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ON THE UNIQUENESS OF TRICOMI PROBLEM ANALOGUE FOR MIXED EQUATION WITH TWO DEGENERATED PARALLEL LINES

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The paper proves the uniqness of the solution of Tricomi problem analogue for a mixed equation in the domain containing two parallel lines of parabolic degeneration.

Key words: extremum principle, Tricomi problem analogue

Intruduction

The paper considers a mixed type equation of second order

$$signy(y-1)\cdot u_{xx}+u_{yy}=0, \tag{1}$$

in a mixed domain, containing the intervals of two nonintersecting change type lines. The following equations may serve as the models of such a type of equations:

$$signy(1-y)\cdot u_{xx}+u_{yy}=0, \tag{2}$$

$$y(y-1)u_{xx}+u_{yy}=0, \tag{3}$$

$$y(1-y)u_{xx}+u_{yy}=0. \tag{4}$$

Equations (1) and (2) are the natural analogues of Lavrentjev-Bitsadze equation $signy\cdot u_{xx}+u_{yy}=0$,, and equations (3) (4) are in some specified sense the analogues of Tricomi equation $yu_{xx}+u_{yy}=0$.

The first results for equation (3) were obtained by Nakhushev A.M. [1],[2].

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Tricomi problem analogue for mixed equation with two parallel lines of degeneration

We consider equation (1) in the mixed domain Ω, limited by the characteristics $A_0C_0 : x+y=0$, $0 \leq x \leq r/2$, $B_0C_0 : x-y=r$, $r/2 \leq x \leq r$, $A_1C_1 : y-x=1$, $0 \leq x \leq r/2$ and $B_1C_1 : x+y=r+1$, $r/2 \leq x \leq r$; Jordan curves σ_0 with the ends at the points $A_0=(0,0)$ and $A_1=(0,1)$, and σ_1 with the ends at the points $B_0=(r,0)$ and $B_1=(r,1)$ in the band $0<y<1$ of Euclidean plane of points (x,y) (see the figure).

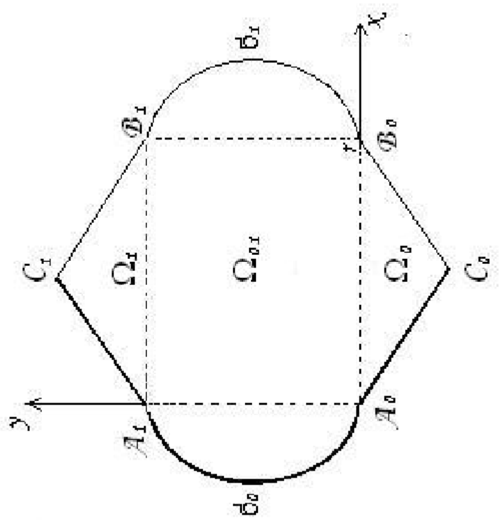

Figure. Domain Ω with characteristics

Equation (1) is a partial derivative mixed equation of second order. It is elliptical in the band $0<y<1$ and hiperbolic within this band. The lines $y=0$ and $y=1$ are parabolic degenerated lines where the coefficient $k(y)=signy(y-1)$ at the higher derivative u_{xx} suffers the discontinuity of the first kind.

By Ω_0 and Ω_1 we denote the parts of the domain Ω, lying in the half-plane $y<0$ and $y>1$, respectively, where equation (1) coincides with the wave equation

$$u_{xx}-u_{yy}=0. \tag{5}$$

By Ω_{01} we denote a part of the domain Ω, lying in the plane $0<y<1$, where equation (1) coincides with Laplace equation

$$u_{xx}+u_{yy}=0. \tag{6}$$

The analogue of Tricomi problem is

Problem 1. *Find in the domains Ω_{01}, Ω_0, Ω_1 a regular solution $u(x,y)$ of equation (1) from the class $C^1(\Omega)\cap C(\overline{\Omega})$, satisfying the following boundary conditions:*

$$u\,|_{\sigma_0}=\varphi_0(x,y), \quad u\,|_{\sigma_1}=\varphi_1(x,y), \tag{7}$$

$$u\,|_{A_1C_1}=\psi_1(x,y), \;\; 0 \leq x \leq r/2, \tag{8}$$

$$u|_{A_0C_0} = \psi_0(x,y), \ \ 0 \le x \le r/2, \tag{9}$$

where $\varphi_0(x,y)$, $\varphi_1(x,y)$, $\psi_0(x,y)$, $\psi_1(x,y)$ *are the specified functions.*

There is an

Analogue of extremum principle of Bitsadze A.V.:

Solution $u(x,y)$ *of problem 1 is equal to zero on the characteristics* A_1C_1 *and* A_0C_0*, and the positive maximum (negative minimum) in the closure of the domain* $\overline{\Omega}_{01}$ *takes on* $\sigma_1 \cup \sigma_0$.

