DOI: 10.26117/2079-6641-2020-30-1-42-58

Поступила в редакцию / Original article submitted: 22.02.2020

В окончательном варианте / Revision submitted: 28.03.2020

Vestnik KRAUNC. Fiz.-Mat. Nauki. 2020. vol. 30. no.1. pp. 42–58.

DOI: 10.26117/2079-6641-2020-30-1-42-58

УДК 517.55+517.33

О НЕКОТОРЫХ НОВЫХ ТЕОРЕМАХ ДЕКОМПОЗИЦИИ АНАЛИТИЧЕСКИХ МУЛЬТИФУНКЦИНАЛЬНЫХ ПРОСТРАНСТВ ГЕРЦА И БЕРГМАНА В ЕДИНИЧНОМ ШАРЕ И В ОГРАНИЧЕННЫХ ПСЕВДОВЫПУКЛЫХ ОБЛАСТЯХ В $\mathbb{C}^n$

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Приведены новые теоремы декомпозиции для аналитических многофункциональных пространств Герца и Бергмана в единичном шаре и в ограниченных строго псевдовыпуклых областях в C^n, обобщающие некоторые ранее известные результаты для многофункциональных аналитических пространств Бергмана. Эти теоремы также обобщают в различных направлениях некоторые известные ранее результаты об атомическом разложении классических аналитических однофункциональных пространств Бергмана в единичном шаре и в ограниченных псевдовыпуклых областях в C^n.

Ключевые слова: пространства Герца, пространства BMOA, Бергмана, единичный шар, псевдовыпуклые области, аналитические функции,теоремы декомпозиции.