In the domain Ω_1 the function $u(x,y)$ as solution of equation (5) may be presented in the form $u(x,y) = f_1(x-y) - f_1(-1)$, where $f_1(x) \in C[-1, r-1] \cap C^2]-1, r-1[$. Thus, the functions $\tau_1(x) = u(x,1)$ and $\nu_1(x) = u_y(x,1)$ are related by the equation

$$\tau_1'(x) + \nu_1(x) = 0, \ \ 0 < x < r. \tag{10}$$

On the compact $\overline{\Omega}_{01}$ the positive maximum (negative minimum) of the function $u(x,y)$ may be reached only on the border. We assume that the positive maximum (negative minimum) $u(x,y)$ on the compact $\overline{\Omega}_{01}$ is reached at the point $(x_1,1)$, $0 < x < 1$ of the segment A_1B_1. Then $\tau_1'(x_1) = 0$, and it follows from (10) that $\nu_1(x_1) = 0$. Nevertheless, according to Zaremba's principle [3, p.85] for equation (5) $\nu_1(x_1) > 0$ ($\nu_1(x_1) < 0$). Thus, the positive maximum (negative minimum) $u(x,y)$ on the compact $\overline{\Omega}_{01}$ at the points $(x_1,1)$, $0 < x_1 < 1$ is not reached.

In the domain Ω_0 the function $u(x,y)$, as the solution of wave equation (5), may be presented in the form $u(x,y) = f_0(x+y) - f_0(0)$, where $f_0(x) \in C[0,r] \cap C^2]0,r[$. So, the functions $\tau_0(x) = u(x,0)$ and $\nu_0(x) = u_y(x,0)$ are related by the equation

$$\tau_0'(x) - \nu_0(x) = 0, \ \ 0 < x < r. \tag{11}$$

We assume that the positive maximum (negative minimum) of the function $u(x,y)$ on the compact Ω_{01} is reached at the point x_0, $0 < x_0 < r$ on the segment A_0B_0. Then $\tau_0'(x_0) = 0$, and it follows from (11) that $\nu_0(x_0) = 0$. Nevertheless, according to Zaremba's principle for equation (5) $\nu_0(x_0) < 0$.

Thus, the positive maximum (negative minimum) $u(x,y)$ may be reached only on $\sigma_0 \cup \sigma_1$.

The following theorem on the solution uniqueness of problem 1 is true.

Theorem. *Problem 1 has only one solution.*

We assume that $u_1(x,y)$ and $u_2(x,y)$ is the solution of problem 1. Then the difference $u(x,y) = u_1(x,y) - u_2(x,y)$ is the solution of a homogeneous problem. It follows from the proved analogue of the extremum principle that the positive maximum (negative minimum) of the function $u(x,y)$ is reached on σ_0, σ_1, where $u(x,y) = 0$, therefore, $u_1(x,y) = u_2(x,y)$.

Problem 2. *Find in the domains* Ω_{01}, Ω_0, Ω_1 *a regular solution* $u(x,y)$ *of the equation (1) from the class* $C^1(\Omega) \cap C(\overline{\Omega})$ *satisfying the boundary conditions (7), (9) and the condition*

$$u|_{B_1C_1} = \psi_1(x,y), \ \ 0 \le x \le r/2. \tag{12}$$

In a similar way to problem 1, we prove that problem 2 has only one solution.

Existence of the solutions for problems 1 and 2 may be proved by the method of their reduction to Riemann-Hilbert problem for analytical function of a complex variable

$z = x + iy$ in the domain Ω_1, a method suggested by A.V. Bitsadze [4, p.8] when solving Tricomi problem for Lavrentjev-Bitsadze equation.

Existence of the solutions for problems 1 and 2 under the supplementary assumptions of smoothness on the curves σ_0, σ_1 may be also proved by the reduction method to the system of Fredholm integral equations of second kind with respect to the functions $\tau_0(x)$, $\nu_0(x)$, $\tau_1(x)$, $\nu_1(x)$.

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